Theory of the tunneling spectroscopy of ferromagnetic superconductors

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We study tunneling conductance in normal metal-insulator-ferromagnetic superconductor junctions. The tunneling spectra show a clear difference between spin-singlet s-wave pairing, spin-triplet opposite spin pairing, and spin-triplet equal spin pairing: These pairings exhibit, respectively, gap structure, double peak structure, and zero-bias peak in the spectra. The obtained result may serve as a tool for determining the pairing symmetry of ferromagnetic superconductors.

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Magnetism and superconductivity have been under intensive pursuit in the field of low-temperature physics. Recently, the interplay between them has also attracted much attention because nontrivial phenomena are predicted or found experimentally. Such phenomena are expected to occur in ferromagnet-superconductor junctions[1–3] and also in ferromagnetic superconductors (FS). Up to now, several bulk materials, e.g., UGe$_2$,[4] ZrZn$_2$,[5] and URhGe,[6] are identified as FS. How Cooper pairs are formed in FS or under the coexistence of ferromagnetism and superconductivity is an interesting problem. However, the pairing symmetries of FS are still controversial.

Ferromagnetic superconductors seem to be triplet superconductors because singlet pairing and ferromagnetism are contrasting, while triplet pairing have a uniform magnetic moment. However, the possibility of s-wave pairing cannot be excluded.[7–14] For example, it is predicted that UGe$_2$ can have s-wave superconductivity mediated by local ferromagnetic spins.[10,11] The study of the nuclear relaxation rate cannot rule out the possibility of s-wave pairing in UGe$_2$.[12,13] A weak ferromagnetic Fermi liquid theory also suggests the possibility of s-wave superconductivity.[14] Therefore, detailed comparison between theoretical predictions and experimental data is required to settle this problem. Then, the properties of thermodynamic quantities should be noted: For example, equilibrium thermodynamic quantities for Balian-Werthamer state of p-wave pairing, which is realized in B phase of ^{3}He, are expected to show s-wave property because its gap is constant.[15] In this way, equilibrium thermodynamic quantities for p-wave pairing could not be clearly distinguished from those of s-wave pairing. Therefore, nonequilibrium quantities are more desirable to compare with experimental data. Although some predictions are made on the properties of junctions with equal spin pairing (ESP) FS,[16–19] the study of tunneling spectra for possible candidate pairings of FS is insufficient.

Tunneling spectroscopy provides important information on the superconducting gap and its pairing symmetry. In normal metal-superconductor junctions, Andreev reflection[20] (AR) is a key concept for low-energy transport. Blonder, Tinkham, and Klapwijk (BTK) formulated the tunneling conductance where the AR is taken into account.[21] This enables us to study the energy gap of superconductors. The generalization of the BTK formula for normal metal–unconventional

superconductor junctions is also useful to study the properties of unconventional superconductors[22–24] because the tunneling conductance is sensitive to the pairing symmetry due to the formation of midgap Andreev resonant states.[22,23]

In the present paper, we study the tunneling conductance in normal metal-insulator-ferromagnetic superconductor (N/FS) junctions. The tunneling spectra show a clear difference between spin-singlet s-wave pairing, spin-triplet opposite spin pairing (OSP), and spin-triplet ESP. This result may be useful in determining the pairing symmetry of ferromagnetic superconductors.

Let us start with an effective Hamiltonian for the Bogoliubov–de Gennes (BdG) equation. The Hamiltonian reads

$$\check{H} = \begin{pmatrix} \hat{H}(\mathbf{k}) & \hat{\Delta}(\mathbf{k}) \\ -\hat{\Delta}^*(-\mathbf{k}) & -\hat{H}^*(-\mathbf{k}) \end{pmatrix}, \tag{1}$$

with $\hat{H}(\mathbf{k}) = \xi_k + \mathbf{h} \cdot \sigma$ and $\hat{\Delta}(\mathbf{k}) = i\Delta \sigma_y$ for singlet pairing or $\hat{\Delta}(\mathbf{k}) = [\mathbf{d}(\mathbf{k}) \cdot \sigma] i\sigma_y$ for triplet pairing. Here, ξ_k, $\mathbf{k}$, $\mathbf{h}$, and σ denote electron band energy measured from the Fermi energy, electron momentum, applied magnetic field, and Pauli matrices, respectively. In this paper, we consider three types of pairings: spin-singlet s-wave pairing, spin-triplet OSP, and spin-triplet ESP. OSP and ESP are characterized by the relations $\mathbf{h} \times \mathbf{d}(\mathbf{k}) = 0$ and $\mathbf{h} \cdot \mathbf{d}(\mathbf{k}) = 0$, respectively.[25]

We consider a two-dimensional ballistic N/FS junction at zero temperature. The N/FS interface located at $x=0$ (along the y axis) has an infinitely narrow insulating barrier described by the delta function $U(x) = U\delta(x)$. We first consider OSP. The BdG equation reads

$$\check{H} \begin{pmatrix} \hat{u}_\pm \\ \hat{v}_\pm \end{pmatrix} = E_\pm \begin{pmatrix} \hat{u}_\pm \\ \hat{v}_\pm \end{pmatrix} \tag{2}$$

for electronlike quasiparticles and

$$\check{H} \begin{pmatrix} \sigma_y \hat{v}_\pm \sigma_y \\ \sigma_y \hat{u}_\pm \sigma_y \end{pmatrix} = -E_\pm \begin{pmatrix} \sigma_y \hat{v}_\pm \sigma_y \\ \sigma_y \hat{u}_\pm \sigma_y \end{pmatrix} \tag{3}$$

for holelike quasiparticles, with

$$E_\pm = \sqrt{(\xi_k)^2 + |\Delta|^2} \pm |\mathbf{h}|, \tag{4}$$

$$\hat{u}_\pm = u_0^\pm (1 \pm \hat{\mathbf{h}} \cdot \sigma)/2, \tag{5}$$

$$\hat{v}_\pm = v_0^\pm \frac{\hat{\Delta}^\dagger}{|\Delta|}(1 \pm \hat{\mathbf{h}} \cdot \sigma)/2, \tag{6}$$

$$u_0^\pm = \sqrt{\frac{1}{2}\left(1 + \frac{\sqrt{E_\pm^2 \mp h - |\Delta^2|}}{E_\pm \mp h}\right)}, \tag{7}$$

$$v_0^\pm = \sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{E_\pm^2 \mp h - |\Delta^2|}}{E_\pm \mp h}\right)}, \tag{8}$$

$\hat{\mathbf{h}} = \mathbf{h}/|\mathbf{h}|$ and $|\Delta|^2 = \frac{1}{2}\mathrm{Tr}\,\hat{\Delta}\hat{\Delta}^\dagger$. We assume $\Delta > h$ because otherwise the gap vanishes for the "$-$" state, as can be seen in Eq. (4). The solution of the BdG equation for s-wave pairing has the same form as that of OSP and is obtained by choosing $\hat{\Delta}(\mathbf{k}) = i\Delta\sigma_y$. Below, we consider unitary state for triplet superconductors and choose, as a model calculation, $\mathbf{h} = -h\hat{\mathbf{z}}$, $\mathbf{d}(\mathbf{k}) = \Delta(k_x + ik_y)/k\hat{\mathbf{z}}$ for OSP, and $\mathbf{d}(\mathbf{k}) = \Delta(k_x + ik_y)/k\hat{\mathbf{x}}$ for ESP. Here, $\hat{\mathbf{x}}$ and $\hat{\mathbf{z}}$ are unit vectors oriented to the x and z axes, respectively. For ESP, eigenfunctions for the Hamiltonian are given by

$$\begin{pmatrix} u_0^- \\ 0 \\ -v_0^- e^{-i\theta} \\ 0 \end{pmatrix},\ \begin{pmatrix} v_0^- \\ 0 \\ -u_0^- e^{-i\theta} \\ 0 \end{pmatrix},\ \begin{pmatrix} 0 \\ u_0^+ \\ 0 \\ v_0^+ e^{-i\theta} \end{pmatrix},\ \begin{pmatrix} 0 \\ v_0^+ \\ 0 \\ u_0^+ e^{-i\theta} \end{pmatrix}, \tag{9}$$

$$u_0^\pm = \sqrt{\frac{1}{2}\left(1 + \frac{\sqrt{E_\pm^2 - |\Delta^2|}}{E_\pm}\right)}, \tag{10}$$

$$v_0^\pm = \sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{E_\pm^2 - |\Delta^2|}}{E_\pm}\right)}, \tag{11}$$

$$E_\pm = \sqrt{(\xi_k \pm |\mathbf{h}|)^2 + |\Delta|^2}, \tag{12}$$

where θ is an angle with respect to the interface normal. Note that the magnitude of h can be larger than that of Δ for ESP because Cooper pairs are insensitive to the exchange field.

We will calculate the tunneling conductance, following the BTK method.[21,22] Wave function $\psi(x)$ for $x \leq 0$ (N region) is represented as

$$\psi(x \leq 0) = \left[\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{ik_F \cos\theta x} + a \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{ik_F \cos\theta x} \right.$$
$$\left. + b \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-ik_F \cos\theta x} \right] e^{ik_F \sin\theta x} \tag{13}$$

for an injection wave in up-spin state, for s-wave pairing, and OSP. a is AR coefficient and b is normal reflection (NR) coefficient. For an injection wave in down-spin state and the junction with ESP, wave functions are given in a similar form.

Similarly, for $x \geq 0$ (FS region), $\psi(x)$ is given by the linear combination of the eigenfunctions. Note that since the translational symmetry holds for the y direction, the momenta parallel to the interface are conserved.

The wave function follows the boundary conditions,

$$\psi(+0) = \psi(-0), \tag{14}$$

$$\frac{\partial}{\partial x}\psi(+0) - \frac{\partial}{\partial x}\psi(-0) = \frac{2mU}{\hbar^2}\psi(+0). \tag{15}$$

Applying BTK theory with AR and NR coefficients for electron injections with up and down spin states, we can calculate the angle-resolved dimensionless conductance for OSP represented in the form

$$\sigma_{S\sigma} = \frac{4(4 + Z_\theta^2) + 16|\Gamma_\sigma^p|^4 - 4Z_\theta^2|\Gamma_\sigma^p\Gamma_\sigma^m|^2}{|4 + Z_\theta^2 - Z_\theta^2\Gamma_\sigma^p\Gamma_\sigma^m|^2}, \tag{16}$$

$$\Gamma_\sigma^p = \Gamma_\sigma e^{-i\theta}, \quad \Gamma_\sigma^m = -\Gamma_\sigma e^{-i\theta}, \tag{17}$$

$$\Gamma_\sigma = \frac{\Delta}{E + \sigma h + \sqrt{(E + \sigma h)^2 - |\Delta|^2}}, \tag{18}$$

$\sigma = \pm$, $Z_\theta = \frac{Z}{\cos\theta}$, and $Z = \frac{2mU}{\hbar^2 k_F}$ with quasiparticle energy $E \equiv E_+ = E_-$, effective mass m, Fermi wave number k_F, and Fermi energy E_F. For s-wave pairing, the conductance is given by just replacing Γ_σ^p and Γ_σ^p with Γ_σ in Eq. (16). We define $\sigma_{N\sigma}$ as the conductance in the normal state which is given by

$$\sigma_{N\sigma} = \frac{4}{4 + Z_\theta^2}. \tag{19}$$

The normalized conductance is represented as

$$\sigma_T = \frac{\displaystyle\int_{-\pi/2}^{\pi/2} d\theta \cos\theta(\sigma_{S+} + \sigma_{S-})}{\displaystyle\int_{-\pi/2}^{\pi/2} d\theta \cos\theta(\sigma_{N+} + \sigma_{N-})}. \tag{20}$$
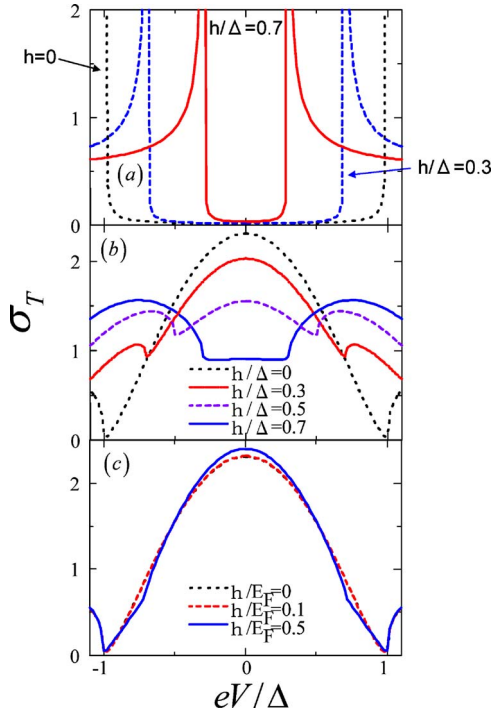
For ESP, the conductances are given by

FIG. 1. (Color online) Normalized tunneling conductance with $Z=10$ for (a) s-wave pairing, (b) OSP, and (c) ESP.

$$\sigma_{S\sigma} = 4\lambda_\sigma$$

$$\times \frac{[Z_\theta^2 + (\lambda_\sigma + 1)^2] + 4\lambda_\sigma |\Gamma^p|^2 - [Z_\theta^2 + (\lambda_\sigma - 1)^2]|\Gamma^p \Gamma^m|^2}{|(\lambda_\sigma + 1)^2 + Z_\theta^2 - [Z_\theta^2 + (\lambda_\sigma - 1)^2]\Gamma^p \Gamma^m|^2},$$

$$(21)$$

$$\Gamma^p = \Gamma e^{-i\theta}, \quad \Gamma^m = -\Gamma e^{-i\theta}, \tag{22}$$

$$\Gamma = \frac{\Delta}{E + \sqrt{E^2 - |\Delta|^2}}, \tag{23}$$

$$\sigma_{N\sigma} = \frac{4\lambda_\sigma}{(1 + \lambda_\sigma)^2 + Z_\theta^2}, \quad \lambda_\sigma = \sqrt{1 - \frac{\sigma h}{E_F \cos^2 \theta}}. \tag{24}$$

Note that $\Theta(\theta_C - |\theta|)$ have to be multiplied for $\sigma = +$ (minority spin) in Eq. (20) with $\theta_C = \cos^{-1}\sqrt{\frac{U}{E_F}}$ because of the mismatch of Fermi surfaces of majority and minority spins.[26] Here, $\Theta(x)$ is the Heaviside step function.

In the above, we choose the same effective mass in N and FS. In most cases, the effective mass in N is much smaller than that in FS. However, it is expected that this effect does not change the results qualitatively for large Z ($Z>1$).[27] Therefore, we choose the same effective mass. The inclusion of the difference of effective masses is straightforward.[28,27] Although it is known that other characteristics, e.g., the shape of Fermi surfaces should be taken into account in some phase-sensitive tests,[29] we use a cylindrical Fermi surface in this paper for simplicity because the Fermi surface of FS has a very complicated structure.[30–32]
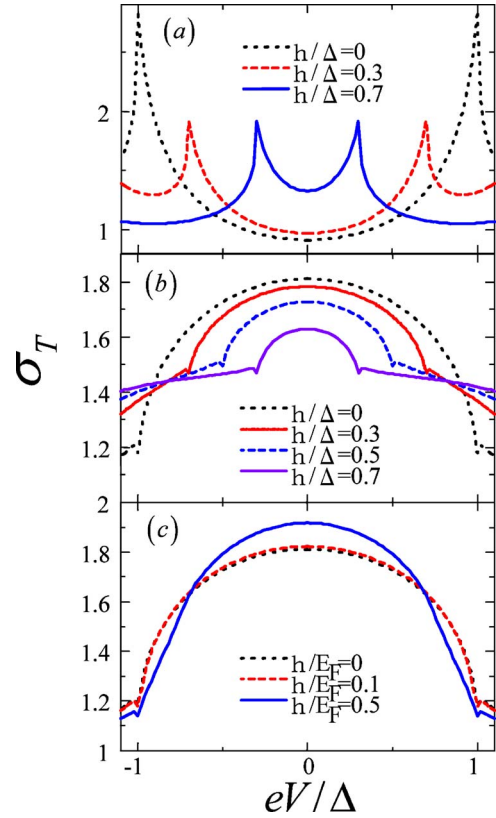

FIG. 2. (Color online) Normalized tunneling conductance with $Z=1$ for (a) s-wave pairing, (b) OSP, and (c) ESP.

We study the normalized tunneling conductance σ_T as a function of bias voltage V. The conductances with $Z=10$ are shown in Figs. 1(a)–1(c) for s-wave pairing, OSP, and ESP, respectively. For s-wave pairing, a gaplike structure appears at $h=0$.[21] With the increase of h, the magnitude of the gap is reduced from 2Δ to $2\Delta - 2h$ [Fig. 1(a)]. For OSP, a zero-bias peak appears at $h=0$, as shown in Fig. 1(b), which stems from the formation of midgap Andreev resonant states.[22] We find a splitting of peak for OSP as h increases. These shifted structures are attributed to the h dependence of wave function in Eqs. (7) and (8), and hence expected to emerge for all OSP [not restricted to the present choice of $\mathbf{d}(\mathbf{k})$]. On the other hand, the tunneling conductance has a zero-bias peak and is almost independent of the exchange field for ESP, as shown in Fig. 1(c). This is because there is no energy shift in the eigenfunctions, as shown in Eqs. (10) and (11). We also find that the absence of the shifted structure is expected for all ESP by calculating the eigenfunctions of the Hamiltonian with ESP. Therefore, a clear difference between three types of pairings can be seen. Especially when the magnitude of the gap Δ is comparable to h, the tunneling spectra are characterized by gap structure, double peak structure, and zero-bias peak for s-wave pairing, OSP, and ESP, respectively.

A corresponding plot for $Z=1$ is shown in Fig. 2. As shown in Fig. 2(a), the reduced dip structure appears for s-wave pairing, the width of which is given by $2\Delta - 2h$. When $\Delta \sim h$, the dip transforms into a single peak. As for OSP, a zero-bias peak is formed and its width is reduced by

the increase of h [see Fig. 2(b)]. A zero-bias peak remains with the increase of h for ESP, as shown in Fig. 2(c). Thus, there is no qualitative difference between OSP and ESP. This is because the effect of midgap Andreev resonant states becomes weak for small Z and hence the zero-bias anomaly is smeared for small Z. Therefore, we find that the difference between s-wave pairing, OSP, and ESP becomes clear for large Z.

In summary, we have studied the tunneling conductance in normal metal-insulator-ferromagnetic superconductor junctions. We have found a clear difference in tunneling spectra between spin-singlet s-wave pairing, spin-triplet OSP, and spin-triplet ESP. The difference is clear for large barrier parameter Z. This result may serve as a tool for de-termining the pairing symmetry of ferromagnetic superconductors.

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