Quasiclassical Green's function theory of the Josephson effect in chiral *p*-wave superconductor/diffusive normal metal/chiral *p*-wave superconductor junctions

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We study the Josephson effect in chiral *p*-wave superconductor/diffusive normal metal (DN)/chiral *p*-wave superconductor (CP/DN/CP) junctions using quasiclassical Green's function formalism with proper boundary conditions. The p_x+ip_y-wave symmetry of superconducting order parameter is chosen which is believed to be the pairing state in Sr_2RuO_4. We show that the superconducting pair amplitude induced in DN has an odd-frequency spin-triplet *s*-wave symmetry and is an odd function of Matsubara frequency. Despite the peculiar symmetry properties of the Cooper pairs, the behavior of the Josephson current is rather conventional. We have found that the current-phase relation is almost sinusoidal and the Josephson current is proportional to $\exp(-L/\xi)$, where ξ is the coherence length in DN and L is the thickness of the DN layer. We also calculate the Josephson current in CP/diffusive ferromagnet metal/CP junctions and show that the 0-π transition can be realized by varying temperature or thickness L similar to the case of conventional *s*-wave junctions. The obtained results may help to explore the properties of superconducting state in Sr_2RuO_4.

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I. INTRODUCTION

An exploration of unconventional superconducting pairing is one of the central issues in the physics of superconductivity. Possible realization of spin-triplet superconductivity in Sr_2RuO_4 is currently widely discussed.[1] A number of experimental results is consistent with the spin-triplet chiral *p*-wave symmetry state in this material.[2–4] On the other hand, it is well-known that the midgap Andreev resonant state (MARS)[5–8] is induced near interfaces in unconventional superconducting junctions where pair potential changes its sign across the Fermi surface. The MARS manifests itself in quasiparticle tunneling experiments as zero bias conductance peak (ZBCP). Tunneling data in Sr_2RuO_4 junctions show ZBCP[9] in accordance with theoretical predictions.[10] The Josephson effect in Sr_2RuO_4 was also studied both theoretically[11] and experimentally.[12] A recent superconducting quantum interference device (SQUID) experiment by Nelson[13] is consistent with the realization of the chiral *p*-wave superconducting state.[14]

At the same time, there are important recent achievements in theoretical study of the proximity effect in junctions between unconventional superconductors. It was predicted that in the diffusive normal metal (DN)/triplet superconductor (TS) junctions the MARS formed at the DN/TS interface can penetrate into the DN.[15] This proximity effect is very unusual since it generates zero energy peak (ZEP) in the local density of states (LDOS) in contrast to the conventional proximity effect where the resulting LDOS has a minigap.[16] It was also shown theoretically that the ZEP appears in the chiral *p*-wave state. Thus to explore the ZEP in the DN region of DN/Sr_2RuO_4 heterostructures is an intriguing topic.[15] Very recently, it was predicted that the Cooper pairs induced in the DN belong to the unconventional odd-frequency symmetry state, in contrast to the usual even-frequency pairing.[17] Since this proximity effect specific to TS is a completely new phenomenon, the Josephson effect in TS/DN/TS junctions deserves a separate study.

Recently, it was shown that the Josephson current in TS/DN/TS junctions[18] is strongly enhanced at low temperatures and is proportional to $\sin(\Psi/2)$, where Ψ is a superconducting phase difference between left and right superconductors.[19] These results are quite different from those for *d*-wave superconductor/DN/*d*-wave superconductor junctions.[20] However, in most of the previous theories of TS/DN/TS junctions, only the *p*-wave state in the presence of the time reversal symmetry was considered. The existing knowledge of the Josephson effect in TS/DN/TS junctions for chiral *p*-wave symmetry is very limited.[21] It is important to study the Josephson effect in the chiral *p*-wave junctions in much more detail because this symmetry is most probably realized in the superconducting state in Sr_2RuO_4.

The quasiclassical Green's function theory is the useful method to study the above problem. In the diffusive regime, the quasiclassical Green's function obeys the Usadel equations.[22] The circuit theory[23] enables one to treat the case of arbitrary interface transparency in *s*-wave superconductor (S) junctions. This theory was recently generalized for unconventional superconductor (US) junctions.[24–27] In this approach, the effect of MARS is naturally included. The theory was extended to the cases of US/DN/US and US/diffusive ferromagnet (DF)/US junctions where the time reversal symmetry is present in US.[28,29] However, these theories cannot be applied for calculation of the Josephson current in chiral *p*-wave superconductor/DN/chiral *p*-wave superconductor (CP/DN/CP) junctions with p_x+ip_y-wave symmetry of the pair wave function in chiral *p*-wave superconductor. The purpose of this paper is to generalize the above approach and to apply it to the interface between the DN (DF)/superconductor with broken time reversal symmetry.

In the present paper, we derive the boundary conditions for the quasiclassical Green's functions at DN (DF)/CP inter-

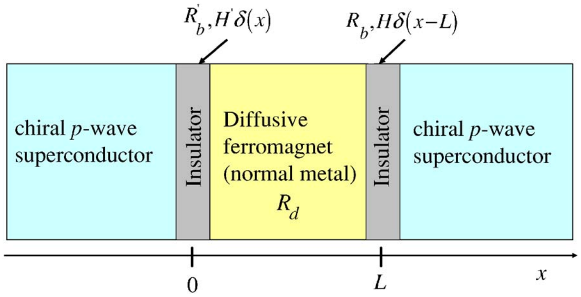

FIG. 1. (Color online) Schematic illustration of the model.

faces in the CP/DN (DF)/CP Josephson junctions. Using these boundary conditions, we calculate the Josephson current in the CP/DN (DF)/CP junctions by solving the Usadel equations. We show that the odd-frequency pairing state is induced in DN. The magnitude of the calculated Josephson current is larger than that in the *s*-wave superconductor/DN/*s*-wave superconductor (S/DN/S) junctions, but smaller than that in p_x-wave superconductor/DN/p_x-wave superconductor (P/DN/P) junctions. The obtained temperature dependence of the Josephson current is similar to that in the conventional *s*-wave junctions. The current-phase relation is almost sinusoidal and the Josephson current is proportional to $\exp(-L/\xi)$, where ξ is the coherence length of the Cooper pair in DN and L is the thickness of DN. We also calculate the Josephson current in the CP/DF/CP junctions. Similar to the case of the S/DF/S junctions, the 0-π transition occurs as a function of the thickness of DF. As follows from these results, it is difficult to extract the unusual properties of proximity effect specific to spin-triplet *p*-wave superconductor junctions from the study of the dc Josephson effect only. The obtained results may help to explore novel properties of superconducting Sr_2RuO_4.

II. FORMULATION

A. θ-ψ representation

In the following sections, the units with $\hbar = k_B = 1$ are used. The model of the CP/DF (DN)/CP junction is illustrated in Fig. 1. We consider the flat interface, where the momentum parallel to the interface is conserved. Here R_b' is the resistance of the insulating barrier located at $x=0$, R_d is the resistance of the DN, R_b is the resistance of the insulating barrier located at $x=L$, and the thickness of DN L is much larger than the mean free path. The infinitely thin insulating barriers are modeled as $U(x)=H'\delta(x)+H\delta(x-L)$. In this case the barrier transparency $T_\phi^{(\prime)}$ is expressed by $T_\phi^{(\prime)} = 4\cos^2\phi/(4\cos^2\phi + Z^{(\prime)2})$ with $Z^{(\prime)} = 2H^{(\prime)}/v_F$. Here ϕ is an injection angle measured from the direction perpendicular to the interface between DF (DN) and CP, and v_F is the Fermi velocity.

First, we concentrate on the retarded part of Nambu-Keldysh (NK) Green's function in DF (DN) within the quasiclassical approximation. We define the retarded part of NK Green's function $\hat{R}_N(x)$ given by

$$\hat{R}_N = \sin\theta\cos\psi\hat{\tau}_1 + \sin\theta\sin\psi\hat{\tau}_2 + \cos\theta\hat{\tau}_3, \tag{1}$$

where $\hat{\tau}_j$ $(j=1,2,3)$ are the Pauli matrices in electron-hole space.

The functions θ and ψ for majority (minority) spins obey the Usadel equation:

$$D\left[\frac{\partial^2}{\partial x^2}\theta - \left(\frac{\partial\psi}{\partial x}\right)^2\cos\theta\sin\theta\right] + 2i[\varepsilon + (-)h]\sin\theta = 0,$$

$$\frac{\partial}{\partial x}\left[\sin^2\theta\left(\frac{\partial\psi}{\partial x}\right)\right] = 0, \tag{2}$$

with the diffusion constant D and the exchange field h. If we choose $h=0$, DF is reduced to be DN. Since the flat interface is assumed, we use the boundary condition of the Numbu-Keldysh Green's function.[24] This boundary condition is an extended version of the circuit theory.[23]

The boundary condition for $\hat{R}_N(x)$ at $x=L$ has the form

$$\frac{L}{R_d}\left[\hat{R}_N(x)\frac{\partial\hat{R}_N(x)}{\partial x}\right]\bigg|_{x=L_-} = -\frac{\langle\hat{I}_\phi\rangle}{R_b},$$

$$\hat{I}_\phi = 2[\hat{R}_1, \hat{B}_\phi],$$

$$\hat{B}_\phi = (-T_{1\phi}[\hat{R}_1, \hat{H}_-^{-1}] + \hat{H}_-^{-1}\hat{H}_+ - T_{1\phi}^2\hat{R}_1\hat{H}_-^{-1}\hat{H}_+\hat{R}_1)^{-1}$$

$$\times (T_{1\phi}(1-\hat{H}_-^{-1}) + T_{1\phi}^2\hat{R}_1\hat{H}_-^{-1}\hat{H}_+), \tag{3}$$

with $\hat{R}_1 = \hat{R}_N(x=L_-)$, $\hat{H}_\pm = (\hat{R}_{2+} \pm \hat{R}_{2-})/2$, and $T_{1\phi} = T_\phi/(2 - T_\phi + 2\sqrt{1-T_\phi})$. $\hat{R}_{2\pm}$ is the asymptotic Green's function in the superconductor[24] defined as

$$\hat{R}_{2\pm} = (f_{1\pm}\cos\Psi + f_{2\pm}\sin\Psi)\hat{\tau}_1 + (f_{1\pm}\sin\Psi - f_{2\pm}\cos\Psi)\hat{\tau}_2 + g_\pm\hat{\tau}_3, \tag{4}$$

with $f_{1\pm} = \mathrm{Re}(f_\pm)$, $f_{2\pm} = \mathrm{Im}(f_\pm)$, $g_\pm = \epsilon/\sqrt{\epsilon^2 - |\Delta_\pm|^2}$, $f_\pm = \Delta_\pm/\sqrt{|\Delta_\pm|^2 - \epsilon^2}$, and the macroscopic phase of the superconductor Ψ. Here, $\Delta_+ = \Delta(\phi) = \Delta e^{i\phi}$ and $\Delta_- = \Delta(\pi - \phi)$ are the pair potentials corresponding to the injection angles ϕ and π-ϕ, respectively. Note that $|\Delta_+| = |\Delta_-|$ is satisfied in the present case, then we can put $g_+ = g_- \equiv g$.

The boundary condition for $\hat{R}_N(x)$ at $x=0$ has the form

$$\frac{L}{R_d}\left[\hat{R}_N(x)\frac{\partial\hat{R}_N(x)}{\partial x}\right]\bigg|_{x=0_+} = \frac{\langle\hat{I}'_\phi\rangle}{R'_b},$$

$$\hat{I}'_\phi = 2[\hat{R}'_1, \hat{B}'_\phi],$$

$$\hat{B}'_\phi = (-T'_{1\phi}[\hat{R}'_1, \hat{H}'^{-1}_-] + \hat{H}'^{-1}_-\hat{H}'_+ - T'^2_{1\phi}\hat{R}'_1\hat{H}'^{-1}_-\hat{H}'_+\hat{R}'_1)^{-1}$$

$$\times (-T'_{1\phi}(1+\hat{H}'^{-1}_-) + T'^2_{1\phi}\hat{R}'_1\hat{H}'^{-1}_-\hat{H}'_+), \tag{5}$$

with $\hat{R}'_1 = \hat{R}_N(x=0_+)$, $\hat{H}'_\pm = (\hat{R}'_{2+} \pm \hat{R}'_{2-})/2$, and $T'_{1\phi} = T'_\phi/(2 - T'_\phi + 2\sqrt{1-T'_\phi})$. $\hat{R}'_{2\pm}$ is defined as

$$\hat{R}'_{2\pm} = f_{1\pm}\hat{\tau}_1 - f_{2\pm}\hat{\tau}_2 + g\hat{\tau}_3. \tag{6}$$

In the above, the average over the various angles of an injected particle at the interface is defined as

$$\langle \hat{I}^{(\prime)}_\phi \rangle = \frac{\int_{-\pi/2}^{\pi/2} d\phi \cos\phi \hat{I}^{(\prime)}_\phi}{\int_{-\pi/2}^{\pi/2} d\phi \cos\phi T^{(\prime)}_\phi}. \tag{7}$$

The resistance of the interface $R_b^{(\prime)}$ is given by

$$R_b^{(\prime)} = \frac{2R_0^{(\prime)}}{\int_{-\pi/2}^{\pi/2} d\phi \cos\phi T^{(\prime)}_\phi}, \tag{8}$$

where $R_0^{(\prime)}$ is the Sharvin resistance, which in the three-dimensional case is expressed by $R_0^{(\prime)-1}=e^2k_F^2S_c^{(\prime)}/4\pi^2$. Here, k_F is the Fermi wave number and $S_c^{(\prime)}$ is the constriction area. The derivation of the boundary condition at $x=L$ is given in the Appendix.

Finally the boundary condition at $x=L$ is obtained as

$$\frac{LR_b}{R_d}\frac{\partial\theta}{\partial x}\bigg|_{x=L_-} = -I_1\sin\theta - I_2\cos\theta\sin(\psi-\Psi),$$

$$\frac{LR_b}{R_d}\left[\frac{\partial\psi}{\partial x}\sin\theta\right]\bigg|_{x=L_-} = -I_2\cos(\psi-\Psi),$$

$$I_1 = \left\langle \frac{2T_\phi g_s}{\Lambda_\phi}\right\rangle, \quad I_2 = \left\langle \frac{2T_\phi f_s}{\Lambda_\phi}\right\rangle,$$

$$\Lambda_\phi = 2 - T_\phi + T_\phi[g_s\cos\theta - f_s\sin\theta\sin(\psi-\Psi)],$$

$$g_s = \frac{2g + i(f_{1+}f_{2-} - f_{2+}f_{1-})}{1+g^2+f_{1+}f_{1-}+f_{2+}f_{2-}},$$

$$f_s = \frac{ig(f_{1+}-f_{1-}) + f_{2+} + f_{2-}}{1+g^2+f_{1+}f_{1-}+f_{2+}f_{2-}}. \tag{9}$$

The boundary condition at $x=0$ is obtained as

$$\frac{LR_b'}{R_d}\frac{\partial\theta}{\partial x}\bigg|_{x=0_+} = I_1'\sin\theta + I_2'\cos\theta\sin\psi,$$

$$\frac{LR_b'}{R_d}\left[\frac{\partial\psi}{\partial x}\sin\theta\right]\bigg|_{x=0_+} = I_2'\cos\psi,$$

$$I_1' = \left\langle \frac{2T_\phi' g_s'}{\Lambda_\phi'}\right\rangle, \quad I_2' = \left\langle \frac{2T_\phi' f_s'}{\Lambda_\phi'}\right\rangle,$$

$$\Lambda_\phi' = 2 - T_\phi' + T_\phi'[g_s'\cos\theta - f_s'\sin\theta\sin\psi],$$

$$g_s' = \frac{2g - i(f_{1+}f_{2-} - f_{2+}f_{1-})}{1+g^2+f_{1+}f_{1-}+f_{2+}f_{2-}},$$

$$f_s' = \frac{-ig(f_{1+}-f_{1-}) + f_{2+} + f_{2-}}{1+g^2+f_{1+}f_{1-}+f_{2+}f_{2-}}. \tag{10}$$

B. Φ parametrization

To calculate thermodynamic quantities, it is convenient to use Matsubara representation by changing $\epsilon\to i\omega$. We parametrize the quasiclassical Green's functions G_ω and F_ω with a function Φ_ω,[30,31]

$$G_\omega = \frac{\omega}{\sqrt{\omega^2+\Phi_\omega\Phi^*_{-\omega}}}, \quad F_\omega = \frac{\Phi_\omega}{\sqrt{\omega^2+\Phi_\omega\Phi^*_{-\omega}}}, \tag{11}$$

where ω is Matsubara frequency. The following relations are satisfied:

$$\frac{G_\omega}{2\omega}(\Phi_\omega + \Phi^*_{-\omega}) = \sin\theta\cos\psi,$$

$$\frac{iG_\omega}{2\omega}(\Phi_\omega - \Phi^*_{-\omega}) = \sin\theta\sin\psi. \tag{12}$$

Then the Usadel equation for majority (minority) spin has the form[31]

$$\xi^2\frac{\pi T_C}{G_\omega}\frac{\partial}{\partial x}\left(G_\omega^2\frac{\partial}{\partial x}\Phi_\omega\right) - [\omega-(+)ih]\Phi_\omega = 0, \tag{13}$$

with the coherence length $\xi=\sqrt{D/2\pi T_C}$, the diffusion constant D, the exchange field h, and the transition temperature T_C.

The boundary condition at $x=L$ is expressed by

$$\frac{G_\omega}{\omega}\frac{\partial}{\partial x}\Phi_\omega = \frac{R_d}{R_bL}\left(-\frac{\Phi_\omega}{\omega}I_1 + ie^{-i\Psi}I_2\right),$$

$$I_1 = \left\langle \frac{2T_\phi g_s}{\Lambda_\phi}\right\rangle, \quad I_2 = \left\langle \frac{2T_\phi f_s}{\Lambda_\phi}\right\rangle,$$

$$g_s = \frac{2g + i(f_{1+}f_{2-} - f_{2+}f_{1-})}{1+g^2+f_{1+}f_{1-}+f_{2+}f_{2-}},$$

$$f_s = \frac{ig(f_{1+}-f_{1-}) + f_{2+} + f_{2-}}{1+g^2+f_{1+}f_{1-}+f_{2+}f_{2-}},$$

$$\Lambda_\phi = 2 - T_\phi + T_\phi[g_sG_\omega + f_s(B\sin\Psi - C\cos\Psi)],$$

$$B = \frac{G_\omega}{2\omega}(\Phi_\omega + \Phi^*_{-\omega}), \quad C = \frac{iG_\omega}{2\omega}(\Phi_\omega - \Phi^*_{-\omega}). \tag{14}$$

The boundary condition at $x=0$ is expressed by

$$\frac{G_\omega}{\omega}\frac{\partial}{\partial x}\Phi_\omega = -\frac{R_d}{R_b'L}\left(-\frac{\Phi_\omega}{\omega}I_1' + iI_2'\right). \tag{15}$$

Here I_1' and I_2' are obtained by adding subscript ϕ to $\pi-\phi$, and putting $\Psi=0$ for I_1 and I_2 at $x=L$. Then the macroscopic phase differences between the left and right superconductor becomes Ψ.

The Josephson current is given by[31]

$$\frac{eIR}{\pi T_C} = i\frac{RTL}{4R_d T_C}\sum_{\uparrow,\downarrow,\omega} \mathrm{Tr}\left[\hat{\tau}_3 \hat{G}_\omega \frac{\partial}{\partial x}\hat{G}_\omega\right]$$
$$= i\frac{RTL}{4R_d T_C}\sum_{\uparrow,\downarrow,\omega} \frac{G_\omega^2}{\omega^2}\left(\Phi_\omega \frac{\partial}{\partial x}\Phi_{-\omega}^* - \Phi_{-\omega}^*\frac{\partial}{\partial x}\Phi_\omega\right), \quad (16)$$

where T is temperature and $R=R_b+R_b'+R_d$ is the total resistance of the junction with $\hat{G}_\omega=\hat{R}_N(i\omega)$.

C. Proximity effect in CP/DN/CP junctions

Before calculating the Josephson current, we consider the proximity effect in DN. To discuss the features of the proximity effect, in the following we will study the frequency dependence of the induced pair amplitude in DN by choosing $h=0$. Before proceeding with a formal discussion, let us present qualitative arguments. Two constraints should be satisfied in the considered system: (1) only the s-wave even-parity state is possible in the DN due to isotropization by impurity scattering, and (2) the spin structure of induced Cooper pairs in the DN is the same as in an attached superconductor. Then the Pauli principle provides the unique relations between the pairing symmetry in a superconductor and the resulting symmetry of the induced pairing state in the DN.[17] Since there is no spin flip at the interface, it is natural to expect that the odd-frequency pairing state is generated in DN.

It is possible to show that

$$f_\pm(-\omega) = f_\pm(\omega), \quad g_\pm(-\omega) = -g_\pm(\omega). \quad (17)$$

Using these equations,

$$g_s(-\omega,-\phi) = -g_s(\omega,\phi), \quad f_s(-\omega,-\phi) = -f_s(\omega,\phi) \quad (18)$$

are satisfied. For $h=0$, the Usadel equation has the form[31]

$$\xi^2\frac{\pi T_C}{G_\omega}\frac{\partial}{\partial x}\left(G_\omega^2\frac{\partial}{\partial x}\Phi_\omega\right) - \omega\Phi_\omega = 0, \quad (19)$$

and the boundary condition at $x=L$ is

$$\frac{G_\omega}{\omega}\frac{\partial}{\partial x}\Phi_\omega = \frac{R_d}{R_b L}\left(-\frac{\Phi_\omega}{\omega}I_1(\omega,\phi) + ie^{-i\Psi}I_2(\omega,\phi)\right),$$

$$I_1 = \left\langle \frac{2T_\phi g_s}{\Lambda_\phi}\right\rangle, \quad I_2 = \left\langle \frac{2T_\phi f_s}{\Lambda_\phi}\right\rangle,$$

$$\Lambda_\phi = 2 - T_\phi + T_\phi[g_s G_\omega + f_s(B\sin\Psi - C\cos\Psi)],$$

$$B = \frac{G_\omega}{2\omega}(\Phi_\omega + \Phi_{-\omega}^*), \quad C = \frac{iG_\omega}{2\omega}(\Phi_\omega - \Phi_{-\omega}^*). \quad (20)$$

By changing ω and ϕ into $-\omega$ and $-\phi$ in Eqs. (19) and (20), the following equations are obtained:

$$\xi^2\frac{\pi T_C}{G_{-\omega}}\frac{\partial}{\partial x}\left(G_{-\omega}^2\frac{\partial}{\partial x}\Phi_{-\omega}\right) + \omega\Phi_{-\omega} = 0, \quad (21)$$

$$\frac{G_{-\omega}}{\omega}\frac{\partial}{\partial x}\Phi_{-\omega} = \frac{R_d}{R_b L}\left[\frac{\Phi_{-\omega}}{\omega}I_1(-\omega,-\phi) + ie^{-i\Psi}I_2(-\omega,-\phi)\right]. \quad (22)$$

To check the consistency of the four above equations, we consider the ω dependence of several quantities. One can show that

$$f_{1\pm}(-\omega) = f_{1\pm}(\omega), \quad f_{2\pm}(-\omega) = f_{2\pm}(\omega),$$

$$g(-\omega) = -g(\omega). \quad (23)$$

As a result,

$$g_s(-\omega,-\phi) = -g_s(\omega,\phi), \quad f_s(-\omega,-\phi) = -f_s(\omega,\phi). \quad (24)$$

By comparing Eq. (19) with Eq. (21), we can derive

$$G_{-\omega} = -G_\omega. \quad (25)$$

Two cases can be considered:

(1)

$$\Phi_{-\omega} = \Phi_\omega, \quad I_1(-\omega,-\phi) = -I_1(\omega,\phi),$$

$$I_2(-\omega,-\phi) = I_2(\omega,\phi). \quad (26)$$

(2)

$$\Phi_{-\omega} = -\Phi_\omega, \quad I_1(-\omega,-\phi) = -I_1(\omega,\phi),$$

$$I_2(-\omega,-\phi) = -I_2(\omega,\phi). \quad (27)$$

For case (1), the relations $B(-\omega)=B(\omega)$ and $C(-\omega)=C(\omega)$ hold, while for case (2) the relations $B(-\omega)=-B(\omega)$ and $C(-\omega)=-C(\omega)$ hold. For case (1), since $\Lambda(-\omega,-\phi) \neq \Lambda(\omega,\phi)$, then it is impossible to satisfy $I_1(-\omega,-\phi)=-I_1(\omega,\phi)$ and $I_2(-\omega,-\phi)=I_2(\omega,\phi)$ simultaneously, thus this case cannot be realized. For case (2),

$$\Lambda(-\omega,-\phi) = \Lambda(\omega,\phi) \quad (28)$$

is satisfied and this relation is consistent with $I_1(-\omega,-\phi)=-I_1(\omega,\phi)$, $I_2(-\omega,-\phi)=-I_2(\omega,\phi)$. Since $\Phi_{-\omega}=-\Phi_\omega$ is satisfied, we can show

$$\sin\theta(-\omega)\cos\psi(-\omega) = -\sin\theta(\omega)\cos\psi(\omega),$$

$$\sin\theta(-\omega)\sin\psi(-\omega) = -\sin\theta(\omega)\sin\psi(\omega). \quad (29)$$

Then $F_{-\omega}=-F_\omega$ is satisfied. This indicates the realization of the odd-frequency pairing state in DN. In the presence of h, i.e., CP/DF/CP junctions, the admixture of an even-frequency spin-singlet even-parity state is also present.

In the following, we calculate numerically the Josephson current in CP/DN (DF)/CP junctions.

III. RESULTS

A. CP/DN/CP junctions

Here, we fix $R_b'=R_b$, $T_\phi'=T_\phi$. We define Δ_0 as $\Delta_0\equiv\Delta(0)$. First, we consider the temperature dependence of a maxi-

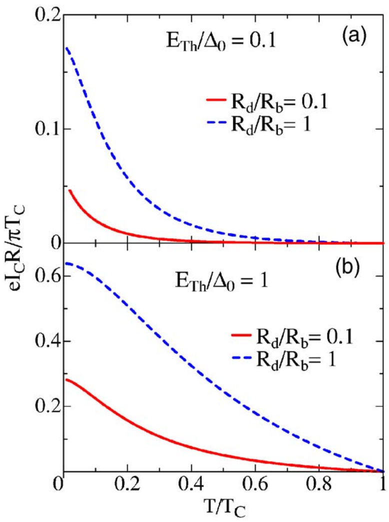

FIG. 2. (Color online) Temperature dependence of the maximum Josephson current for $Z=10$. The solid lines are the results for $R_d/R_b=0.1$ and broken lines are the results for $R_d/R_b=1$. (a) $E_{Th}/\Delta_0=0.1$ and (b) $E_{Th}/\Delta_0=1$.

mum Josephson current I_C for $Z=10$ as shown in Fig. 2. The magnitude of I_C is enhanced for large E_{Th}/Δ_0 and large R_d/R_b. It is enhanced at low temperatures in both cases (a) and (b). These features are consistent with the conventional case of the S/DN/S junctions.

Next, we consider the dependence of I_C on Z, the transparency parameter at the interface. Figure 3 shows the temperature dependence of the critical Josephson current for $R_d/R_b=1$. The magnitude of RI_C is enhanced for large Z, i.e., low transparent interface for both $E_{Th}/\Delta_0=0.1$ and 1. This result is specific for junctions between triplet superconductors, where proximity effect is enhanced by MARS formed at the interface. It is known that the degree of the influence of MARS on the charge transport becomes prominent for low

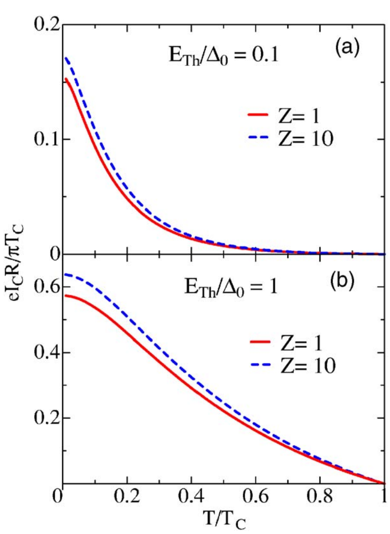

FIG. 3. (Color online) Temperature dependence of the maximum Josephson current for $R_d/R_b=1$. The solid lines are the results for $Z=1$ and the broken lines are the results for $Z=10$. (a) $E_{Th}/\Delta_0=0.1$ and (b) $E_{Th}/\Delta_0=1$.

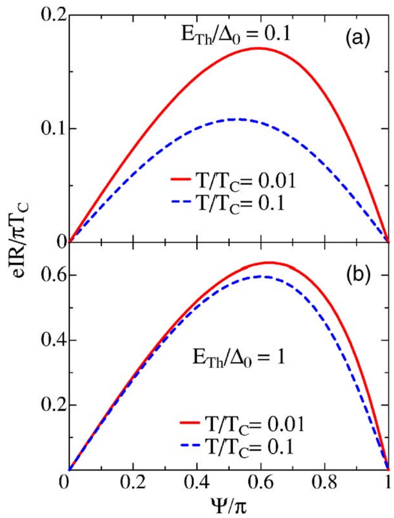

FIG. 4. (Color online) The current-phase relation for $Z=10$ and $R_d/R_b=1$. The solid lines are the results for $T/T_C=0.01$ and the broken lines are the results for $T/T_C=0.1$. (a) $E_{Th}/\Delta_0=0.1$ and (b) $E_{Th}/\Delta_0=1$.

transparent junctions with large Z.[15] On the contrary, in S/DN/S junctions the maximum Josephson current is suppressed for large Z.[19]

Next, we study the current-phase relation in order to examine the unusual proximity effect specific to CP/DN/CP junctions. Figure 4 shows the current-phase relation for $Z=10$ and $R_d/R_b=1$. We find that the peak is shifted to $\Psi > 0.5\pi$ at low temperatures, and this effect becomes rather strong in particular for large E_{Th}/Δ_0. This result indicates that the magnitude of the Josephson current is enhanced by the proximity effect, and the Josephson current is not proportional to $\sin\Psi$. However, this effect is not as pronounced as in the case of p_x-wave/DN/p_x-wave (P/DN/P) junctions.[19]

The dependence of the I_C on the thickness of DN is shown in Fig. 5 for $Z=10$ and $R_d/R_b=1$. We find that the I_C is proportional to $\exp(-L/\xi)$ in agreement with existing theoretical results.

In order to compare our results with the existing theories, we also calculate I_C in a S/DN/S junction and P/DN/P junctions.[27,28] The results are shown in Fig. 6 for $Z=10$ and $R_d/R_b=1$. We find that the magnitude of I_C in the CP/DN/CP junction is larger than that in the S/DN/S junction, and less than that in the P/DN/P junction at low temperatures for both (a) and (b). These results indicate that the maximum Josephson current is enhanced due to the unusual proximity effect coexisting with MARS in CP/DN/CP junctions. However, it is known that MARS is induced only for the particle with an injection angle $\phi=0$ in CP/DN/CP junctions, thus the I_C is smaller than in P/DN/P junctions. We also find that qualitative temperature dependence of the critical current in CP/DN/CP junctions is quite similar to that in S/DN/S junctions. The result is consistent with the experiment in Sr_2RuO_4-$Sr_3Ru_2O_7$ eutectic junctions.[39] As follows from these calculations, if we focus on the temperature dependence and current phase relation of the Josephson current of CP/DN/CP junctions, the obtained results are rather conventional.

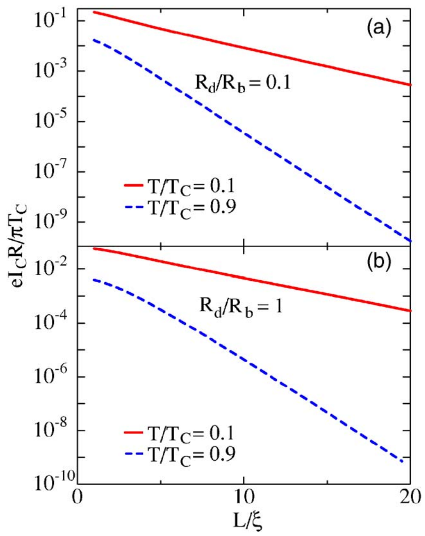

FIG. 5. (Color online) The critical current as a function of DN thickness for $Z=10$. The solid lines are the results for $T/T_C=0.1$ and the broken lines are the results for $T/T_C=0.9$. (a) $R_d/R_b=0.1$ and (b) $R_d/R_b=1$.

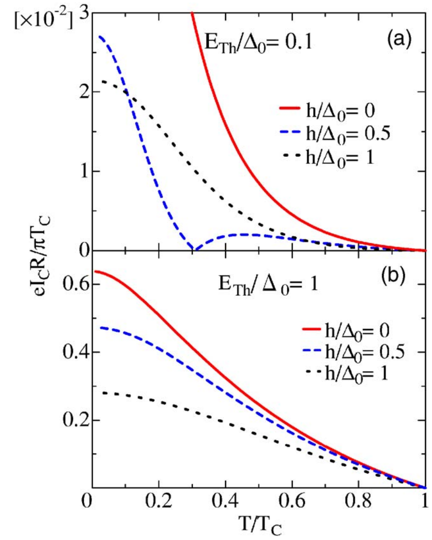

FIG. 7. (Color online) Temperature dependence of the critical current for $Z=10$ and $R_d/R_b=1$. The solid lines are the results for $h/\Delta_0=0$, the broken lines are the results for $h/\Delta_0=0.5$, and the dotted lines are the results for $h/\Delta_0=1$. (a) $E_{Th}/\Delta_0=0.1$ and (b) $E_{Th}/\Delta_0=1$.

B. CP/DF/CP junctions

It is well-known that in the S/DF/S junctions, the 0-π transition[32,33] can be induced as a result of the peculiar proximity effect in DF. In DF, the Cooper pairs have finite center of mass momentum and the pair amplitude is spatially oscillating. As a result, various interesting phenomena are predicted in these junctions.[29,34–38] The 0-π transition is a typical example. It also exists in $d(p)$-wave superconductor/DF/$d(p)$-wave superconductor junctions.[28] Here we study the Josephson effect in CP/DF/CP junctions.

Figure 7 shows the temperature dependence of the critical current for $Z=10$ and $R_d/R_b=1$. In all cases, the exchange field suppresses the magnitude of I_C. For $E_{Th}/\Delta_0=0.1$ and $h/\Delta_0=0.5$, the nonmonotonic temperature dependence of I_C is realized.

To clarify that this nonmonotonic temperature dependence originates from the 0-π transition, we focus on the current-phase relation as shown in Fig. 8 for $Z=10$ and $R_d/R_b=1$ at $T/T_C=0.1$. With the increase of the exchange field, the maximum of the Josephson current is shifted to $\Psi<0.5\pi$ for $E_{Th}/\Delta_0=1$. Especially, for $E_{Th}/\Delta_0=0.1$, the Josephson current changes its sign for $h/\Delta_0=0.5$. These results indicate that the exchange field induces the 0-π transition in this case.

In Fig. 9, I_C is plotted as a function of the thickness L of DF. In the presence of the exchange field h in DF, the I_C

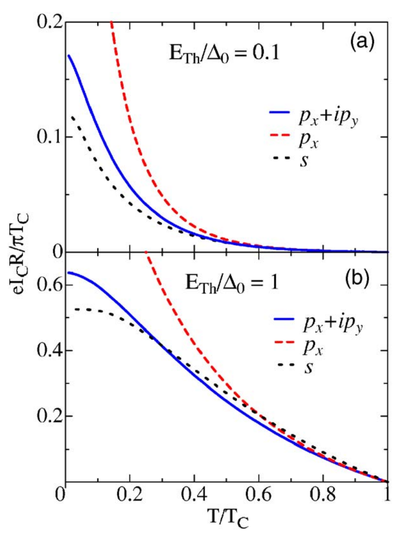

FIG. 6. (Color online) Temperature dependence of the critical current for $Z=10$ and $R_d/R_b=1$. The solid lines are the results for CP/DN/CP junctions, the broken lines are the results for P/DN/P junctions, and the dotted lines are the results for S/DN/S junctions. (a) $E_{Th}/\Delta_0=0.1$ and (b) $E_{Th}/\Delta_0=1$.

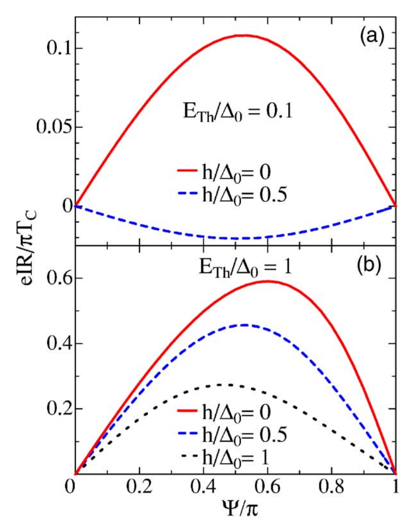

FIG. 8. (Color online) Current-phase relation for $Z=10$ and $R_d/R_b=1$ at $T/T_C=0.1$. The solid lines are the results for $h/\Delta_0=0$, the broken lines are the results for $h/\Delta_0=0.5$, and the dotted line is the result for $h/\Delta_0=1$. (a) $E_{Th}/\Delta_0=0.1$ and (b) $E_{Th}/\Delta_0=1$.

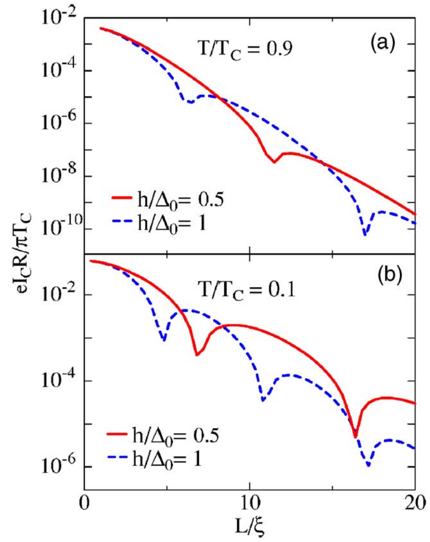

FIG. 9. (Color online) The critical current as a function of DF thickness for $Z=10$ and $R_d/R_b=1$. (a) $T/T_C=0.9$ and (b) $T/T_C=0.1$. The solid lines are the results for $h/\Delta_0=0.5$ and the broken lines are the results for $h/\Delta_0=1$.

oscillates as a function of L. The period of this oscillation becomes shorter with the increase of the magnitude of h.

We have shown that the 0-π transition also exists in the CP/DN/CP junctions. The nonmonotonic temperature dependence of I_C and the oscillatory dependence of I_C as a function of L are consistent with S/DN/S junctions or $d(p)$-wave superconductor/DF/$d(p)$-wave superconductor junctions.[28] It is shown that the 0-π transition specific to DF junctions is robust against the change of the symmetry of the Cooper pair.

IV. CONCLUSIONS

We have derived generalized boundary conditions for the DN (DF)/CP interface including the macroscopic phase of the superconductor. The Josephson effect in the CP/DN (DF)/CP junctions has been studied by solving the Usadel equations with the above boundary conditions. Here, we choose the p_x+ip_y-wave as a symmetry of the CP superconductor. The results obtained in the present paper can be summarized as follows.

(1) It is shown that the odd-frequency spin triplet s-wave pairing symmetry is induced in DN due to the proximity effect. The Josephson current is carried by the odd-frequency pairing state.

(2) Almost all of the obtained results are qualitatively similar to those in the S/DN/S junctions. I_C is proportional to $\exp(-L/\xi)$ where L and ξ is the thickness and the coherence length in DN, respectively. The temperature dependence of the maximum Josephson current in the CP/DN/CP junction is qualitatively similar to that in the S/DN/S junctions.

(3) Although the magnitude of the I_C is enhanced at low temperatures as compared to the S/DN/S junctions, this enhancement is not as strong as in the case of the P/DN/P junctions.

(4) In the CP/DF/CP junctions, the current-phase relation changes drastically with the decrease of the temperature due to the 0-π transition. The resulting I_C oscillates as a function of the thickness of DF. These properties are similar to those of the S/DN/S junctions.

Recently, the Josephson effect in the Sr_2RuO_4-$Sr_3Ru_2O_7$ eutectic junction is experimentally observed.[39] There is no qualitative difference of the temperature dependence as compared to that of the S/DN/S junctions. The present theoretical result is consistent with this experiment. Surprisingly, although the proximity effect is unusual due to the presence of an odd-frequency pairing state, the resulting Josephson current is not much different compared to the conventional junctions. The reason is that in the present case, the magnitude of the odd-frequency pair amplitude is small compared to that in the P/DN/P junctions. Especially, the magnitude of the pair amplitude in DN for low Matsubara frequency in the considered CP/DN/CP junctions is much smaller than that in the P/DN/P junctions. It should be stressed that even though there is no qualitative difference between the actual experimentally observed Josephson current[39] and that in the S/DN/S junctions, it means neither the absence of the spin-triplet pairing state in Sr_2RuO_4 nor the absence of the odd-frequency pairing state in DN.

In the present paper, the spatial dependence of the order parameter (pair potential) of a chiral p-wave state is not determined self-consistently. As shown in Ref. 41, the p_x-wave component is suppressed while the p_y-wave component is enhanced at the interface. However, as regards to the proximity effect in a diffusive normal metal region (DN), the resulting pair amplitude is an odd-frequency spin-triplet s-wave state. This important conclusion is robust even if we take into account the spatial dependence of the pair potential. In the present paper, we only focus on the diffusive limit. Recently, theory of proximity effect in the clean limit case is presented.[40] In such a case, it is expected that not only the odd-frequency spin-triplet s-wave pairing state but also the even-frequency spin-triplet p-wave state exists in the normal region.[42] It is an interesting issue to study the transition from the clean limit to the diffusive limit systematically.

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APPENDIX: BOUNDARY CONDITION AT DF (DN)/CP INTERFACE

We consider the boundary condition for the retarded part of the NK Green's functions at the DF (DN)/CP interface.

The left side of the boundary condition of Eq. (3) is expressed by

$$\frac{L}{R_d}\hat{R}_N(x)\frac{\partial}{\partial x}\hat{R}_N(x)\bigg|_{x=L_-} = \frac{Li}{R_d}\left[\left(-\frac{\partial\theta}{\partial x}\sin\psi - \frac{\partial\psi}{\partial x}\sin\theta\cos\theta\cos\psi\right)\hat{\tau}_1 + \left(\frac{\partial\theta}{\partial x}\cos\psi - \frac{\partial\psi}{\partial x}\sin\theta\cos\theta\sin\psi\right)\hat{\tau}_2 + \frac{\partial\psi}{\partial x}\sin^2\theta\hat{\tau}_3\right]. \tag{A1}$$

In the right side of Eq. (3), $\hat{I}_\phi$ is expressed by

$$\hat{I}_\phi = 4iT_{1\phi}(\mathbf{d}_R\cdot\mathbf{d}_R)^{-1}\times\Big(-\frac{1}{2}(1+T_{1\phi}^2)(\mathbf{s}_{2+}-\mathbf{s}_{2-})^2[\mathbf{s}_1\times(\mathbf{s}_{2+} + \mathbf{s}_{2-})]\cdot\hat{\tau} + 2T_{1\phi}\mathbf{s}_1\cdot(\mathbf{s}_{2+}\times\mathbf{s}_{2-})[\mathbf{s}_1\times(\mathbf{s}_{2+}\times\mathbf{s}_{2-})]\cdot\hat{\tau} + 2T_{1\phi}\mathbf{s}_1\cdot(\mathbf{s}_{2+}-\mathbf{s}_{2-})[\mathbf{s}_1\times(\mathbf{s}_{2+}-\mathbf{s}_{2-})]\cdot\hat{\tau} - i(1+T_{1\phi}^2)(1 - \mathbf{s}_{2+}\cdot\mathbf{s}_{2-})[\mathbf{s}_1\times(\mathbf{s}_{2+}\times\mathbf{s}_{2-})]\cdot\hat{\tau} + 2iT_{1\phi}(1-\mathbf{s}_{2+}\cdot\mathbf{s}_{2-}) \times[\mathbf{s}_1\cdot(\mathbf{s}_{2+}-\mathbf{s}_{2-})\mathbf{s}_1 - (\mathbf{s}_{2+}-\mathbf{s}_{2-})]\cdot\hat{\tau}\Big), \tag{A2}$$

$$\mathbf{d}_R = (1+T_{1\phi}^2)(\mathbf{s}_{2+}\times\mathbf{s}_{2-}) - 2T_{1\phi}\mathbf{s}_1\times(\mathbf{s}_{2+}-\mathbf{s}_{2-}) - 2T_{1\phi}^2\mathbf{s}_1\cdot(\mathbf{s}_{2+}\times\mathbf{s}_{2-})\mathbf{s}_1, \tag{A3}$$

with $\hat{R}_1 = \mathbf{s}_1\cdot\hat{\tau}$, and $\hat{R}_{2\pm} = \mathbf{s}_{2\pm}\cdot\hat{\tau}$. The spectral vector $\mathbf{s}_1$ and $\mathbf{s}_{2\pm}$ are expressed by

$$\mathbf{s}_1 = \begin{pmatrix}\sin\theta\cos\psi\\ \sin\theta\sin\psi\\ \cos\theta\end{pmatrix},$$

$$\mathbf{s}_{2\pm} = \begin{pmatrix}f_{1\pm}\cos\Psi + f_{2\pm}\sin\Psi\\ f_{1\pm}\sin\Psi - f_{2\pm}\cos\Psi\\ g\end{pmatrix}. \tag{A4}$$

Here, Ψ is the macroscopic phase of the right superconductor. After some algebra, the matrix current is reduced to be

$$\hat{I}_\phi = 2iT_\phi\{(2-T_\phi) + T_\phi[g_s\cos\theta + f_s\sin\theta\sin(\psi-\Psi)]\}^{-1} \times\{[-g_s\sin\theta\sin\psi - f_s\cos\theta\cos\Psi]\hat{\tau}_1 + [g_s\sin\theta\cos\psi - f_s\cos\theta\sin\Psi]\hat{\tau}_2 + f_s\sin\theta\cos(\psi-\Psi)\hat{\tau}_3\}. \tag{A5}$$

Then the resulting $\langle\hat{I}_\phi\rangle$ can be expressed as

$$\langle\hat{I}_\phi\rangle = \begin{pmatrix}-I_1\sin\theta\sin\psi - I_2\cos\theta\cos\Psi\\ I_1\sin\theta\cos\psi - I_2\cos\theta\sin\Psi\\ I_2\sin\theta\cos(\psi-\Psi)\end{pmatrix}\cdot\hat{\tau},$$

$$I_1 = \left\langle\frac{2T_\phi g_s}{\Lambda_\phi}\right\rangle,\quad I_2 = \left\langle\frac{2T_\phi f_s}{\Lambda_\phi}\right\rangle,$$

$$\Lambda_\phi = 2 - T_\phi + T_\phi[g_s\cos\theta - f_s\sin\theta\sin(\psi-\Psi)],$$

$$g_s = \frac{2g + i(f_{1+}f_{2-} - f_{2+}f_{1-})}{1+g^2+f_{1+}f_{1-}+f_{2+}f_{2-}},$$

$$f_s = \frac{ig(f_{1+}-f_{1-}) + f_{2+} + f_{2-}}{1+g^2+f_{1+}f_{1-}+f_{2+}f_{2-}}. \tag{A6}$$

Finally the boundary condition at $x=L$ is obtained as

$$\frac{LR_b}{R_d}\frac{\partial\theta}{\partial x} = -I_1\sin\theta - I_2\cos\theta\sin(\psi-\Psi),$$

$$\frac{LR_b}{R_d}\frac{\partial\psi}{\partial x}\sin\theta = -I_2\cos(\psi-\Psi). \tag{A7}$$

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