Theory of Tunneling Spectroscopy in the Larkin-Ovchinnikov State

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We present a theory of tunneling spectroscopy for normal metal/Larkin-Ovchinnikov state junctions in which the spatial periodic modulation in the pair potential amplitude is taken into account. The tunneling spectra show the characteristic line shapes reflecting the minigap structures under the periodic pair potentials depending on the boundary condition of the pair potentials at the junction interface. These features are qualitatively different from the tunneling spectra of the Fulde-Ferrell state. We propose an experimental setup which identifies the superconducting state of $CeCoIn_5$.

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Fulde and Ferrell [1] and Larkin and Ovchinnikov [2] proposed superconducting states in which spin-singlet superconducting pair potentials are periodically modulated in real space under high magnetic fields. The Zeeman spin splitting results in a total momentum $2\boldsymbol{q}$ of Cooper pairs. Fulde and Ferrell (FF) discussed that the pair potential becomes $\Delta \exp(i\boldsymbol{q}\cdot\boldsymbol{r})$, where the phase of the pair potential changes periodically in real space. Larkin and Ovchinnikov (LO) proposed independently an alternative scenario in which the order parameter is real, but its amplitude varies periodically in real space like $\Delta\cos(\boldsymbol{q}\cdot\boldsymbol{r})$ [2]. Although the two states are collectively called the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state [3], basic properties of the LO state are expected to be very different from those in the FF state. For instance, the supercurrent is expected in the FF state, whereas it is absent in the LO state.

Recently, the FFLO state has suddenly become a hot topic [4] because several experiments [5] and theories [6] suggested the realization of the FFLO state in a heavy fermionic compound $CeCoIn_5$. According to the phase diagram of $CeCoIn_5$, the LO state is considered to be more stable than the FF state [6]. At present, however, no experiment can distinguish the LO state from the FF state in $CeCoIn_5$. In addition to the supercurrent, the energy spectrum of the quasiparticles can be qualitatively different in the two states. In the LO state, the minigap structures are expected, because the amplitude of the pair potential is periodically modulated in real space. On the other hand, in the FF state, such a minigap structure may be absent. Tunneling spectroscopy is a promising tool to detect such differences in the energy spectra. In fact, experimental research in this direction is becoming accessible now [7]. Thus, a theory of tunneling spectroscopy of the FF and LO states is desired to interpret the tunneling spectra in experiments.

As shown in a previous paper [8], it is easy to calculate the tunneling conductance of normal metal (N)/insulator (I)/FF state junctions by using the standard method [9] with a simple gauge transformation. On the other hand, the theory of tunneling spectroscopy in the LO state has not been established yet, because it is not easy to calculate the tunneling conductance under the periodic modulation of the pair potential amplitude. In the LO state, we have to consider the sign change of the pair potential in two spaces: real space and momentum space. The sign change of the pair potential in real space [10] drastically changes the local density of states (LDOS). In addition, we also have to consider the sign change of the pair potential in momentum space, because $CeCoIn_5$ is a strongly correlated material and its promising symmetry of the pair potential is considered to be d-wave. In such a pairing symmetry, charge transport of the junctions is governed by a midgap Andreev resonant state (MARS) formed at the junction interface [9,11,12]. Thus, we must take into account the two kinds of sign change in the pair potentials on an equal footing.

In this Letter, we present a theory of tunneling spectra in a normal metal/insulator/semi-infinite superconductor in the LO state (N/I/LO) in the ballistic regime at zero temperature. We choose that $\boldsymbol{q}$ is parallel to the normal of the junction interface. We consider two types of junction: (i) node contact junctions, where the amplitude of the LO pair potentials vanishes at the junction interface, and (ii) maximum contact junctions, where the amplitude of the pair potentials takes its maximum at the junction interface. We will show that calculated results of tunneling spectra strongly depend on the type of the junction. In both cases, the tunneling spectra have complex structures reflecting the miniband due to the spatial modulation of pair potentials. On the other hand, in N/I/FF junctions, the line shapes of the tunneling conductance do not have such fine structures. The calculated results give us useful information to identify the superconducting state of the LO state in $CeCoIn_5$.

Let us consider an N/I/LO junction in two dimensions as shown in Fig. 1, where ballistic semi-infinite N and LO correspond to regions for $x < 0$ and $x > 0$, respectively. We assume a flat interface in the y direction and an isotropic Fermi surface. External magnetic fields H are applied in the x direction so that effects of magnetic fields on the orbital part of the wave function can be neglected. The insulating barrier is expressed by the delta-function model, and its potential is given by $H_b\delta(x)$. The effective mass m and Fermi wave number k_F are chosen to be common in N and LO. Here we neglect spin-orbit coupling in LO.

To calculate the tunneling conductance, we first obtain the quasiclassical Green's function $\hat{g}^{\uparrow(\downarrow)}_{\pm,\pm}(x)$ in the bulk LO state [13]. When the period of the oscillations in the pair potential is L, the Green's function is expressed by

$$\hat{g}\,^{\nu}_{\pm,\pm}(x) = \frac{\pm i\{U^{\nu}_{\pm}(x, x+L) - \frac{1}{2}\mathrm{Tr}[U^{\nu}_{\pm}(x, x+L)]\}}{\sqrt{\{\frac{1}{2}\mathrm{Tr}[U^{\nu}_{\pm}(x, x+L)]\}^2 - \det[U^{\nu}_{\pm}(x, x+L)]}}, \tag{1}$$

where $U^{\nu}_{\pm}$ is the evolution operator of the Green's function with spin $\nu = \uparrow$ ($\downarrow$). The Eilenberger equation for $U^{\nu}_{\pm}$ reads

$$iv_{Fx}\partial_x U^{\nu}_{\pm}(x, x') = \mp[\varepsilon_s\hat{1} - \hat{\Delta}(\theta_{\pm}, x)]\hat{\tau}_3 U^{\nu}_{\pm}(x, x'), \tag{2}$$

with $\hat{\Delta}(\theta_{\pm}, x) = \hat{\tau}_1\Delta(\theta_{\pm}, x)$ and $\varepsilon_s = \varepsilon - s\mu_B H$, where s is 1 and -1 for $\nu = \uparrow$ and $\downarrow$, respectively. The Pauli matrices are denoted by $\hat{\tau}_j$, with $j = 1, 2, 3$, and $\hat{1}$ is the 2×2 unit matrix. The injection angle θ of a quasiparticle at the interface is measured from the x direction. The two angles are defined by $\theta_+ = \theta$ and $\theta_- = \pi - \theta$. In Eq. (2), ε denotes a quasiparticle energy measured from the Fermi energy, and v_{Fx} is the x component of the Fermi velocity. The spatial dependence of the pair potential is described by $\Delta(\theta, x) = \Delta(x)\Theta(x)f(\theta)$, where $\Theta(x)$ is a step function. The form factor $f(\theta)$ depends on pairing symmetries and orientations of crystalline axis in $CeCoIn_5$: $f(\theta) = 1$ for s-wave symmetry, $f(\theta) = \cos 2\theta$ for $d_{x^2-y^2}$-wave symmetry, and $f(\theta) = \sin 2\theta$ for d_{xy}-wave symmetry. It is sufficient to solve $U^{\nu}_{\pm}(x, x+L)$ for $0 \le x \le L$ due to the periodicity of the LO state.

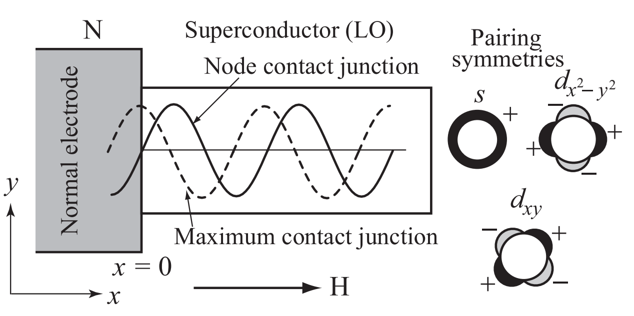

FIG. 1. A normal metal/insulator/superconductor in the LO state junction. In a node contact junction and a maximum contact junction, Q_1 is set to be $-\pi/2$ and 0, respectively. We also show pairing symmetries in this coordinate.

Next we calculate the conductance of N/I/LO junctions by using Eq. (1), which is decomposed into $\hat{g}^{\nu}_{\pm,\pm}(0) = g^{\nu}_{\pm}\hat{\tau}_3 + f^{\nu}_{1\pm}\hat{\tau}_1 + f^{\nu}_{2\pm}\hat{\tau}_2$. From the components of $\hat{g}^{\nu}_{\pm,\pm}(0)$, we define $\Gamma_{\pm,\nu} = -(if^{\nu}_{2\pm} \pm f^{\nu}_{1\pm})/(1 + g^{\nu}_{\pm})$. The tunneling conductance σ_S for a bias voltage V at zero temperature becomes [9]

$$\sigma_S(\varepsilon = eV) = (\sigma_\uparrow + \sigma_\downarrow)/2, \tag{3}$$

$$\sigma_\nu = \int_{-\pi/2}^{\pi/2} d\theta \cos\theta \sigma_N F_\nu, \tag{4}$$

$$F_\nu = \frac{1 + \sigma_N|\Gamma_{+,\nu}|^2 + (\sigma_N - 1)|\Gamma_{+,\nu}\Gamma_{-,\nu}|^2}{|1 + (\sigma_N - 1)\Gamma_{+,\nu}\Gamma_{-,\nu}|^2}. \tag{5}$$

The transparency at the interface is given by $\sigma_N(\theta) = 4\cos^2\theta/(4\cos^2\theta + Z^2)$, with $Z = 2mH_b/\hbar k_F$. The barrier parameter Z is chosen to be $Z = 5$ throughout this Letter. The resulting transparency of the junction is about 0.1. The normal conductance of the junction is $\bar{\sigma}_N = \int d\theta \cos\theta \sigma_N$. In what follows, we discuss the normalized tunneling conductance $\sigma_T = \sigma_S(eV)/\bar{\sigma}_N$.

We describe the pair potential in the LO state as $\Delta(x) = \Delta_0\cos(Qx + Q_1)$, with $Q = 2\pi/L$. In what follows, we fix the magnetic field $\mu_B H$ at $0.4\pi T_C$, because the previous study [13] suggested the stable LO state around $\omega_L = 2\mu_B H \sim 0.8\pi T_C$, with Larmor frequency ω_L. The resulting effective magnitude of the Zeeman splitting is about $0.7\Delta_0$, with $\Delta_0 \sim 0.56\pi T_C$ from the BCS relation. The period L is measured in units of $L_0 = 2\pi/Q_0$, with $Q_0 = \pi T_C/\hbar v_F$. The previous study also showed that the value of L changes drastically for a slight change of H [13]. Thus, it is instructive to see the L dependence without changing other parameters. Here we choose two typical values for L as $L = L_0$ and $10L_0$ as the short and sufficient long limit of the actual calculation in the following. The shorter L corresponds to the larger magnetic field in the phase diagram. In addition to the conductance, we also calculate the LDOS given by $\rho_T = \frac{1}{4\pi}\sum_\nu \int_{-\pi/2}^{\pi/2} d\theta(g^{\nu}_{+} + g^{\nu}_{-})$, where the LDOS is normalized by its value in the normal state.

First, we focus on the LO state in the s-wave symmetry. In Figs. 2(a) and 2(c), σ_T of the node contact junction and that of the maximum contact junction are shown, respectively. The results for $L = L_0$ and $L = 10L_0$ are represented by a dotted line a and a solid line b, respectively. In Figs. 2(b) and 2(d), ρ_T at a nodal point and that at a maximum point of bulk superconductors are shown. As shown in Figs. 2(b) and 2(d), the LDOS of the bulk LO state has fine structures, which reflect the miniband structures due to the periodic pair potential. The results also show that the LDOS at the nodal and maximum points of

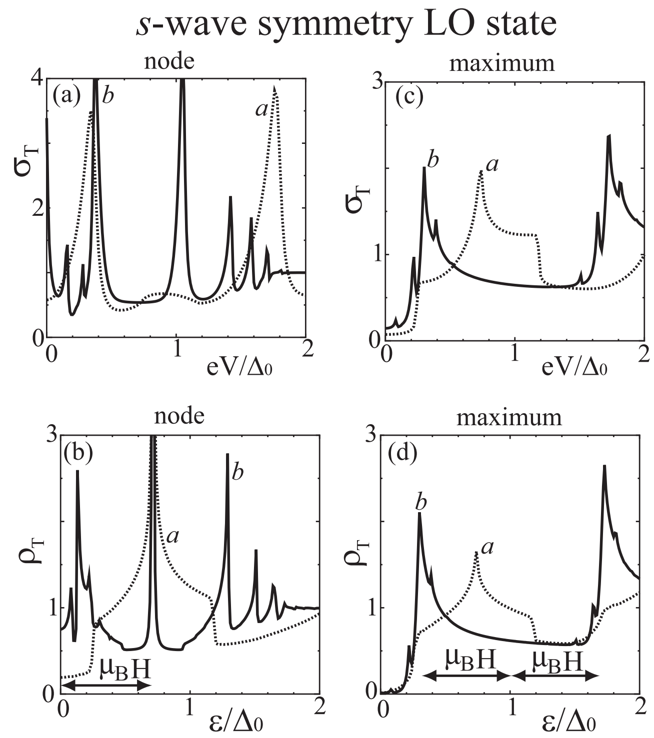

FIG. 2. Calculated results for s-wave N/I/LO junctions. Normalized tunneling conductance σ_T is plotted as a function of bias voltages for the node contact junction in (a) and for the maximum contact junction in (c). Local density of states ρ_T at a nodal point and that at a maximum point of the bulk LO state are shown in (b) and (d), respectively. In all panels, a: $L = L_0$ and b: $L = 10L_0$. By the Zeeman shift, the zero bias peak is shifted to $\varepsilon = \pm\mu_B H$ in (b), and peaks of the gap edge are split to $\varepsilon = \Delta_0 \pm \mu_B H$ in (d).

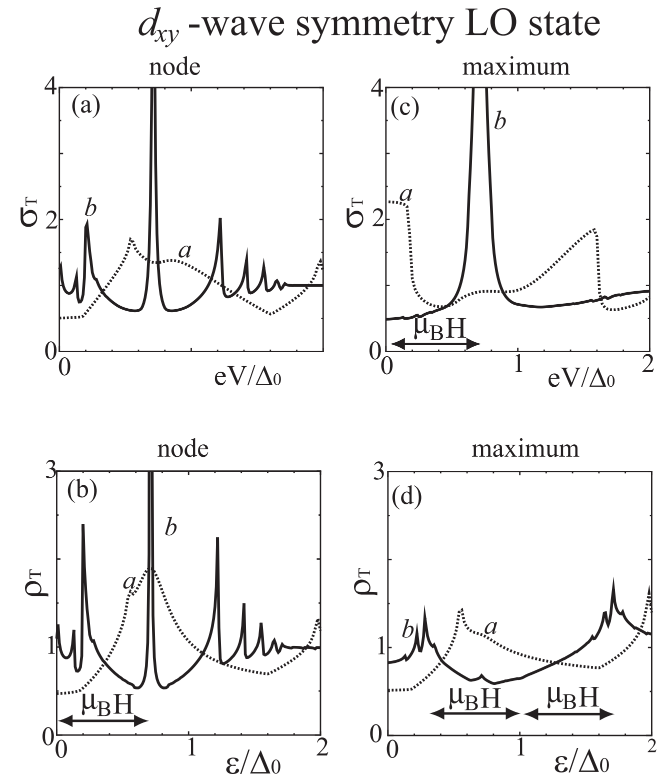

FIG. 3. Calculated results for d_{xy}-wave N/I/LO junctions. σ_T is plotted as a function of bias voltages for the node contact junction in (a) and for the maximum contact junction in (c). ρ_T at a nodal point and that at a maximum point of the bulk LO state are shown in (b) and (d), respectively. In all panels, a: $L = L_0$ and b: $L = 10L_0$.

the pair potential are different from each other [14]. These fine structures can be also seen in σ_T and are more remarkable for a longer period of oscillations. In the maximum contact junctions, the line shapes of σ_T and those of ρ_T are very similar to each other. This feature is seen more clearly for a longer period of oscillations. On the other hand, in the node contact junctions, the line shapes of σ_T are very different from those of ρ_T. Here we note that results for the $d_{x^2-y^2}$-wave symmetry are qualitatively the same as those in the s-wave junctions in Fig. 2.

Second, we show calculated results of N/I/LO junctions in the d_{xy}-wave symmetry in Fig. 3. Differently from the s-wave symmetry, the MARS forming at the interface changes the spectra in the d_{xy}-wave symmetry. When spatial modulation of pair potentials is absent, the MARS results in a zero bias conductance peak (ZBCP) at $H = 0$ [9]. We see this peak at $\mu_B H$ in Figs. 3(a) and 3(c) for $L = 10L_0$. In magnetic fields, it is known that the ZBCP splits into two peaks at $eV = \pm\mu_B H$. The line shapes of the σ_T in the maximum contact junction in Fig. 3(c) are very different from those of ρ_T in Fig. 3(d). On the other hand, in the node contact junctions, the line shapes of the σ_T in Fig. 3(a) are similar to those of ρ_T in Fig. 3(b). This tendency in the d_{xy} symmetry is opposite to that in the s-wave symmetry. The correspondence between σ_T and ρ_T is seen more clearly for longer L. A number of peaks and dips due to the miniband structures appear in curve b in Figs. 3(a) and 3(b). The results in Figs. 2 and 3 indicate that σ_T is sensitive to the boundary condition of the pair potential at the interface. It should be remarked that ρ_T does not always express σ_T.

Third, we look into σ_T for the FF state as shown in Fig. 4. Here we use the pair potential $\Delta(x) = \Delta_0 \exp[i(Qx + Q_1)]$. The tunneling conductance is calculated from Eq. (5) with

$$\Gamma_{\pm,\nu} = \frac{\Delta_\pm}{(\varepsilon_s \mp Qv_{Fx}/2) + \sqrt{(\varepsilon_s \mp Qv_{Fx}/2)^2 - \Delta_\pm^2}} \quad (6)$$

and $\Delta_\pm = \Delta_0 f(\theta_\pm)$. Thus, a preexistence formula for uniform superconductors [9] can be applied to the N/I/FF junctions even in the presence of the Zeeman splitting. In contrast to the N/I/LO junctions, the resulting σ_T in the FF junction is independent of Q_1, which characterizes the boundary condition of the LO state. At the same time, ρ_T does not have spatial dependence. This is because the pair

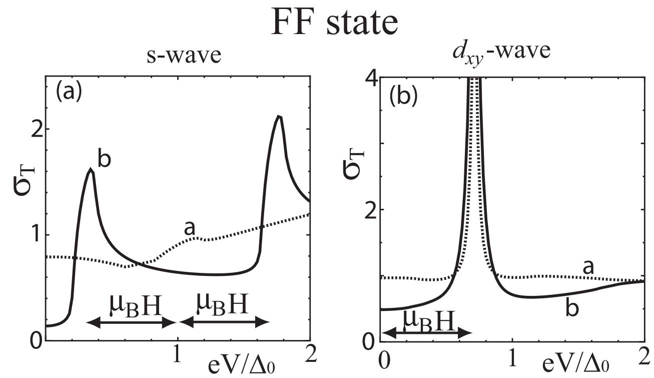

FIG. 4. Calculated results for N/I/FF junctions. σ_T is plotted as a function of bias voltages for the s wave in (a) and for the d_{xy} wave in (b). a: $L = a_0$ and b: $L = 10a_0$.

amplitudes are uniform in real space in the FF state. In both the s- and d_{xy}-wave symmetries, the tunneling conductance does not have any fine structures as shown in Fig. 4. The two peaks appear at $eV = \Delta_0 \pm \mu_B H$ in the s-wave symmetry. The large peak in the d_{xy}-wave symmetry is one of the splitting ZBCP. The overall features of conductance in the FF state are very different from those in the LO state.

Finally, on the basis of the obtained results, we predict the experimental tunneling conductance at a sufficiently low temperature. Since $CeCoIn_5$ is a strongly correlated electron system, realization of d-wave symmetry is more plausible than that of s-wave symmetry [15,16]. When the (110) crystalline axis of $CeCoIn_5$ is perpendicular to the junction interface, which corresponds to the d_{xy}-wave symmetry in this Letter, the ZBCP is expected in tunneling spectra in the absence of a magnetic field. In the uniform superconducting state, the height of the ZBCP would decrease with the increase of magnetic fields H. At the same time, the ZBCP would split into two peaks at $\varepsilon = \pm\mu_B H$ because of the Zeeman effect. Thus, the splitting width of the ZBCP increases with the increase of magnetic fields [17]. At the FFLO critical magnetic field, $CeCoIn_5$ undergoes a transition to the LO or FF state. If the node contact is realized at the junction interface, the σ_T first show the fine structures with increasing H, and then these fine structures vanish. When the maximum contact is realized, the fine structures are not clearly visible in the conductance. The line shapes of the conductance would be very sensitive to changing H. In the case of the FF state, on the other hand, the fine structures are absent and the split ZBCP is rather robust against the change of H. It is possible to discriminate the LO state with the node contact junction, the LO state with the maximum contact junction, and the FF state from one another by observing the tunneling conductance near the phase boundary between the uniform superconducting state and the LO or FF state.

In conclusion, a theory of tunneling spectroscopy of a normal metal/LO state is presented by fully taking account of the periodic modulation of the pair potentials in real space. The tunneling spectra show several maxima and minima reflecting the minigap structures in the density of states. These features are not expected in the FF state, because the amplitude of pair potentials is uniform in the FF state. The present results are not changed even in the presence of the impurity scattering, if the amplitude of the normal scattering rate $\hbar/(2\tau)$ is smaller than the energy which characterizes the fine structures of the tunneling conductance in the LO state. Our results serve as a guide to identify the LO state in experiments.

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