Transition from a matter-dominated era to a dark energy universe

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We develop a general program of the unification of a matter-dominated era with an acceleration epoch for scalar-tensor theory or a dark fluid. The general reconstruction of the scalar-tensor theory is fulfilled. The explicit form of the scalar potential for which the theory admits a matter-dominated era, a transition to an acceleration, and an (asymptotically de Sitter) acceleration epoch consistent with Wilkinson Microwave Anisotropy Probe data is found. The interrelation of the epochs of deceleration-acceleration transition and matter dominance-dark energy transition for dark fluids with a general equation of state (EOS) is investigated. We give several examples of such models with explicit EOS (using redshift parametrization) where matter-dark energy domination transition may precede the deceleration-acceleration transition. As a by-product, the reconstruction scheme is applied to scalar-tensor theory to define the scalar potentials which may produce the dark matter effect. The obtained modification of Newton potential may explain the rotation curves of galaxies.

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I. INTRODUCTION

The origin of the dark energy era remains to be one of the challenges of modern cosmology. A number of various models aiming at the description of a dark energy universe have been proposed (for a recent review, see [1]). However, even being consistent with recent Wilkinson Microwave Anisotropy Probe (WMAP) data, such models turn out to be problematic at the early and intermediate universe. In other words, we are lacking the unified theory which describes at once the sequence of the well-known cosmological epochs: inflation, radiation/matter-dominated epoch, and the current universe speed-up. In such a situation, the attempt to unify several known cosmological phases may be considered as some approximation for such a complete theory. From another side, it may rule out the number of dark energy models which, being consistent with recent WMAP data, cannot correctly describe the earlier cosmological phases.

In the present paper, we develop the general program of the search of dark energy models (scalar-tensor theory or ideal fluid) where the sequence of matter-dominated era and acceleration era is realized. The paper is organized as follows. Section II is devoted to the reconstruction program of scalar-tensor theory admitting the matter-dominated era before the current acceleration. In the class of theories with single scalar, it is explicitly demonstrated for which scalar potentials the matter-dominated stage precedes the acceleration era. Clearly, the method may be generalized for the multiscalar case (compare with [2]). In Sec. III we study the interrelation of the epochs of deceleration-acceleration transition and the matter-dark energy domination transition in the class of dark energy fluids with a general equation of state (EOS). In Sec. III A we develop the general setting for this class of models, present some general results, and the interesting special cases. In Sec. III B we give examples of dark energy fluids, defined by the concrete EOS, in which in certain parametric regimes the epoch of the matter-dark energy domination transition may precede the epoch of the deceleration-acceleration transition. In Sec. III C we analyze a redshift parametrization of the dark energy EOS and elaborate the conditions on the parameters to observe the matter-dark energy domination transition before the deceleration-acceleration transition. In Sec. IV we show that the same reconstruction method of Sec. II may be applied to explain the qualitatively different phenomena of dark matter from scalar-tensor theory with specific potential. In particular, it is shown which types of scalar potential lead to the requested modification of Newton

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potential at large distances so that rotation curves of galaxies may be explained. Some summary and outlook are given in the Discussion.

II. MATTER-DOMINATED AND ACCELERATION ERA IN SCALAR-TENSOR THEORY

In the present section, the reconstruction program in scalar-tensor theory is developed in such a way that the sequence of matter-dominated and acceleration phases may be achieved for some potentials. We follow the approach developed in Ref. [2]. One may begin with the following action of scalar-tensor theory:

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R - \frac{1}{2}\omega(\phi)\partial_\mu\phi\partial^\mu\phi - V(\phi)\right\} + S_m. \tag{1}$$

Here $\omega(\phi)$ and $V(\phi)$ are proper functions of the scalar field ϕ, and S_m is the action of matter fields. We now assume the spatially flat Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a(t)^2\sum_{i=1}^{3}(dx^i)^2. \tag{2}$$

Let the scalar field ϕ only depend on the time coordinate t. Then the FRW equations are given by

$$\frac{3}{\kappa^2}H^2 = \rho + \rho_m, \qquad -\frac{2}{\kappa^2}\dot{H} = p + \rho + p_m + \rho_m. \tag{3}$$

Here ρ_m and p_m are the energy density and the pressure of the matter fields, respectively. The energy density ρ and the pressure p for the scalar field ϕ are given by

$$\rho = \tfrac{1}{2}\omega(\phi)\dot{\phi}^2 + V(\phi), \qquad p = \tfrac{1}{2}\omega(\phi)\dot{\phi}^2 - V(\phi). \tag{4}$$

We also have the scalar field equation given by the variation of the scalar ϕ:

$$0 = \omega(\phi)\ddot{\phi} + \tfrac{1}{2}\omega'(\phi)\dot{\phi}^2 + 3H\omega(\phi)\dot{\phi} + V'(\phi). \tag{5}$$

First, we consider the case that the matter fields can be neglected by putting $\rho_m = p_m = 0$. Let $\omega(\phi)$ and $V(\phi)$ be expressed in terms of the single function $f(\phi)$ [2] as follows:

$$\omega(\phi) = -\frac{2}{\kappa^2}f'(\phi), \qquad V(\phi) = \frac{1}{\kappa^2}(3f(\phi)^2 + f'(\phi)). \tag{6}$$

Then we can easily find the following solution for Eqs. (3)–(5) [2]:

$$\phi = t, \qquad H = f(t). \tag{7}$$

We should note ϕ inside the function f is replaced by t since t is associated with ϕ as in Eq. (7). In the case that $\omega(\phi)$ is always positive, the function ω can be absorbed into the redefinition of the scalar field

$$\varphi \equiv \int^\phi d\phi\sqrt{\omega(\phi)} \tag{8}$$

and the action (1) has the canonical form:

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \tilde{V}(\varphi)\right\}. \tag{9}$$

Here the potential $\tilde{V}(\varphi)$ is defined by

$$\tilde{V}(\varphi) \equiv V(\phi(\varphi)). \tag{10}$$

The matter may be included into the action (1). Such a matter, in general, interacts with the dark energy. Then the separation of the total energy density to the contribution from the dark energy and that from the matter $\rho_{\rm total} = \rho + \rho_m$ is not unique. However, if it is defined as

$$p_m \equiv -\rho_m + \frac{\dot{\rho}}{3H}, \qquad p \equiv p_{\rm total} - p_m, \tag{11}$$

then the matter and the dark energy satisfy the conservation law, separately:

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0, \qquad \dot{\rho} + 3H(\rho + p) = 0. \tag{12}$$

Especially if $w_m = p_m/\rho_m$ is constant, it follows that

$$\rho_m = \rho_{m0}a^{-3(1+w_m)}. \tag{13}$$

Here ρ_{m0} is a constant. Hence, for the case that $\omega(\phi)$ and $V(\phi)$ are given by a single function $g(\phi)$ as follows,

$$\omega(\phi) = -\frac{2}{\kappa^2}g''(\phi) - (w_m + 1)g_0 \mathrm{e}^{-3(1+w_m)g(\phi)},$$
$$V(\phi) = \frac{1}{\kappa^2}(3g'(\phi)^2 + g''(\phi)) + \frac{w_m - 1}{2}g_0\mathrm{e}^{-3(1+w_m)g(\phi)}, \tag{14}$$

we find the solution:

$$\phi = t, \qquad H = g'(t),$$
$$\left(a = a_0\mathrm{e}^{g(t)},\ a_0 \equiv \left(\frac{\rho_{m0}}{g_0}\right)^{1/[3(1+w_m)]}\right). \tag{15}$$

The present universe is expanding with acceleration. On the other hand, there occurred the earlier matter-dominated period, where the scale factor a behaves as $a \sim t^{2/3}$. Such behavior could be generated by dust in the Einstein gravity. The baryons are dust and (cold) dark matter could be a dust. The ratio of the baryons and the dark matter in the present universe could be 1:5 or 1:6, which should not be changed even in the matter dominant era. It is not clear what the dark matter is. For instance, the dark matter might not be the real matter but some (effective) artifact which appears in the modified/scalar-tensor gravity. In the present paper, it is assumed that not only dark energy but also the dark matter originates from the scalar field ϕ.

We now investigate that the transition from the matter dominant period to the acceleration period could be realized in the present formulation. In the following, the contribution from matter is neglected since the ratio of the matter with the (effective) dark matter could be small.

The first example is

$$H = f(t) = g_0 + \frac{g_1}{t}. \tag{16}$$

When t is large, the first term in (16) dominates and the Hubble rate H becomes a constant. Therefore, the universe is asymptotically de Sitter space, which is an accelerating universe (for recent examples of late-time accelerating cosmology in scalar-tensor theory, see [3,4] and for earlier study and a list of references, see [5,6]). On the other hand, when t is small, the second term in (16) dominates and the scale factor behaves as $a \sim t^{g_1}$. Therefore, if $g_1 = 2/3$, the matter-dominated period could be realized. By substituting (16) into (6), one finds

$$\omega(\phi) = \frac{2}{\kappa^2}\frac{g_1}{\phi^2}, \qquad V(\phi) = \frac{1}{\kappa^2}\left(3g_0^2 + \frac{6g_0g_1}{\phi} + \frac{3g_1^2 - g_1}{\phi^2}\right). \tag{17}$$

Hence, (8) shows

$$\varphi = \frac{\sqrt{2g_1}}{\kappa \ln\frac{\phi}{\phi_0}}. \tag{18}$$

Here ϕ_0 is a constant. In terms of the canonical field φ, the potential $\tilde{V}(\varphi)$ (10) is given by

$$\tilde{V}(\varphi) = \frac{1}{\kappa^2}\left(3g_0^2 + \frac{6g_0g_1}{\phi_0}e^{-\kappa\varphi/\sqrt{2g_1}} + \frac{3g_1^2 - g_1}{\phi_0^2}e^{-2\kappa\varphi/\sqrt{2g_1}}\right). \tag{19}$$

Before going to the second example, we consider the Einstein gravity with cosmological constant and with matter characterized by the EOS parameter w. The FRW equation has the following form:

$$\frac{3}{\kappa^2}H^2 = \rho_0 a^{-3(1+w)} + \frac{3}{\kappa^2 l^2}. \tag{20}$$

Here l is the length parameter coming from the cosmological constant. The solution of (20) is given by

$$a = a_0 e^{g(t)}, \qquad g(t) = \frac{2}{3(1+w)}\ln\left(\alpha\sinh\left(\frac{3(1+w)}{2l}(t - t_0)\right)\right). \tag{21}$$

Here t_0 is a constant of the integration and

$$\alpha^2 \equiv \tfrac{1}{3}\kappa^2 l^2 \rho_0 a_0^{-3(1+w)}. \tag{22}$$

As a second example, we consider (21) without matter. In this case, (6) shows

$$\omega(\phi) = \frac{3(1+w)}{\kappa^2 l^2}\sinh^{-2}\left(\frac{3(1+w)}{2l}(\phi - t_0)\right),$$
$$V(\phi) = \frac{1}{\kappa^2}\left(\frac{3}{l^2}\coth^2\left(\frac{3(1+w)}{2l}(\phi - t_0)\right) - \frac{3(1+w)}{2l^2}\sinh^{-2}\left(\frac{3(1+w)}{2l}(\phi - t_0)\right)\right). \tag{23}$$

Equations (8) and (10) also indicate

$$\varphi = \frac{2}{\kappa\sqrt{3(1+w)}}\ln\tanh\left(\frac{3(1+w)}{4l}(\phi - t_0)\right),$$
$$V(\varphi) = \frac{1}{\kappa^2}\left(\frac{3}{l^2}\cosh^2\left(\frac{\varphi}{\varphi_0}\right) - \frac{3(1+w)}{2l^2}\sinh^2\left(\frac{\varphi}{\varphi_0}\right)\right), \tag{24}$$
$$\frac{1}{\varphi_0} \equiv \frac{\kappa}{2}\sqrt{\frac{1+w}{\alpha}}.$$

Thus, in both examples, (16) and (21), there occurs the transition from the matter-dominated phase to the acceleration phase. In the acceleration phase, in the above examples, the universe asymptotically approaches to de Sitter space. This does not conflict with WMAP data. Indeed, three years of WMAP data have been analyzed in Ref. [7]. The combined analysis of WMAP with supernova Legacy survey (SNLS) constrains the dark energy equation of state $w_{\rm DE}$ pushing it towards the cosmological constant. The marginalized best fit values of the equation of state parameter at 68% confidence level are given by $-1.14 \leq w_{\rm DE} \leq -0.93$. In the case of a prior that the universe is flat, the combined data gives $-1.06 \leq w_{\rm DE} \leq -0.90$. In the examples (16) and (21), the universe goes to asymptotically de Sitter space, which gives $w_{\rm DE} \to -1$, which does not, of course, conflict with the above constraints. Note, however, one needs to fine-tune g_0 in (16) and $1/l$ in (21) to be $g_0 \sim 1/l \sim 10^{-33}$ eV, in order to reproduce the observed Hubble rate $H_0 \sim 70\ {\rm km\,s^{-1}\,Mpc^{-1}} \sim 10^{-33}$ eV.

In principle, more complicated examples of multiscalar-tensor theory reconstruction may be considered. These (radiation/matter-dominated) regimes may be related with dark energy stages proposed in [2]. In the next section, it will be demonstrated that a similar scenario may be realized for dark fluids.

Before ending this section, we consider what could happen in the Jordan frame. By transforming the metric by a function $\sigma(\phi)$ of the scalar field ϕ as $g_{\mu\nu} \to e^{\sigma(\phi)}g_{\mu\nu}$, the action (1) is transformed as

$$S \to S_J = \int d^4x\sqrt{-g}e^{\sigma(\phi)}\left\{\frac{1}{2\kappa^2}R - \left(\frac{1}{2}\omega(\phi) - \frac{3}{2}\sigma'(\phi)^2\right) \times \partial_\mu\phi\partial^\mu\phi - e^{\sigma(\phi)}V(\phi)\right\}. \tag{25}$$

Especially, if we consider the case $\sigma(\phi) = 2\phi/\phi_0$ with a positive constant ϕ_0, we find

$$S_J = \int d^4x\sqrt{-g}e^{2\phi/\phi_0}\left\{\frac{1}{2\kappa^2}R - \left(\frac{1}{2}\omega(\phi) - \frac{6}{\phi_0^2}\right) \times \partial_\mu\phi\partial^\mu\phi - e^{2\phi/\phi_0}V(\phi)\right\}. \tag{26}$$

Then we obtain the Jordan frame action.

We should note, however, that the solution is the Jordan frame does not correspond to the real cosmology. As an example, we consider the case (16) by choosing $\phi = t$. Then we have the following relations between the quantities in the original Einstein frame and those in the Jordan frame, which are distinguished by the suffix J:

$$a_J = e^{\phi/\phi_0}a, \qquad t_J = \phi_0 e^{t/\phi_0}. \tag{27}$$

Then the Hubble rate in the Jordan frame is given by

$$H_J = \frac{1}{a_J}\frac{da_J}{dt_J} = \frac{1 + \phi_0 g_0 + \frac{g_1}{\ln(t_J/\phi_0)}}{t_J}. \tag{28}$$

Different from the case of (16), the solution does not go to asymptotically de Sitter space in the limit of $t_J \to +\infty$,

$$H_J \to \frac{1 + \phi_0 g_0}{t_J}. \tag{29}$$

Then the cosmology looks to change in the frame. If we neglect the matter, we cannot say which frame should be physical. If we couple the matter, in the original Einstein frame (1), the matter does not couple with the scalar field ϕ, but in the Jordan frame the matter couples with ϕ through the scale transformation $g_{\mu\nu} \to e^{2\phi/\phi_0}g_{\mu\nu}$. In the present model, the Jordan frame would not be physical.

III. MATTER-DARK ENERGY DOMINATION TRANSITION VERSUS DECELERATION-ACCELERATION TRANSITION

A. The general setting

In the present section, we consider a wide class of cosmological models to study the interrelation of two epochs, the epoch of the transition from the matter-dominated to the dark energy dominated universe and the epoch of the deceleration-acceleration transition. Unlike in the previous section, we limit ourselves by the consideration of the FRW universe filled with dark energy ideal fluid of a general form. We assume that the universe is spatially flat and that it contains two components, a dark energy component, and a matter component, which is presently nonrelativistic. In the study of this class of models, we are especially interested in the possibility that the parameter of dark energy EOS may be variable with redshift and that dark energy at high redshifts may have different properties than at low redshifts (i.e. that at higher redshifts it could even be matterlike). The dependence of the nonrelativistic matter component on the redshift (scale factor) is the standard one, $\rho_m(z) = \rho_{m,0}(1+z)^3$ $[\rho_m(a) = \rho_{m,0}(a/a_0)^{-3}]$, whereas the behavior of the dark energy component is defined by its equation of state (EOS):

$$p_d(z) = w_d(z)\rho_d(z), \tag{30}$$

$$p_d(a) = w_d(a)\rho_d(a), \tag{31}$$

where $w_d(z)$ $[w_d(a)]$ is in general a function of redshift (scale factor), i.e. we discuss a very general class of dark energy models. The dark energy density can be written in the form

$$\rho_d(z) = \rho_{d,0}\exp\left(3\int_0^z \frac{1 + w_d(z')}{1+z'}dz'\right) = \rho_{d,0}(1+z)^3\exp\left(3\int_0^z \frac{w_d(z')}{1+z'}dz'\right), \tag{32}$$

$$\rho_d(a) = \rho_{d,0}\exp\left(-3\int_{a_0}^a \frac{1 + w_d(a')}{a'}da'\right) = \rho_{d,0}\left(\frac{a}{a_0}\right)^{-3}\exp\left(-3\int_{a_0}^a \frac{w_d(a')}{a'}da'\right). \tag{33}$$

This expression allows us to write the ratio $r(z) = \rho_d(z)/\rho_m(z)$ $[r(a) = \rho_d(a)/\rho_m(a)]$ as

$$r(z) = \frac{1-\Omega_m^0}{\Omega_m^0}\exp\left(3\int_0^z \frac{w_d(z')}{1+z'}dz'\right), \tag{34}$$

$$r(a) = \frac{1-\Omega_m^0}{\Omega_m^0}\exp\left(-3\int_{a_0}^a \frac{w_d(a')}{a'}da'\right). \tag{35}$$

The description of the cosmological evolution is completed with the Hubble equation,

$$H^2 = \frac{8\pi G}{3}\rho_m(1+r), \tag{36}$$

and the expression for the acceleration of the expansion of the universe,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_m(1 + r(1+3w_d)). \tag{37}$$

The main objects of our present study are two redshifts (scale factor values), the redshift z_{EQ} (scale factor a_{EQ}) at which the matter-dominated epoch ends and the dark energy dominated epoch begins, and the redshift $z_{D/A}$ (scale factor $a_{D/A}$) at which the universe transits from the decelerated expansion to the accelerated one. The redshift (scale factor) of the dark energy domination onset z_{EQ} (a_{EQ}) is characterized by $r(z_{EQ}) = 1$ $[r(a_{EQ}) = 1]$. This requirement can be further elaborated using (34)

$$r(z_{EQ}) = \frac{1-\Omega_m^0}{\Omega_m^0}\exp\left(3\int_0^{z_{EQ}} \frac{w_d(z')}{1+z'}dz'\right) = 1, \tag{38}$$

$$r(a_{EQ}) = \frac{1-\Omega_m^0}{\Omega_m^0}\exp\left(-3\int_{a_0}^{a_{EQ}} \frac{w_d(a')}{a'}da'\right) = 1. \tag{39}$$

The deceleration-acceleration transition redshift $z_{D/A}$

(scale factor $a_{D/A}$) is the point where the acceleration of the expansion of the universe vanishes which leads to

$$1+(1+3w_d(z_{D/A}))\frac{1-\Omega_m^0}{\Omega_m^0}\exp\left(3\int_0^{z_{D/A}}\frac{w_d(z')}{1+z'}dz'\right)=0, \tag{40}$$

$$1+(1+3w_d(a_{D/A}))\frac{1-\Omega_m^0}{\Omega_m^0}\exp\left(-3\int_{a_0}^{a_{D/A}}\frac{w_d(a')}{a'}da'\right)=0. \tag{41}$$

Expressions (38) and (40) can be conveniently combined into a compact expression depending only on the evolution of the function $w_d(z)$ $[w_d(a)]$ in the interval between z_{EQ} and $z_{D/A}$ (a_{EQ} and $a_{D/A}$):

$$\exp\left(3\int_{z_{D/A}}^{z_{EQ}}\frac{w_d(z')}{1+z'}dz'\right)=-(1+3w_d(z_{D/A})), \tag{42}$$

$$\exp\left(-3\int_{a_{D/A}}^{a_{EQ}}\frac{w_d(a')}{a'}da'\right)=-(1+3w_d(a_{D/A})). \tag{43}$$

This expression, gives a (trivial) condition $w_d(z_{D/A})< -1/3$ $[w_d(a_{D/A})<-1/3]$.

All results obtained so far do not depend on the ordering of z_{EQ} and $z_{D/A}$ (a_{EQ} and $a_{D/A}$). For a large class of dark energy models, including the benchmark ΛCDM model, the deceleration-acceleration transition happens before the onset of the dark energy dominated epoch. Here we focus our attention on an alternative scenario in which the transition from the matter domination to the dark energy domination precedes the deceleration-acceleration transition, i.e. $z_{EQ}\geq z_{D/A}$ ($a_{EQ}\leq a_{D/A}$). Physically, this ordering of the aforementioned epochs suggests that the dark energy component may acquire matterlike properties at higher redshifts, i.e. might be a dominant component but still nonaccelerating in character and only at the small redshifts become a truly accelerating component, as indicated in Sec. II.

For this scenario, using the expressions given above, further general information can be obtained. At z_{EQ} (a_{EQ}) the expansion of the universe is decelerated. From (37) one obtains the condition $1+r(z_{EQ})(1+3w_d(z_{EQ}))\geq 0$ $[1+r(a_{EQ})(1+3w_d(a_{EQ}))\geq 0]$, which, using the fact that $r(z_{EQ})=1$ $[r(a_{EQ})=1]$ leads to the condition $w_d(z_{EQ})\geq -2/3$ $[w_d(a_{EQ})\geq -2/3]$. On the other hand, for $z>z_{EQ}$ ($a<a_{EQ}$) the universe is matter dominated and $r<1$. Therefore, at z_{EQ} (a_{EQ}), $r(z)$ is a decreasing function of redshift [$r(a)$ is an increasing function of scale factor]. This fact can be further expressed as

$$\left.\frac{dr}{dz}\right|_{z=z_{EQ}}=3\frac{w_d(z_{EQ})}{1+z_{EQ}}<0, \tag{44}$$

$$\left.\frac{dr}{da}\right|_{a=a_{EQ}}=-3\frac{w_d(a_{EQ})}{a_{EQ}}>0, \tag{45}$$

which leads to the condition $w_d(z_{EQ})<0$ $[w_d(a_{EQ})<0]$. Combining this result with the one coming from the deceleration of the universe at z_{EQ} (a_{EQ}) we finally obtain $-2/3\leq w_d(z_{EQ})<0$ $[-2/3\leq w_d(a_{EQ})<0]$. Since at $z_{D/A}$ ($a_{D/A}$) the condition $w_d(z_{D/A})<-1/3$ $[w_d(a_{D/A})<-1/3]$ must be satisfied, for models with the monotonous variation of $w_d(z)$ $[w_d(a)]$ in the $(z_{EQ}, z_{D/A})$ interval $[(a_{EQ}, a_{D/A})$ interval], it follows that $w_d(z)$ $[w_d(a)]$ is negative in the same interval. The expression

$$r(z_{D/A})=\exp\left(-3\int_{z_{D/A}}^{z_{EQ}}\frac{w_d(z')}{1+z'}dz'\right), \tag{46}$$

$$r(a_{D/A})=\exp\left(-3\int_{a_{EQ}}^{a_{D/A}}\frac{w_d(a')}{a'}da'\right), \tag{47}$$

which immediately leads to $r(z_{D/A})>1$ $[r(a_{D/A})>1]$.

An interesting limiting case is the one when $z_{EQ}=z_{D/A}$ ($a_{EQ}=a_{D/A}$). From (42) it is straightforward to obtain $w_d(z_{EQ}=z_{D/A})=-2/3$ $[w_d(a_{EQ}=a_{D/A})=-2/3]$. In general, the redshift dependence of $w_d(z)$ $[w_d(a)]$ will contain a number of parameters. This condition on $w_d(z_{EQ}=z_{D/A})$ $[w_d(a_{EQ}=a_{D/A})]$ determines the boundary of the part of the parametric space where the studied scenario of the expansion of the universe is realized.

Finally, let us study the simplest case when $w_d(z)=w_d=\text{const}$ $[w_d(a)=w_d=\text{const}]$. In this case we have

$$\left(\frac{1+z_{EQ}}{1+z_{D/A}}\right)^{3w_d}=-(1+3w_d), \tag{48}$$

$$\left(\frac{a_{D/A}}{a_{EQ}}\right)^{3w_d}=-(1+3w_d). \tag{49}$$

Since in the studied scenario we have $z_{EQ}\geq z_{D/A}$ ($a_{EQ}\leq a_{D/A}$), the relation (48) gives a constraint $w_d\geq -2/3$. In the limiting case $z_{EQ}=z_{D/A}$ ($a_{EQ}=a_{D/A}$) we obtain $w_d=-2/3$, in accordance with the general result obtained in the previous paragraph.

B. Examples of dark energy models

We further discuss two concrete examples of the dark energy models which result in the studied scenario. Let us first discuss the dark energy model, first introduced in [8] and then analyzed in detail in [4,9], defined by the EOS

$$p_d=-\rho_d-A\rho_d^{\alpha}. \tag{50}$$

This model exhibits a rich variety of interesting phenomena [4,8,9] in different parameter regimes, including the cosmological singularity at the finite value of the scale factor, Big rip-like singularities [10] (for a classification of future cosmological singularities, see [4]). The dependence of the dark energy density on scale factor can be given as a simple analytic expression,

$$\rho_d = \rho_{d,0}\left(1 + 3\tilde{A}(1-\alpha)\ln\frac{a}{a_0}\right)^{1/(1-\alpha)}, \qquad (51)$$

where $\tilde{A} = A\rho_d^{\alpha-1}$. Moreover, in the situation where the matter density or the spatial curvature term in the Hubble equation are negligible compared to the dark energy density, an analytic expression for the time evolution of the scale factor can be obtained [9]. For $\alpha \neq 1/2$ we have

$$\left(1 + 3\tilde{A}(1-\alpha)\ln\frac{a_1}{a_0}\right)^{(1-2\alpha)/[2(1-\alpha)]} - \left(1 + 3\tilde{A}(1-\alpha)\ln\frac{a_2}{a_0}\right)^{(1-2\alpha)/[2(1-\alpha)]} = \frac{3}{2}\tilde{A}(1-2\alpha)\Omega_{d,0}^{1/2}H_0(t_1 - t_2), \qquad (52)$$

whereas for $\alpha = 1/2$ the following expression is valid:

$$\ln\frac{1 + \frac{3}{2}\tilde{A}\ln\frac{a_1}{a_0}}{1 + \frac{3}{2}\tilde{A}\ln\frac{a_2}{a_0}} = \frac{3}{2}\tilde{A}\Omega_{d,0}^{1/2}H_0(t_1 - t_2). \qquad (53)$$

The scenario $z_{EQ} > z_{D/A}$ ($a_{EQ} < a_{D/A}$) can be realized for the values $\alpha > 1$ and $\tilde{A} = A\rho_{d,0}^{\alpha} < 0$. For instance, for $\alpha = 2$ and $\tilde{A} = -0.27$ one obtains $z_{EQ} = 0.52$ ($a_{EQ}/a_0 = 0.66$) and $z_{D/A} = 0.42$ ($a_{D/A}/a_0 = 0.70$). However, in the parameter regime $\alpha > 1$ and $\tilde{A} < 0$, the dark energy model (50) is burdened with a following problem: for redshifts larger than some limiting redshift z_l (scale factor values smaller than some limiting scale value a_l), the dark energy is no longer a well-defined function of redshift (scale factor) (for the concrete values of $\alpha = 2$ and $\tilde{A} = -0.27$ we have $z_l = 2.44$ ($a_l/a_0 = 0.29$). Clearly, the realization of the studied scenario in the dark energy model (50) is physically acceptable only if the description given by (50) is valid up to some redshift $z_{\rm lim} < z_l$ (for values of the scale factor bigger than $a_{\rm lim} > a_l$). One theoretical possibility might be that the dark energy component (50) suddenly appears at the redshift $z_{\rm lim}$ (the scale value $a_{\rm lim}$), i.e. that it is negligible before that epoch.

Next we turn to the dark energy model with the following scaling of the dark energy density with redshift:

$$\rho_d = \rho_{d,0}\left[\frac{(1+z)^{3(1+w_1)/b} + C_2(1+z)^{3(1+w_2)/b}}{1 + C_2}\right]^b, \qquad (54)$$

$$\rho_d = \rho_{d,0}\left[\frac{(\frac{a}{a_0})^{-3(1+w_1)/b} + C_2(\frac{a}{a_0})^{-3(1+w_2)/b}}{1 + C_2}\right]^b, \qquad (55)$$

where $C_2 = (w_{d,0} - w_1)/(w_2 - w_{d,0})$. This dark energy model has an implicitly defined equation of state and it can also exhibit the phenomenon of the cosmological constant boundary crossing. The studied scenario may be realized in this dark energy model. For illustration purposes, for $w_1 = -0.2$, $w_2 = -1.3$, $w_{d,0} = -1.05$, and $b = 0.6$, we obtain $z_{EQ} = 0.41$ ($a_{EQ}/a_0 = 0.71$), $z_{D/A} = 0.35$ ($a_{D/A}/a_0 = 0.74$), and the CC boundary crossing at $z_{\rm cross} = 0.045$ ($a_{\rm cross}/a_0 = 0.96$).

The first example demonstrates how the studied scenario can be realized in a dark energy model with an explicitly defined equation of state of the type $p_d = -\rho_d - f(\rho_d)$, whereas the second example demonstrates that the behavior of interest can also be obtained in the dark energy models with the implicitly defined EOS of the type $F(p_d, \rho_d) = 0$ [11–13] (the EOS may contain also inhomogeneous terms [12–14]). These facts show that the above scenario can be realized in different classes of dark energy models (even in modified gravity as explicitly shown in [15]) as already studied in the general framework of the previous subsection.

C. Example of a dark energy parametrization

We further consider a parametrization of the redshift dependence which has recently attracted a lot of attention in the analysis of the observational data [16,17]

$$w_d(z) = w_0 + w_0'\frac{z}{1+z}. \qquad (56)$$

To find the part of the parametric space where our scenario is realized, we determine the boundary of that part from the condition $z_{EQ} = z_{D/A}$. We obtain

$$z_{EQ} = -\frac{2/3 + w_0}{2/3 + w_0 + w_0'}, \qquad (57)$$

which in combination with (38) and (56) leads to the condition on w_0 and w_0':

$$\frac{1-\Omega_m^0}{\Omega_m^0}\left(\frac{w_0'}{2/3 + w_0 + w_0'}\right)^{3(w_0+w_0')} e^{3w_0+2} = 1. \qquad (58)$$

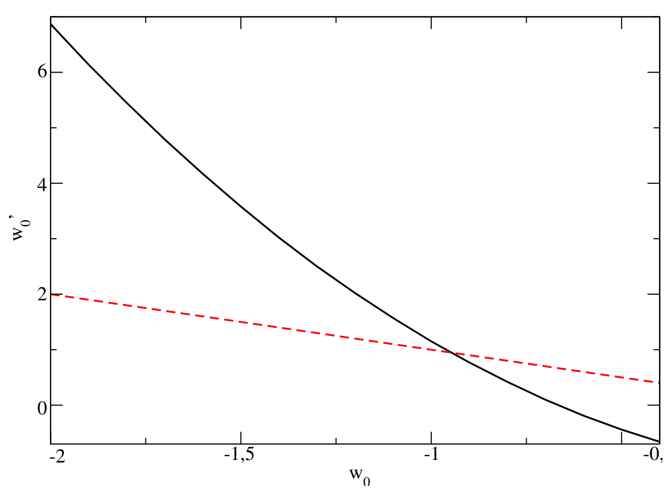

FIG. 1 (color online). The regions with $z_{EQ} > z_{D/A}$ (above the full line curve) and $w_0 + w_0' < 0$ (below the dashed line curve) for $\Omega_m^0 = 0.3$.

Solving this equation numerically, we obtain the boundary of the part of the parametric space where the studied scenario is realized. This boundary is depicted in Fig. 1. It is also possible to impose the condition $w_0 + w_0' < 0$ to obtain the part of the parametric space where our scenario is possible and the dark energy does not dominate at high redshifts, see Fig. 1. Thus, the principal possibility of sequence:matter-dominated and acceleration era is demonstrated for dark energy ideal fluid.

IV. THE RECONSTRUCTION OF DARK MATTER FROM SCALAR-TENSOR THEORY

The existence of the dark matter (for recent review from modified gravity point of view, see [18]) could be conjectured from the rotation curves of the galaxies and the formation of galaxy clusters. This means, if the dark matter is not some strange matter, but, for instance, modified gravity/scalar-tensor theory produces the dark matter, the Newton law should be modified at large (astrophysical) scales. Here as an example, we consider scalar-tensor gravity (1) or (9). It will be shown (by analogy with the method of second section) that for some scalar-tensor gravity with specific potentials the requested modification of Newton law could be achieved.

For the action (1), the Einstein equation has the following form:

$$0 = \frac{1}{2}\left\{\frac{R}{2\kappa^2} - \frac{1}{2}\omega(\phi)\partial_\rho\phi\partial^\rho\phi - V(\phi)\right\}g_{\mu\nu} - \frac{1}{2\kappa^2}R_{\mu\nu} + \frac{1}{2}\omega(\phi)\partial_\mu\phi\partial_\nu\phi, \tag{59}$$

and the field equation is

$$0 = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\omega(\phi)\partial^\mu\phi) - \frac{1}{2}\omega'(\phi)\partial_\rho\phi\partial^\rho\phi - V'(\phi). \tag{60}$$

We now assume a 4-dimensional spherically symmetric and static metric:

$$ds^2 = e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2\sum_{i,j=1,2}\tilde{g}_{ij}dx^i dx^j. \tag{61}$$

Here $\tilde{g}_{ij}$ is the metric of the unit sphere and ϕ only depends on r. Then the (t, t), (r, r), and (i, j) components of the Einstein equation (59) and the field equation (60) have the following forms:

$$0 = \frac{e^{2(\nu-\lambda)}}{2\kappa^2}\left(-\frac{2\lambda'}{r} + \frac{e^{2\lambda}-1}{r^2}\right) + \frac{1}{4}\omega(\phi)(\phi')^2 e^{2(\nu-\lambda)} + \frac{1}{2}V(\phi)e^{2\nu}, \tag{62}$$

$$0 = \frac{1}{2\kappa^2}\left(\frac{2\nu'}{r} + \frac{e^{2\lambda}-1}{r^2}\right) + \frac{1}{4}\omega(\phi)(\phi')^2 - \frac{1}{2}V(\phi)e^{2\lambda}, \tag{63}$$

$$0 = \frac{e^{-2\lambda}r^2}{2\kappa^2}\left(-\nu'' - (\nu'-\lambda')\nu' - \frac{\nu'-\lambda'}{r} + \frac{2(e^{2\lambda}-1)}{r^2}\right) + r^2\left(-\frac{1}{4}\omega(\phi)(\phi')^2 e^{-2\lambda} - \frac{1}{2}V(\phi)\right), \tag{64}$$

$$0 = e^{-2\lambda}\left\{\omega(\phi)\phi'' + \frac{1}{2}\frac{d\omega(\phi)}{d\phi}(\phi')^2 + \frac{2\omega(\phi)}{r}\phi' - 2\lambda'\omega(\phi)\phi'\right\} - \frac{dV(\phi)}{d\phi}. \tag{65}$$

By using the ambiguity of the redefinition of the scalar field ϕ, one may identify ϕ with the radial coordinate r:

$$\phi = r. \tag{66}$$

We should note that this is a mathematical trick. The scalar field should not be always identified with the radial coordinate r, but we should note that there is an ambiguity of the redefinition of ϕ like $\phi \to \tilde{\phi} = \Phi(\phi)$ by a proper function Φ. If we redefine $\omega(\phi)$ and $\tilde{V}(\tilde{\phi})$ by $\omega(\phi) \to \tilde{(}\,\tilde{\phi}) \equiv (\Phi'(\phi(\tilde{\phi})))^2\omega(\phi(\tilde{\phi}))$ and $\tilde{V}(\tilde{\phi}) \equiv V(\phi(\tilde{\phi}))$, the form of the action (1) is invariant. Then if the scalar field ϕ is not constant, at least locally, we may choose to identify the scalar field with r.

By choosing $\phi = r$, Eqs. (62)–(65) reduce to

$$0 = \frac{1}{2\kappa^2}\left(-\frac{2\lambda'}{r} + \frac{e^{2\lambda}-1}{r^2}\right) + \frac{1}{4}\omega(\phi) + \frac{1}{2}V(\phi)e^{2\lambda}, \tag{67}$$

$$0 = \frac{1}{2\kappa^2}\left(\frac{2\nu'}{r} + \frac{e^{2\lambda}-1}{r^2}\right) + \frac{1}{4}\omega(\phi) - \frac{1}{2}V(\phi)e^{2\lambda}, \tag{68}$$

$$0 = \frac{1}{2\kappa^2}\left(-\nu'' - (\nu'-\lambda')\nu' - \frac{\nu'-\lambda'}{r} + \frac{2(e^{2\lambda}-1)}{r^2}\right) - \frac{1}{4}\omega(\phi) - \frac{1}{2}V(\phi)e^{2\lambda}, \tag{69}$$

$$0 = \frac{1}{2}\frac{d\omega(\phi)}{d\phi} + \frac{2\omega(\phi)}{r} - 2\lambda'\omega(\phi) - e^{2\lambda}\frac{dV(\phi)}{d\phi}. \tag{70}$$

Equations (67)–(69) give

$$0 = \nu'' + (\nu'-\lambda')\nu' + \frac{\nu'+\lambda'}{r} - \frac{3(e^{2\lambda}-1)}{r^2} \tag{71}$$

$$\omega(\phi = r) = -\frac{4}{\kappa^2}\left\{\frac{\nu'-\lambda'}{r} + \frac{e^{2\lambda}-1}{r^2}\right\}, \tag{72}$$

$$V(\phi = r) = \frac{e^{-2\lambda}(\nu'+\lambda')}{\kappa^2 r}. \tag{73}$$

Let us assume there could be some desired form of $\nu(\lambda)$. Solving (71), one finds the form of $\lambda(\nu)$. Then by using Eqs. (72) and (73), we get the explicit form of $\omega(\phi)$ and $V(\phi)$, which generates the requested form of $\mu(\lambda)$ as a solution, as follows:

$$\omega(\phi) = -\frac{4}{\kappa^2}\left\{\frac{\nu'(\phi) - \lambda'(\phi)}{\phi} + \frac{e^{2\lambda(\phi)} - 1}{\phi^2}\right\}, \tag{74}$$

$$V(\phi) = \frac{e^{-2\lambda(\phi)}(\nu'(\phi) + \lambda'(\phi))}{\kappa^2 \phi}. \tag{75}$$

As an example, we consider the case

$$e^{-2\lambda} = 1 - \frac{M}{r} + \alpha r^{-\beta}. \tag{76}$$

where it is chosen $0 < \beta < 1$. If r is small, the last term can be neglected but if r is large, the last term dominates if compared with the second term. In the limit $r \to \infty$, $e^{-\lambda} \to 1$, which is necessary for the spacetime to be asymptotically flat. Let us consider the region where r is large. One can approximate (75) as

$$e^{-2\lambda} \sim 1 + \alpha r^{-\beta}. \tag{77}$$

Solving (71), we find

$$e^{2\nu} = 1 - \frac{\alpha(\beta + 6)}{\beta(\beta + 1)} r^{-\beta}, \tag{78}$$

which gives

$$\omega(\phi) = \omega_0 \phi^{-\beta-2}, \qquad \omega_0 \equiv -\frac{2\alpha\{-(\beta+1)^2 + 5\}}{\kappa^2(\beta+1)},$$
$$V(\phi) = V_0 \phi^{-\beta-2}, \qquad V_0 \equiv \frac{\alpha(\beta^2 - 6)}{2\kappa^2(\beta+1)}. \tag{79}$$

Equation (78) shows that, if the matter does not directly couple with the scalar field ϕ, the matter particle receives the potential proportional to $r^{-\beta}$, that is, the force whose strength is proportional to $r^{-1-\beta}$. As $0 < \beta < 1$, the force is stronger than the one produced by the Newton potential for large r. We should also note that, if $\alpha < 0$, the force is attractive. Hence, the above potential might explain the rotation curves of the galaxies and the formation of the galaxy clusters. It is remarkable that the above potential contains the powers of scalar in close analogy with cosmologically viable potential (17).

We should also note that ω_0 is positive if α is negative. Therefore $\omega(\phi)$ is positive since $\phi = r > 0$. Then by using (8), one finds

$$\varphi = -\frac{2\sqrt{\omega_0}}{\beta}\phi^{-(\beta/2)}, \tag{80}$$

which gives the potential in (10) as

$$V = V_0\left(-\frac{\beta\varphi}{2\sqrt{\omega_0}}\right)^{2(1+2/\beta)}. \tag{81}$$

As ϕ is always positive, φ is negative. Thus, using the method of Ref. [2] (second section) it is demonstrated that scalar-tensor theory with specific potential may produce the dark matter effect. It is remarkable that the corresponding reconstruction may be fulfilled for any requested modification of the Newton potential.

V. DISCUSSION

In summary, we demonstrated that the matter-dominated era may be combined with the acceleration era within some class of scalar-tensor theory. The explicit program of scalar-tensor theory reconstruction is presented. Clearly, this makes a nonrealistic single scalar-tensor theory with other types of potentials as dark energies due to appearance of cosmological bounds. Nevertheless, the situation may be improved for several scalars (several dark energy types) where the realization of sequence of matter-dominated and acceleration era should be investigated from the very beginning. This will be studied in another place.

The same problem is discussed for dark energy fluid in terms of redshift parametrization. It is shown that for some examples of EOS the matter domination-dark energy transition may occur prior to the deceleration-acceleration transition. Hence, intermediate and late-time stages of the universe may be unified also in frames of dark energy fluid. As a by-product, the reconstruction method which was used to get the form of scalar potential from cosmological bounds is applied for the same scalar-tensor theory to get the dark matter effect from it. The scalar-tensor theory with requested modified Newton law at large scales is constructed. Such a theory may explain the rotation curves of the galaxies and the formation of galaxy clusters. It is interesting that the obtained scalar potential seems to be qualitatively similar to the one obtained from the cosmological bounds.

It seems that we are still at the beginning of the road to a complete theory which unifies all the known epochs being compatible with astrophysical/cosmological data and solar system tests. Nevertheless, the fact that such unification is possible, at least for the intermediate-time and the late-time universe, is quite promising.

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