Jet quenching in a three-dimensional hydrodynamic medium

Thorsten Renk

Department of Physics, P.O. Box 35 FIN-40014, University of Jyväskylä, Jyväskylä, Finland and
Helsinki Institute of Physics, P.O. Box 64 FIN-00014, University of Helsinki, Helsinki, Finland

Jörg Ruppert

Department of Physics, McGill University, Montreal, Quebec, Canada H3A 2T8

Chiho Nonaka

Department of Physics, Nagoya University, Nagoya 464-8602, Japan

Steffen A. Bass

Department of Physics, Duke University, Durham, North Carolina 27708, USA

We study the radiative energy loss of hard partons in a soft medium in the multiple soft scattering approximation. The soft medium is described by a 3D hydrodynamical model and we treat the averaging over all possible parton paths through the medium without approximation. While the nuclear suppression factor R_{AA} does not reflect the high quality of the medium description (except in a reduced systematic uncertainty in extracting the quenching power of the medium), the hydrodynamical model also allows to study different centralities and in particular the angular variation of R_{AA} with respect to the reaction plane, allowing for a controlled variation of the in-medium path-length. We study the angular dependence of R_{AA} for different centralities, discuss the influence of hydrodynamical expansion and flow and comment on the comparison with preliminary data.

Experiments at the Relativistic Heavy Ion Collider (RHIC) have established a significant suppression of high-p_T hadrons produced in central $A + A$ collisions compared to those produced in peripheral $A + A$ or binary scaled $p + p$ reactions, indicating a strong nuclear medium effect [1,2]. The origin of this phenomenon, commonly referred to as *jet quenching*, can be understood in the following way: during the early preequilibrium stage of the relativistic heavy-ion collision, scattering of partons which leads to the formation of deconfined quark-gluon matter often occurs with large momentum transfers which leads to the formation of two back-to-back hard partons. These traverse the dense medium, losing energy and finally fragment into hadrons which are observed by the experiments. Within the framework of perturbative QCD, the leading process of energy loss of a fast parton is gluon radiation induced by elastic collisions of the leading parton or the radiated gluon with color charges in the quasithermal medium [3–5].

Over the past two years, a large amount of jet-quenching related experimental data has become available including but not limited to the nuclear modification factor R_{AA}, the elliptic flow v_2 at high p_T (as a measure of the azimuthal anisotropy of the jet cross section) and a whole array of high p_T hadron-hadron correlations. Computations of such jet modifications have acquired a certain level sophistication regarding the incorporation of the partonic processes involved. However, most of these calculations have been utilizing over-simplified models for the underlying soft medium, e.g., assuming a simple density distribution and its variation with time. Even in more elaborate setups, most jet quenching calculations assume merely a one-dimensional Bjorken expansion.

The availability of a three-dimensional hydrodynamic evolution code [6] allows for a much more detailed study of jet interactions in a longitudinally and transversely expanding medium. The variation of the gluon density in such a medium is very different from that in a simple Bjorken expansion. A previous calculation in this direction [7,8] estimated the effects of three-dimensional (3-D) expansion on the R_{AA}. However, this approach treated the energy loss of jets in a rather simplified and heuristic manner. Here, we shall perform a detailed investigation of the modification of jets in a three dimensionally expanding medium within a formalism of [9].

Relativistic fluid dynamics (RFD) (see, e.g., [10–12]) is ideally suited for the high-density phase of heavy-ion reactions at RHIC, but breaks down in the later, dilute, stages of the reaction when the mean free paths of the hadrons become large and flavor degrees of freedom are important. The biggest advantage of RFD is that it directly incorporates an equation of state as input and thus is so far the only dynamical model in which a phase transition can explicitly be incorporated. Starting point for a RFD calculation is the relativistic hydrodynamic equation

$$\partial_\mu T^{\mu\nu} = 0, \qquad (1)$$

where $T^{\mu\nu}$ is the energy momentum tensor which is given by

$$T^{\mu\nu} = (\epsilon + p)U^\mu U^\nu - pg^{\mu\nu}. \qquad (2)$$

Here ϵ, p, U, and $g^{\mu\nu}$ are energy density, pressure, four velocity and metric tensor, respectively. The relativistic hydrodynamic Eq. (1) is solved numerically using baryon number

n_B conservation

$$\partial_\mu(n_B(T, \mu)U^\mu) = 0. \tag{3}$$

as a constraint and closing the resulting set of partial differential equations by specifying an equation of state (EoS): $\epsilon = \epsilon(p)$. In the ideal fluid approximation (i.e., neglecting off-equilibrium effects) and once the initial conditions for the calculation have been fixed, the EoS is the *only* input to the equations of motion and relates directly to properties of the matter under consideration. Ideally, either the initial conditions or the EoS should be determined beforehand by an *ab initio* calculation (e.g., for the EoS via a lattice-gauge calculation), in which case a fit to the data would allow for the determination of the remaining quantity. Our particular RFD implementation utilizes a Lagrangian mesh and light-cone coordinates (τ, x, y, η) $(\tau = \sqrt{t^2 - z^2})$, in order to optimize the model for ultrarelativistic regime of heavy collisions at RHIC.

We assume that hydrodynamic expansion starts at $\tau_0 = 0.6$ fm. Initial energy density and baryon number density are parametrized by

$$\begin{aligned}\epsilon(x, y, \eta) &= \epsilon_{\max}W(x, y; b)H(\eta), \\ n_B(x, y, \eta) &= n_{B\max}W(x, y; b)H(\eta),\end{aligned} \tag{4}$$

where b and $\epsilon_{\max}$ $(n_{B\max})$ are the impact parameter and the maximum value of energy density (baryon number density), respectively. $W(x, y; b)$ is given by a combination of wounded nuclear model and binary collision model [13] and $H(\eta)$ is given by $H(\eta) = \exp[-(|\eta| - \eta_0)^2/2\sigma_\eta^2 \cdot \theta(|\eta| - \eta_0)]$. RFD has been very successful in describing single soft matter properties at RHIC, especially collective flow effects and particle spectra [6,14–16]. All parameters of our hydrodynamic evolution [6] have been fixed by a fit to the soft sector (elliptic flow, pseudorapidity distributions, and low-p_T single particle spectra), therefore providing us with a fully determined medium evolution for the hard probes to propagate through.

Let us now discuss the treatment of partons propagating through the medium: our calculation follows the BDMPS formalism for radiative energy loss [17] using quenching weights as introduced by Salgado and Wiedemann [9,18]. The probability density $P(x_0, y_0)$ for finding a hard vertex at the transverse position $\mathbf{r_0} = (x_0, y_0)$ and impact parameter $\mathbf{b}$ is given by the product of the nuclear profile functions as

$$P(x_0, y_0) = \frac{T_A(\mathbf{r_0}+\mathbf{b}/2)T_A(\mathbf{r_0}-\mathbf{b}/2)}{T_{AA}(\mathbf{b})}, \tag{5}$$

where the thickness function is given in terms of Woods-Saxon the nuclear density $\rho_A(\mathbf{r}, z)$ as $T_A(\mathbf{r}) = \int dz\rho_A(\mathbf{r}, z)$.

If we call the angle between outgoing parton and the reaction plane ϕ, the path of a given parton through the medium $\xi(\tau)$ is specified by $(\mathbf{r_0}, \phi)$ and we can compute the energy loss probability $P(\Delta E)_{\text{path}}$ for this path. We do this by evaluating

the line integrals

$$\begin{aligned}\omega_c(\mathbf{r_0}, \phi) &= \int_0^\infty d\xi\xi\hat{q}(\xi) \quad \text{and} \\ \langle\hat{q}L\rangle(\mathbf{r_0}, \phi) &= \int_0^\infty d\xi\hat{q}(\xi)\end{aligned} \tag{6}$$

along the path where we assume the relation

$$\hat{q}(\xi) = K \cdot 2 \cdot \epsilon^{3/4}(\xi) \tag{7}$$

between the local transport coefficient $\hat{q}(\xi)$ (specifying the quenching power of the medium) and energy density ϵ. Here, ω_c is the characteristic gluon frequency, setting the scale of the energy loss probability distribution and $\langle\hat{q}L\rangle$ is a measure of the path-length, weighted by the local quenching power. We view the parameter K as a tool to account for the uncertainty in the selection of α_s and possible nonperturbative effects increasing the quenching power of the medium (see discussion in [19]).

Using the numerical results of [9], we obtain $P(\Delta E)_{\text{path}}$ for ω_c and $R = 2\omega_c^2/\langle\hat{q}L\rangle$ as a function of jet production vertex and the angle ϕ (here R is a dimensionless quantity needed as input for the energy loss probability distributions as defined in [9]). The energy loss probability $P(\Delta E)_{\text{path}}$ is derived in the limit of infinite parton energy [9]. In order to account for the finite energy of the partons we truncate $P(\Delta E)$ at $\Delta E = E_{\text{jet}}$ and add $\delta(\Delta - E_{\text{jet}})\int_{E_{\text{jet}}}^\infty d\epsilon P(\epsilon)$. This procedure is known as nonreweighting [20]. We point out that the alternative concept of reweighting to our understanding systematically overestimates $P(\Delta E)$ for $\Delta E \ll E_{\text{jet}}$ and should be disregarded. In fact for a dense medium increasing $\hat{q}$ and employing reweighting leads to an increased escape probability whereas increasing $\hat{q}$ and nonreweighting leads to the expected decrease in escape probability, see also [21].

From the energy loss distribution given a single path, we can define the averaged energy loss probability distribution for a given angle ϕ as

$$\langle P(\Delta E)\rangle_\phi = \int_{-\infty}^\infty dx_0 \int_{-\infty}^\infty dy_0 P(x_0, y_0)P(\Delta E)_{\text{path}} \tag{8}$$

(this is conceptually similar to the angular averaged distribution $P(\Delta E)\rangle_{T_{AA}}$ introduced in [22] for central collisions).

We calculate the momentum spectrum of hard partons in leading order perturbative QCD (LO pQCD) (explicit expressions are given in [19] and references therein). The medium-modified perturbative production of hadrons at angle ϕ can then be computed from the expression

$$\begin{aligned}\frac{d\sigma_{\text{med}}^{AA\to h+X}}{d\phi} &= \sum_f \frac{d\sigma_{\text{vac}}^{AA\to f+X}}{d\phi} \\ &\quad \otimes \langle P(\Delta E)\rangle_\phi \otimes D_{f\to h}^{\text{vac}}\left(z, \mu_F^2\right)\end{aligned} \tag{9}$$

with $D_{f\to h}^{\text{vac}}(z, \mu_F^2)$ the fragmentation function with momentum fraction z at scale μ_F^2 [23], and from this we compute the nuclear modification function R_{AA} vs. reaction plane as

$$R_{AA}(p_T, y, \phi) = \frac{dN_{AA}^h/dP_T dy d\phi}{T_{AA}(\mathbf{b})d\sigma^{\text{pp}}/dP_T dy d\phi}. \tag{10}$$
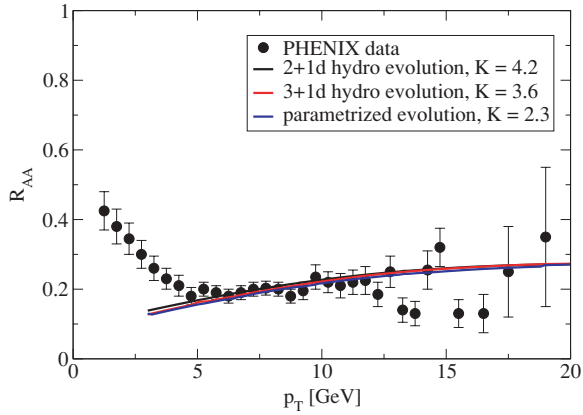
FIG. 1. (Color online) R_{AA} for central collisions as calculated in three different models for the medium evolution [6,24,25] with the overall quenching power scale K adjusted to data.

In [19,22] it has been shown that R_{AA} for central collisions only constrains a scale, but not the detailed functional form of $\langle P(\Delta E) \rangle_{T_{AA}}$. In the approach outlined above, this is manifest in the parameter K which we adjust to the data in central collisions. We illustrate in Fig. 1 that three different dynamical models, a two-dimensional (2-D) hydrodynamical evolution [24], the 3-D hydrodynamical evolution outlined above [6] and a parametrized fireball evolution [25] give almost equal descriptions of R_{AA} once the scale parameter is adjusted, albeit they require different values of K (the chief reason for this being the different longitudinal dynamics). All three dynamical models provide a successful description of the bulk properties of the medium at RHIC in central collisions [6,24,25]. The parametrized evolution is adjusted in such a way that it also describes the Hanbury Brown–Twiss (HBT) correlation measurements correctly. Given this successful description of measured observables within the three evolution models the ±50% spread in the values of K for the different models of the medium can therefore be interpreted as a measure for the systematic error inherent in the tomographic analysis of jet energy-loss via the nuclear modification function R_{AA}.

However, one may gain predictive power in going to collisions at finite impact parameter $\mathbf{b}$. The ϕ dependence of R_{AA} for noncentral collisions constitutes a systematic variation of path-length within a system with fixed overall scale. Hydrodyamical models as [7] are able to provide the best framework for studying collisions at finite impact parameter whereas the application of a parametrized evolution model as [25] to noncentral collisions encounters the difficulty of how to implement elliptic flow appropriately. In the following we therefore exclusively resort to the 3-D hydrodynamical evolution [6] which provides a very successful framework for the description bulk properties also for noncentral collisions.

The average path-length is expected to be smaller for a parton emitted in plane as compared to one emitted out of plane, and hence R_{AA} is expected to be larger at $\phi = 0$ than at $\phi = \pi/2$ with the difference in R_{AA} between these angles increasing with the initial asymmetry (and hence $\mathbf{b}$). Using a simple model for the time-evolution of the medium and
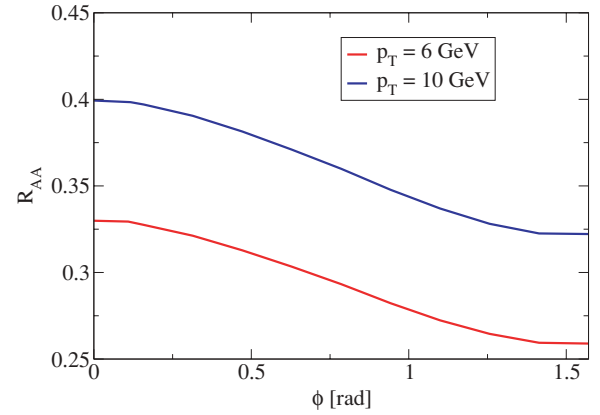

FIG. 2. (Color online) Angular dependence of R_{AA} for $\mathbf{b} = 7.5$ fm for two different values of p_T.

collective flow effects, it has been shown in [26] that the ϕ dependence of R_{AA} is quite sensitive to the initial gluon density distribution and temporal evolution of the medium.

Utilizing the previously discussed 3-D RFD model [6], we study the angular dependence of R_{AA} for two fixed values of p_T at $\mathbf{b} = 7.5$ fm in Fig. 2 and show the p_T dependence of R_{AA} for emission in plane and out of plane at three different impact parameters $\mathbf{b}$ in Fig. 3.

As expected, R_{AA} grows for more peripheral collisions as there is less soft matter produced to induce energy loss. Moreover, there is a smooth angular variation of R_{AA} observed, reflecting the underlying medium asymmetry. The difference between in-plane and out of plane emission grows with impact parameter, at $\mathbf{b} = 2.4$ fm there is hardly angular variation whereas at 7.5 fm differences are of order 20% (see Fig. 4).

It is clear from the observation of elliptic flow that the pressure of the hydrodynamical fluid tends to remove the initial spatial asymmetry from the almond-shaped overlap region in noncentral collisions. Thus, on general grounds, we may expect that in a dynamical model for the medium, the difference between in plane and out of plane emission is less pronounced than in an estimate using a static medium distributed according to Eq. (5). On the other hand, the main
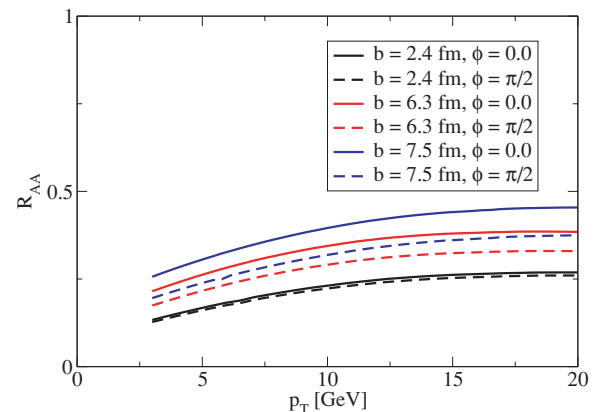

FIG. 3. (Color online) p_T dependence of R_{AA} in plane (solid) and out of plane (dashed) emission at different values of impact parameter $\mathbf{b}$.
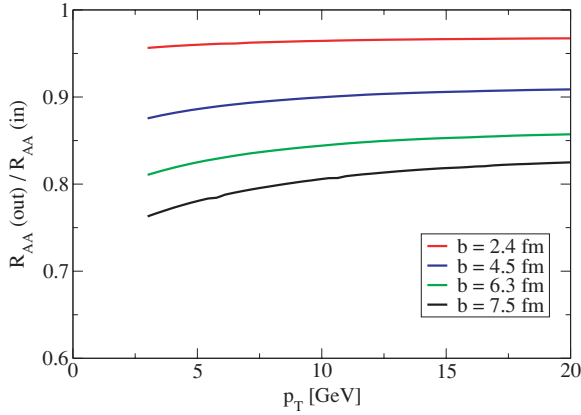
FIG. 4. (Color online) Ratio of R_{AA} for out of plane vs. in plane emission as a function of p_T for four different values of the impact parameter **b**.

fraction of observed hadrons arises from vertices close to the medium surface and the expected energy loss per unit time $\frac{d\Delta E}{d\tau}$ reached a peak value early in the evolution [19], thus it is reasonable to expect that a large number of partons escapes the medium before the spatial asymmetry is completely removed. We investigate this competition of timescales in Fig. 5 where we make the comparison with a scenario in which we keep the medium static at its initial value.

Not altering the value $K = 3.6$ for the quenching scale, keeping the energy density at its value in the initial state vastly overpredicts the quenching power of the medium, leading to R_{AA} of the order of 0.25 for **b** = 7.5 fm. Thus, we have to readjust K to account for the (unphysical) fact that we keep the medium static. $K = 0.65$ leads to a good description for central collisions, and employing this value also at **b** = 7.5 fm allows a fair assessment of the effect of the hydrodynamical evolution. The azimuthal dependence of R_{AA} in the BDMPS-formalism without considering expansion effects using a fixed energy density profile has also been studied in [27].
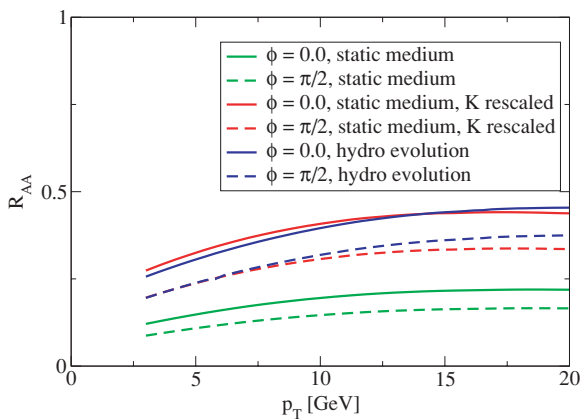

FIG. 5. (Color online) R_{AA} for in plane and out of plane emission as a function of p_T at **b** = 7.5 fm, assuming a static medium given by the hydrodynamical initial state, a static medium with readjusted quenching power given by the hydrodynamical initial state and the full hydrodynamical evolution.
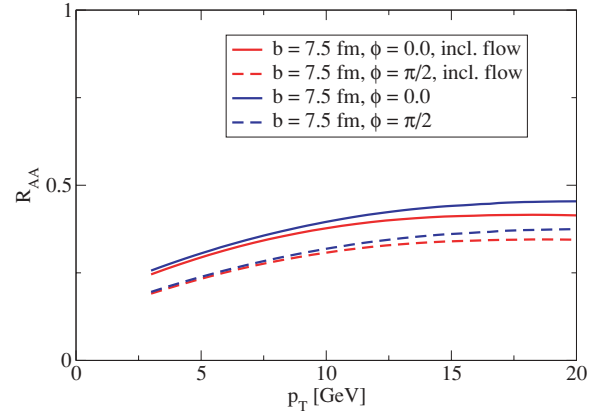

FIG. 6. (Color online) R_{AA} for in plane and out of plane emission as a function of p_T for **b** = 7.5 fm, comparing the result with and without explicit dependence on flow.

One finds that indeed the difference between in plane and out-of plane emission is reduced by some 7% when the hydrodynamical evolution is taken into account properly. In addition, a small change of the shape of the distribution with p_T is induced. Thus, the model shows sensitivity to the size of the spatial asymmetry in the distribution of matter and the timescale at which it is removed. This complements the information found in low p_T v_2 which reflects the asymmetry in momentum space.

However, jet energy loss can also couple to collective flow [28,29], thus potentially blurring the relation to the spatial asymmetry. In order to assess this effect, we implement the effect of flow based on an expression based on the definition of $\hat{q}$ in AdS/CFT. We scale

$$\hat{q}' = \hat{q}(\cosh\rho - \sinh\rho \cos\alpha), \quad (11)$$

where α is the angle between flow and hard parton trajectory and ρ is the flow rapidity [30]. This expression has a straightforward interpretation in terms of the density of scattering partners seen by the hard parton per unit time/length from the c.m. frame. The resulting effect on R_{AA} at **b** = 7.5 fm is shown in Fig. 6. Note that the scenario with transverse flow requires an increase of K by about 30% to describe R_{AA} in central collisions, in agreement with [31] transverse flow indeed decreases the quenching power.

Transverse flow is in general weak during the timescale of jet quenching (i.e., during the early evolution), thus there is no pronounced difference between the curves. While we observe some change in shape, there is no significant change in the asymmetry between in plane and out of plane emission.

Let us now make a rough comparison with the preliminary PHENIX data [32] (since systematic errors due to the reaction plane resolution are large, we refrain from doing a detailed comparison at this point). While the measurement appears to be consistent within errors with the calculation for $p_T > 6$ GeV, the data show a trend toward a greater angular spread between in plane and out of plane emission than found in the calculation. If this trend is confirmed, it would indicate that parametrically the path-length dependence of energy loss is larger than L^2 and would clearly rule out a linear dependence on L (as expected,

e.g., for elastic collisions as the dominant mechanism for energy loss). While the L^2 dependence can be understood from coherence length phenomena (see, e.g., [9]), a dependence on a higher power of L has so far no theoretical explanation. The apparent disagreement between calculation and data for p_T below 6 GeV is yet another hint that fragmentation is not the dominant mechanism for the production of hadrons in this momentum regime and other mechanisms, such as parton recombination [33] are of increasing importance for lower p_T.

In summary, we have discussed jet energy loss within a 3-D hydrodynamical description of the medium. We have made no approximation in computing the average of the energy loss probability $\langle P(\Delta E)\rangle_\phi$ over all possible paths through the medium (in particular, we have not tried to solve the problem by identifying a typical path).

Since the nuclear suppression factor R_{AA} is not sensitive to the detailed form of $\langle P(\Delta E)\rangle_\phi$ as long as the quenching scale is adjusted correctly, using a hydrodynamical medium does not improve the quality of the description of R_{AA} per se. However, once we fix the quenching scale to R_{AA} in central collisions, we gain predictive power when going to more peripheral collisions. In particular, studying the angular dependence of R_{AA} with respect to the reaction plane gives systematic control over the average in-medium path-length.

As expected, the angular dependence of R_{AA} reflects the spatially asymmetric distribution of soft matter. We also observe that the fact that the hydrodynamical evolution removes the asymmetry results in a reduction of the angular spread as compared to a situation in which the asymmetry is kept unchanged. We have also gauged the potential impact of the flow field on the results. The calculation describes preliminary data within errors, however there is a trend that the data show larger angular splitting than the calculation, indicating that the energy loss may scale parametrically with a larger power of the path-length than L^2. It remains to be seen if this trend can be confirmed.

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