Gait Generation for Passive Running via Iterative Learning Control

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***Abstract*— This paper proposes a novel framework to generate optimal passive gait trajectories for a planar one-legged hopping robot via iterative learning control. The proposed method utilizes variational symmetry of the plant model in executing the steepest decent method in the learning algorithm. This allows one to obtain solutions of a class of optimal control problems without using precise knowledge of the plant model. Furthermore, its application to a hopping robot produces a passive running gait trajectory with zero input. Some numerical examples demonstrate the effectiveness of the proposed method.**

I. Introduction

In the last decade, the gait generation problem has been well studied and experimentally demonstrated. McGeer's *passive dynamic walking* [1] shows that a suitably designed biped robot can walk down a gentle slope with no control input and generates a stable periodic gait. This inspires many researchers to work on an optimization problem of gaits with respect to the energy consumption. Behavior analysis of passive walkers were investigated, e.g, in [2], [3]. There are some results on gait generation based on passive dynamic walking [4], [5], [6] by designing appropriate feedback control systems such that the closed loop systems behave like passive walkers. Thompson and Raibert show that a spring-driven one-legged hopping robot can hop with zero control input under appropriate initial conditions in [7]. This implies that the hopping robot can be regarded as a passive walker walking on a horizontal plane. We proposed an adaptive control system for this robot to achieve a passive walking gait, that is, a walking gait with zero input is achieved [8].

Our objective is to generate optimal walking gait trajectories for this hopping robot via iterative learning control. Our approach is robust over modeling error since it does not require precise knowledge of the plant model. Here, we formulate an optimal control type cost function and try to find a control input minimizing it by iterative learning technique based on *variational symmetry* of Hamiltonian control systems [9], which can solve a class of optimal control problems by iteration of experiments. For this purpose, two novel techniques with respect to the iterative learning control are proposed in the authors' preliminary result [10]: One is a technique to take the time derivatives of the output signal into account in the iterative learning control by employing a pseudo adjoint of the time derivative operator. The other is a cost function to achieve time symmetric gait trajectories to guarantee stable walking without a fall. Although we succeed in generating *sub-optimal* gait trajectories minimizing the L_2 norm of the control input, they are *not optimal* in a sense that the running gaits with zero input are not obtained. This is because the algorithm proposed in [10] can not take into account the variation of the initial condition.

In this paper, we propose a novel algorithm to generate *optimal* gait trajectories by employing an update law for the initial conditions as well as that for the feedforward input. This learning framework generates passive walking gaits by iteration of experiments without using precise knowledge of the plant model. Furthermore, the proposed scheme is applied to the hopping robot in [11] and the corresponding numerical simulations demonstrate its advantage.

II. Iterative learning control based on variational symmetry

This section refers to the iterative learning control (ILC) method based on variational symmetry in [9] briefly.

A. Variational symmetry of Hamiltonian systems

Consider a Hamiltonian system with dissipation Σ with a controlled Hamiltonian $H(x, u, t)$ described as $(x^1, y) = \Sigma(x^0, u)$:

$$\left\{ \begin{array}{rcl} \dot{x} &=& (J-R)\,\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}}, \quad x(t^0) = x^0 \\ y &=& -\dfrac{\partial H(x,u,t)}{\partial u}^{\mathrm{T}} \\ x^1 &=& x(t^1) \end{array} \right. . \quad (1)$$

Here $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^r$ describe the state, the input and the output, respectively. The structure matrix $J \in \mathbb{R}^{n\times n}$ and the dissipation matrix $R \in \mathbb{R}^{n\times n}$ are skew-symmetric and symmetric positive semi-definite, respectively. The matrix R represents dissipative elements. For this system, the following theorem holds. This property is called *variational symmetry* of Hamiltonian control systems.

Theorem 1: [9] Consider the Hamiltonian system in (1). Suppose that J and R are constant and that there exists a nonsingular matrix $T \in \mathbb{R}^{n\times n}$ satisfying

$$\begin{aligned} J &= -TJ\,T^{-1} \qquad R = TR\,T^{-1} \\ \frac{\partial^2 H(x,u,t)}{\partial(x,u)^2} &= \begin{pmatrix} T & 0 \\ 0 & I \end{pmatrix} \frac{\partial^2 H(x,u,t)}{\partial(x,u)^2} \begin{pmatrix} T^{-1} & 0 \\ 0 & I \end{pmatrix}. \end{aligned} \tag{2}$$

Then the Fréchet derivative of Σ is described by another linear Hamiltonian system $(x_v^1, y_v) = \mathrm{d}\Sigma((x^0,u),(x_v^0,u_v))$:

$$\begin{cases} \dot{x} = (J-R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}}, & x(t^0) = x^0 \\ \dot{x}_v = (J-R)\dfrac{\partial H_v(x,u,x_v,u_v,t)}{\partial x_v}^{\mathrm{T}}, & x_v(t^0) = x_v^0 \\ y_v = -\dfrac{\partial H_v(x,u,x_v,u_v,t)}{\partial u_v}^{\mathrm{T}} \\ x_v^1 = x_v(t^1) \end{cases} \tag{3}$$

with a controlled Hamiltonian $H_v(x,u,x_v,u_v,t)$

$$H_v(x,u,x_v,u_v,t) = \frac{1}{2}\begin{pmatrix} x_v \\ u_v \end{pmatrix}^{\mathrm{T}} \frac{\partial^2 H(x,u,t)}{\partial(x,u)^2} \begin{pmatrix} x_v \\ u_v \end{pmatrix}.$$

Furthermore, the adjoint of the variational system with zero initial state $u_a \mapsto y_a = (\mathrm{d}\Sigma^{x^0}(u))^*(u_a)$ is given by

$$\begin{cases} \dot{x} &= (J-R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}} \\ \dot{\bar{x}}_v &= -(J-R)\dfrac{\partial H_v(x,u,\bar{x}_v,u_a,t)}{\partial \bar{x}_v}^{\mathrm{T}} \\ y_a &= -\dfrac{\partial H_v(x,u,\bar{x}_v,u_a,t)}{\partial u_a}^{\mathrm{T}} \end{cases} \tag{4}$$

with the initial state $x(t^0) = x^0$ and the terminal state $\bar{x}_v(t^1) = 0$. Suppose moreover that $J-R$ is nonsingular. Then the adjoint $(x_a^1, u_a) \mapsto (x_a^0, y_a) = (\mathrm{d}\Sigma(x^0,u))^*(x_a^1,u_a)$ is given by the same state-space realization (4) with the initial state $x(t^0) = x^0$ and the terminal state $\bar{x}_v(t^1) = -(J-R)T\ x_a^1$ and $x_a^0 = -T^{-1}(J-R)^{-1}\bar{x}_v(t^0)$.

Remark 1: This theorem reveals that the variational system and its adjoint of a Hamiltonian system in the form (1) have almost the same state-space realizations. This means that the input-output mapping of the adjoint can be calculated by the input-output data of the original system as

$$\mathcal{R}\circ(\mathrm{d}\Sigma(u))^*\circ\mathcal{R}(v) = \mathrm{d}\Sigma(\bar{u})(v) \approx \Sigma(\bar{u}+v) - \Sigma(\bar{u}) \tag{5}$$

provided appropriate boundary conditions are selected, where $\mathcal{R}$ is the time reversal operator defined by

$$\mathcal{R}(u)(t-t^0) = u(t^1-t), \quad \forall t \in [t^0,t^1]. \tag{6}$$

This property is utilized for solving the optimal control problems in which the adjoint operator plays an important role.

B. Optimal control via iterative learning

Let us consider the system $\Sigma : X\times U \to X\times Y$ in (1) and a cost function $\Gamma : X^2\times U\times Y \to \mathbb{R}$ with Hillbert spaces X, U and Y. Typically, $X = \mathbb{R}^n$, $U = L_2^m[t^0,t^1]$ and $Y = L_2^r[t^0,t^1]$. The objective is to find the optimal input minimizing the cost function $\Gamma(x^0,u,x^1,y)$. Note that the Fréchet derivative of $\Gamma(x^0,u,x^1,y)$ is $\mathrm{d}\Gamma(x^0,u,x^1,y)$. Here we can calculate

$$\begin{aligned} &\mathrm{d}\left(\Gamma((x^0,u),\Sigma(x^0,u))\right)(\mathrm{d}x^0,\mathrm{d}u) \\ &= \mathrm{d}\Gamma((x^0,u),\Sigma(x^0,u))\left((\mathrm{d}x^0,\mathrm{d}u),\mathrm{d}\Sigma(x^0,u)(\mathrm{d}x^0,\mathrm{d}u)\right) \\ &= \langle \Gamma'((x^0,u),\Sigma(x^0,u)), \begin{pmatrix} \mathrm{id}_{X\times U} \\ \mathrm{d}\Sigma(x^0,u) \end{pmatrix} (\mathrm{d}x^0,\mathrm{d}u)\rangle_{X^2\times U\times Y} \\ &= \langle (\mathrm{id}_{X\times U},(\mathrm{d}\Sigma(x^0,u))^*)\,\Gamma'(x^0,u,x^1,y),(\mathrm{d}x^0,\mathrm{d}u)\rangle_{X\times U} \end{aligned} \tag{7}$$

Well-known Riesz's representation theorem guarantees that there exists an operator $\Gamma'(x^0,u,x^1,y)$ as above. Therefore, if the adjoint $(\mathrm{d}\Sigma(x^0,u))^*$ is available, we can reduce the cost function Γ down at least to a local minimum by an iteration law with a $K_{(i)} > 0$.

$$\begin{aligned} u_{(i+1)} = \ & u_{(i)} - K_{(i)}\,(0_{UX},\mathrm{id}_U)\left(\mathrm{id}_{X\times U},\ (\mathrm{d}\Sigma(x_{(i)}^0,u_{(i)}))^*\right) \\ & \times\Gamma'(x_{(i)}^0,u_{(i)},x_{(i)}^1,y_{(i)}) \end{aligned} \tag{8}$$

Here i denotes the i-th iteration in laboratory experiment.

The results in the previous section enable one to execute this procedure without using the parameters of the original operator Σ by the relation (5), provided Σ is a Hamiltonian system and the boundary conditions are selected appropriately.

III. EXTENSION OF ILC FOR TIME DERIVATIVES

There is a constraint with respect to cost functions in the iterative learning control method in [9]. For the system Σ in (1), the output y is uniquely defined by the definition of the input u. The possible choice of the optimal control type cost function used in the iterative learning control is a functional of u and y, and it is not possible to choose a functional of $\dot{y}$ the time derivative of the output. However, the signal $\dot{y}$ often plays an important role in control systems and, particularly, it is important to check the behavior of $\dot{y}$ for the gait trajectory generation problem. In this section, we extend the iterative learning control method referred in the previous section to take the time derivative $\dot{y}$ into account.

A. Adjoint of the time derivative operator

Here we investigate the adjoint of the time derivative operator to take account of the time derivative of the output signal $\dot{y}$ in the iterative learning control procedure. Consider a differentiable signal $\xi \in L_2[t^0,t^1]$ and an operator $D(\cdot)$ which maps the signal $\xi(t)$ into its time derivative is defined as the time derivative operator.

$$D(\xi)(t) := \frac{\mathrm{d}\xi(t)}{\mathrm{d}t} \tag{9}$$

Let us provide the following lemma to define the adjoint of the time derivative operator.

Lemma 1: *[10] Consider the signal $\xi(t)$ defined above and another differentiable signal $\eta \in L_2[t^0, t^1]$. Suppose that the signal $\xi(t)$ satisfies the following condition*

$$\xi(t^0) = \xi(t^1) = 0. \tag{10}$$

Then the following equation holds.

$$\langle \eta, D(\xi) \rangle_{L_2} = \langle -D(\eta), \xi \rangle_{L_2} \tag{11}$$

Proof: See [10]. ■

This lemma implies

$$D^* = -D$$

for a certain class of input signals.

B. Application to the iterative learning control

Here we take the following cost function $\Gamma(\dot{y})$ to illustrate the proposed method

$$\Gamma(\dot{y}) = \frac{1}{2} \int_{t^0}^{t^1} \Big((\dot{y}(t) - \dot{y}^d(t))^{\mathrm{T}} \Lambda_{\dot{y}} (\dot{y}(t) - \dot{y}^d(t)) \Big) \, \mathrm{d}t. \tag{12}$$

Here $\dot{y}^d$ is a differentiable signal as a desired velocity satisfying $\dot{y}^d \in L_2^r[t^0, t^1]$ and $\Lambda_{\dot{y}} \in \mathbb{R}^{r \times r}$ is a positive definite matrix. Let us consider the Hamiltonian system in (1) and suppose that the following assumption holds.

Assumption 1: Following conditions always hold
$\mathrm{d}y(t^0) = 0$ and $\mathrm{d}y(t^1) = 0$.
Under this assumption, we will derive an iterative learning control method which can handle the time derivative of the output function $\dot{y}$ In the iterative learning control, it is assumed that all the initial conditions are same in each laboratory experiment in general. Therefore the condition $\mathrm{d}y(t^0) = 0$ always holds. But the other one $\mathrm{d}y(t^1) = 0$ does not always hold. In order to let the latter condition $\mathrm{d}y(t^1) = 0$ hold approximately, we can employ an optimal control type cost function such as $\int_{t^1-\epsilon}^{t^1} \|y(t) - y^d(t)\|^2 \mathrm{d}t$ with a small constant $\epsilon > 0$ as in [12].

Suppose that the output y fulfills Assumption 1. Then we have

$$\mathrm{d}(\Gamma(\dot{y})) = \langle \Lambda_{\dot{y}}(\dot{y} - \dot{y}^d), \ \mathrm{d}\dot{y} \rangle_{L_2} \tag{13}$$

The authors' former result [9] can not directly apply to this cost function (12) because it contains $\dot{y}$. Here let us rewrite $\dot{y}$ as $\dot{y} = D(y)$ with the time derivative operator $D(\cdot)$ defined by Equation (9). Then we have

$$\mathrm{d}\dot{y} = \mathrm{d}D(y)(\mathrm{d}y). \tag{14}$$

Note that the time derivative operator is linear, we obtain $\mathrm{d}\dot{y} = D(\mathrm{d}y)$. Assumption 1 and (11) imply

$$\begin{aligned} \mathrm{d}(\Gamma(\dot{y})) &= \langle \Lambda_{\dot{y}}(\dot{y} - \dot{y}^d), D(\mathrm{d}y) \rangle_{L_2} \\ &= \langle -D\Big(\Lambda_{\dot{y}}(\dot{y} - \dot{y}^d)\Big), \ \mathrm{d}y \rangle_{L_2} \\ &= \langle (\mathrm{d}\Sigma(u))^* \Big(-D(\Lambda_{\dot{y}}(\dot{y} - \dot{y}^d)) \Big), \ \mathrm{d}u \rangle_{L_2} \end{aligned} \tag{15}$$

As we mentioned above, this method allows one to obtain an iterative learning control method which a cost function consisting of $\dot{y}$.

TABLE I

PARAMETERS

notation	Meaning	Unit
r_0	natural leg length	m
m	total mass	kg
g	gravity acceleration	$\mathrm{m/s^2}$
K_l	leg spring stiffness	$\mathrm{kgm^2}$
K_h	hip spring stiffness	$\mathrm{kgm^2}$
T_s	stance time	s
T_f	flight time	s

IV. OPTIMAL GAIT GENERATION

In this section, a cost function to generate a symmetric gait is proposed for guaranteeing stable running of a hopping robot without a fall. The iterative learning control with respect to this cost function will generate a passive running gait.

A. Description of the plant

Let us consider a passive running robot in [11], [8] depicted in Figure 1.

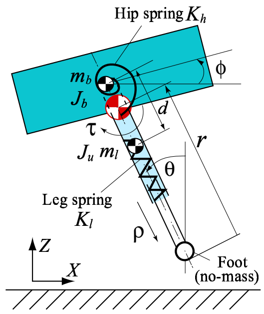

Fig. 1. Description of the plant

Here the body and the leg have mass m_b and m_l and moment of inertia J_b and equivalent leg inertia J_l respectively. Let us also define the control force of the leg ρ and the control torque of the hip joint τ. Table I shows the physical parameters. See [8] for more detail.

Here the stance time represents the time interval during the stance phase and the flight time is defined in a similar way. Furthermore, we suppose the following assumption.

Assumption 2: The foot does not bounce back nor slip on the ground (inelastic impulsive impact).
One locomotion cycle is illustrated in Figure 2. It consists of the *stance phase*, where the leg touches the ground and the leg spring is compressed, and the *flight phase*, where the leg is above the ground and the robot traverses a ballistic trajectory.

In the stance phase, let us define the generalized coordinate $q := (r, \theta, \phi)^{\mathrm{T}} \in \mathbb{R} \times \mathbb{S} \times \mathbb{S}$, the generalized momentum $p := (p_r, p_\theta, p_\phi)^{\mathrm{T}} \in \mathbb{R}^3$, input $u := (\rho, \tau)^{\mathrm{T}} \in \mathbb{R}^2$ and the inertia matrix $M(q) \in \mathbb{R}^{3\times3}$. Then, the dynamics of this robot is described by a Hamiltonian system in (1) with the Hamiltonian

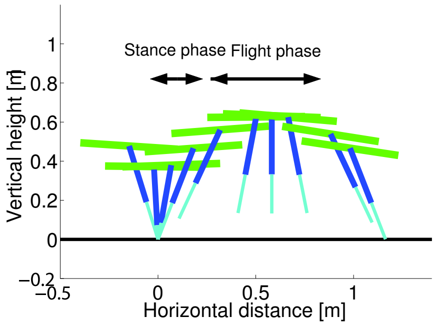

Fig. 2. Locomotion phases during one cycle

$$H(q,p,u) = \frac{1}{2}p^{\mathrm{T}}M(q)^{-1}p - mgr(1-\cos\theta)$$
$$+\frac{1}{2}K_l(r-r_0)^2 + \frac{1}{2}K_h(\theta-\phi)^2 - u^{\mathrm{T}}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & -1\end{pmatrix}q \quad (16)$$

Let us consider the dynamics in the flight phase as below.

$$\begin{cases} \ddot{x} = 0 \\ \ddot{z} = -g \\ J_l\ddot{\theta} + J_b\ddot{\phi} = 0 \\ J_b\ddot{\phi} = K_h(\theta-\phi) - \tau_f \end{cases} \quad (17)$$

Here the variables x and z represent the horizontal and vertical positions of the center of mass and τ_f is the control torque.

B. Generation of optimal symmetric gait via iterative learning control

This section sets the control problem to get the periodic gait based on [13].

The leg angle θ is the most important state variable in controlling running gait, because it has a direct effect to avoid falling. However, it is difficult to control the variable θ in the stance phase, since this robot has no foot and ankle torque is not available. Therefore as in [13], we apply zero input in the stance phase, and try to control the variable θ in the flight phase to get the periodic gait.

We let $t^0 = 0$ for simplicity in what follows. Let us define the desired values of θ and $\dot{\theta}$ as

$$\theta^d \;:=\; \theta\big|_{t=T_s+T_f} = -\theta\big|_{t=T_s} \quad (18)$$
$$\dot{\theta}^d \;:=\; \dot{\theta}\big|_{t=T_s+T_f} = \dot{\theta}\big|_{t=T_s}. \quad (19)$$

As for the model mentioned above, energy dissipation occurs at the touchdown. Let E_- and E_+ represent the energies just before the touchdown and just after the touchdown. Then the variation of the energy between them ΔE can be calculated as below from [8]

$$\Delta E := E_+ - E_- = -\frac{mJ_l}{2(J_l+mr_0^2)}\mu_-^2 \quad (20)$$

where μ_- is defined as follows and is called the energy dissipation coefficient.

$$\mu_- := v_{x-}cos\theta_- + v_{z-}sin\theta_- + \frac{r_0}{J_l+mr_0^2}p_{\theta-} \quad (21)$$

Here v_{x-} and v_{z-} represent the velocity of the center of mass. Suppose that the condition (22) holds at the touchdown

$$\Delta E = 0. \quad (22)$$

This implies that there is no energy transfer except for the control input. If the total mechanical energy is completely preserved, it is expected that a periodic gait trajectories are autonomously generated. In fact, [8] implies that the condition (22) is satisfied if the control objects (18) and (19) are achieved. The initial condition is appropriately chosen according to [8].

In [8], dead-beat control is applied to this problem. It works well, but it requires precise knowledge of the plant system. Here we try to use iterative learning control based on variational symmetry with a special cost function given as follows. It will generate an optimal flow in the flight phase without using precise knowledge of the system.

Now we propose a cost function as

$$\Gamma(\theta,\dot{\theta},u) := \frac{K_\theta}{2}\|\theta-(-\mathcal{R}(\theta))\|_{L_2}^2 \;+\; \frac{K_{\dot{\theta}}}{2}\|\dot{\theta}-\mathcal{R}(\dot{\theta})\|_{L_2}^2 \;+\; \frac{K_u}{2}\|u\|_{L_2}^2 \quad (23)$$

where K_θ, $K_{\dot{\theta}}$ and K_u represent appropriate positive constants. $\mathcal{R}$ is the time-reversal operator as defined in (6). The first term in the right hand side of (23) makes Assumption 1 approximately hold. It is expected that we can generate an optimal trajectory such that it satisfies (18) and (19) while minimizing the L_2 norm of the control input. Furthermore, there is no energy transfer except for the control input.

Let us recall the fact that gait trajectories are essentially periodic, however ILC can not generate a periodic trajectory. Let us connect the stance flow and that generated from (23). Take this connected trajectory as an one period of a periodic gait trajectory. Therefore if (22) holds, we can generate an optimal periodic trajectory.

Now, let us define the input $u = \tau_f$ and rewrite the dynamics of θ in the flight phase. Under the initial condition selected as in [8], we can remove the dynamics of θ from the dynamics in (17) as below

$$\begin{pmatrix}\dot{q}_\theta \\ \dot{p}_\theta\end{pmatrix} = \begin{pmatrix}0 & I \\ -I & 0\end{pmatrix}\begin{pmatrix}\frac{\partial H(q_\theta,p_\theta,u)}{\partial q_\theta}^{\mathrm{T}} \\ \frac{\partial H(q_\theta,p_\theta,u)}{\partial p_\theta}^{\mathrm{T}}\end{pmatrix} \quad (24)$$
$$y = -\frac{\partial H(q_\theta,p_\theta,u)}{\partial u}^{\mathrm{T}} = q_\theta = \theta,$$

where $q_\theta := \theta$, $p_\theta := J_l\dot{\theta}$ and a controlled Hamiltonian

$$H(q_\theta,p_\theta,u) := \frac{1}{2J_l}p_\theta^2 + \frac{1}{2}\frac{K_h(J_b+J_l)}{J_b}q_\theta^2 - q_\theta u.$$

Calculate the iteration law as in (8) provided that the initial input $u_{(0)}$ is equivalent to zero. The pair of iteration laws (25) and (26) implies that this learning procedure needs two steps laboratory experiments, that is, it requires two experiments to execute a single update step in the steepest decent method. In the ($2i$-1)-th iteration, we can get the output signal of $\Sigma(\bar{u}+v)$ in (5) and then can calculate the input and output signals of $(\,\mathrm{d}\Sigma(u))^*$ from (5). The input for the $2i$-th iteration is generated by (8) with these signals. (Some details are omitted due to limitations of space.)

$$
\begin{aligned}
u_{(2i-1)} &= u_{(2i-2)} + 2\mathcal{R}\Big(K_{\theta}(\mathrm{id}+\mathcal{R})(y_{(2i-2)}) \\
&+ K_{\dot\theta}(\mathrm{id}-\mathcal{R})(\ddot y_{(2i-2)})\Big) \qquad (25)\\
u_{(2i)} &= (\mathrm{id}-K_{(2i-2)}K_u)u_{(2i-2)} \\
&- K_{(2i-2)}\mathcal{R}\Big(y_{(2i-1)} - y_{(2i-2)}\Big) \qquad (26)
\end{aligned}
$$

C. Generation of passive running gait

The iteration procedure mentioned in the previous section can generate a *sub-optimal* symmetric gait minimizing the L_2 norm of the control input. But this gait does not generate an *optimal* one in a sense that the corresponding input does not coincide with zero. Under certain initial conditions, the passive running robot depicted in Figure 1 can run with zero input [7]. In this section, we derive a novel update law for the initial condition to generate an *optimal* passive gait.

Since the state space is 6 dimensional, the initial condition to be determined is $(\dot x^0, \dot z^0, \theta^0, \dot\theta^0, \phi^0, \dot\phi^0)$. Here we select $\dot x^0$ and θ^0 as free parameters and let the rest $(\dot z^0, \dot\theta^0, \phi^0, \dot\phi^0)$ be calculated according to $\dot x^0$ and θ^0 as in [8]. In this section, let us update one of the free parameters θ^0 as well as the control input u to achieve the optimal initial conditions under which passive running gait is generated. The other free parameter $\dot x^0$ is not determined, so we can select the horizontal velocity of passive running gait by choosing it appropriately. A more detailed procedure is explained below. Here let ${X^0}_{(j)} \in \mathbb{R}^6$ denote the initial condition in $(2j-1)$-th and $2j$-th Steps.

Step 0: Set the constant positive parameter δ, where δ denotes the desired value of the L_2 norm of the control input. We let δ be sufficiently small. Set the initial free parameters $\dot x^0$ and ${\theta^0}_{(1)}$ and calculate the other initial conditions according to [8]. Then we get the first initial condition ${X^0}_{(1)}$.

Step $2j-1$: With ${X^0}_{(j)}$, executes the $(2j-1)$-th laboratory experiment via the iteration law of (25), then goto Step $2j$.

Step $2j$: With ${X^0}_{(j)}$, executes the $2j$-th laboratory experiment via the iteration law of (26). If $\|u_{(2j)}\|_{L_2} \le \delta$, the procedure terminates. Here $\delta > 0$ is a prescribed sufficiently small constant. Otherwise, update ${\theta^0}_{(j)}$ by the update law

$$
\theta^0_{(j+1)} = \theta^0_{(j)} - K_{\theta^0}\ p_{\theta v(j)}\Big|_{t=t^1}, \qquad (27)
$$

where K_{θ^0} is an appropriate positive gain and $p_{\theta v}$ is the variation of the p_θ in (24). We get the $(j+1)$-th initial condition ${X^0}_{(j+1)}$ and goto Step $(2j+1)$.

Let us derive the update law (27). In Equation (7), we write

$$
\Gamma'(x^0, u, x^1, y) \equiv (\Gamma'_{x^0}\ \Gamma'_u\ \Gamma'_{x^1}\ \Gamma'_y)^{\mathrm{T}}. \qquad (28)
$$

Here we can calculate

$$
\begin{aligned}
&\mathrm{d}\left(\Gamma((x^0,u),\Sigma(x^0,u))\right)(\mathrm{d}x^0,\mathrm{d}u)\\
&= \langle \Gamma'_{x^0} + \pi_{\mathbb{R}^n}\circ(\ \mathrm{d}\Sigma(x^0,u))^*(\Gamma'_{x^1},\Gamma'_y),\ \mathrm{d}x^0\rangle\\
&+\langle \Gamma'_u + \pi_U\circ(\ \mathrm{d}\Sigma(x^0,u))^*(\Gamma'_{x^0},\Gamma'_u),\ \mathrm{d}u\rangle. \qquad (29)
\end{aligned}
$$

Here $\pi_{(\cdot)}$ denotes the projection operator onto $(\cdot)$. The steepest decent method implies that we should update the initial condition such that

$$
\mathrm{d}x^0 = -K_{x^0}\Big(\Gamma'_{x^0} + \pi_{\mathbb{R}^n}\circ(\ \mathrm{d}\Sigma(x^0,u))^*(\Gamma'_{x^1},\Gamma'_y)\Big) \quad (30)
$$

where K_{x^0} is an appropriate positive gain.

Suppose that we apply this method to our novel cost function (23), satisfying $\Gamma'_{x^0} \equiv 0$ and $\Gamma'_{x^1} \equiv 0$. Here the state at $t = t^0$ is said to be an *initial state* and the one at $t = t^1$ is said to be a *terminal state*. Then let us calculate the initial state of $(\ \mathrm{d}\Sigma(x^0,u))^*$ with respect to the input $(0, \Gamma'_y)$, that is, $x^0_a := \pi_{\mathbb{R}^n}\circ(\ \mathrm{d}\Sigma(x^0,u))^*(\Gamma'_{x^1},\Gamma'_y)$. As mentioned in Section II, the initial state is calculated as $x^0_a = -T^{-1}(J-R)^{-1}\bar x_v(t^0)$, where $\bar x_v(t^0)$ is the initial state of the adjoint of the variational system in (4). The state of the dynamics in (4) is identified with the time-reversal version of that in (3), so we can write the initial state of the adjoint of the variational system as $x^0_a = -T^{-1}(J-R)^{-1}x_v(t^1)$, where $x_v(t^1)$ is the terminal state of the variational system in (3) under certain circumstances as explained in Remark 1. For the dynamics of θ in the flight phase (24), $x^0_a = ({q_\theta}^0_a\ ,\ {p_\theta}^0_a)$ is calculated as follows

$$
\begin{aligned}
\begin{pmatrix} {q_\theta}^0_a \\ {p_\theta}^0_a \end{pmatrix} &= -\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^{-1}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^{-1}\begin{pmatrix} {q_\theta}_v(t^1) \\ {p_\theta}_v(t^1) \end{pmatrix}\\
&= \begin{pmatrix} {p_\theta}_v(t^1) \\ {q_\theta}_v(t^1) \end{pmatrix}, \qquad (31)
\end{aligned}
$$

where $({q_\theta}_v(t^1)\ ,\ {p_\theta}_v(t^1)) = x_v(t^1)$ and

$$
T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\ ,\ R = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.
$$

For (30) and (31), we should update the initial condition θ^0 as

$$
\mathrm{d}\theta^0 = -K_{\theta^0}\ {p_\theta}_v(t^1) \qquad (32)
$$

Hence the update law (27) is derived immediately.

In the existing results on the iterative learning control, it is assumed that all the initial conditions are the same. In this paper, we derive a novel update law for initial conditions (27) and combining with the learning procedure proposed in [10]. The proposed algorithm generates *optimal* passive gait trajectories.

V. Simulation

We apply the proposed algorithm to the hopping robot modelled in [11]. We set the parameters mentioned in the previous section as $\delta = 0.025$, $\theta^0_{(1)} = 0.30[\mathrm{rad}]$ and $\dot x^0 = 2.0[\mathrm{m/s}]$. Figure 5 shows the procedure terminates at the 34th Step.

Figure 3 shows the history of the cost function (23) along the iteration. It monotonically decreases, which implies that the output trajectory converges to the *optimal* one

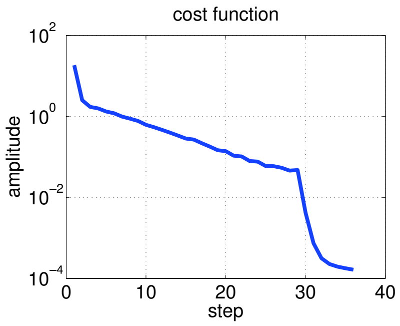

Fig. 3. Cost function

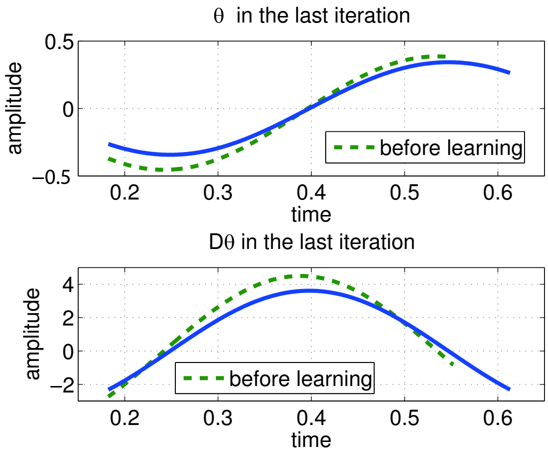

Fig. 4. Responses of θ and $\dot{\theta}$ at the last step

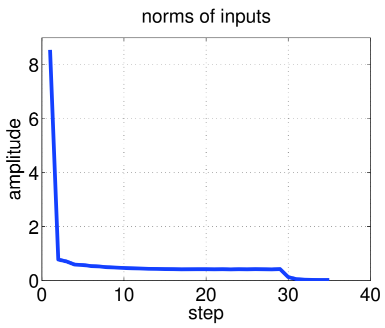

Fig. 5. L_2 norm of the control inputs

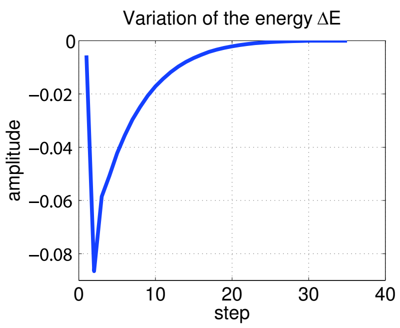

Fig. 6. Variation of the energy at the touch down ΔE

consequently at each learning experiment. Figure 4 shows responses of θ and $\dot{\theta}$ at the last step in the proposed method in solid lines and those of the initial trajectory in dotted lines. This figure shows that both θ and $\dot{\theta}$ converge to the trajectories satisfying the symmetric constraint (18) and (19). Furthermore Figures 5 and 6 show that the variation of the energy at the touchdown (20) and the L_2 norm of the control input converge to zero as well. Those results show that the proposed algorithm generates an *optimal* passive gait by learning.

VI. Conclusion

In this paper, we have proposed a framework to generate a passive walking gait via iterative learning control based on variational symmetry. Adopting a novel update law for initial conditions allows us to obtain this algorithm. A hopping robot modelled in [11] can walk with zero control input after iteration of laboratory experiments. Furthermore, numerical simulations have exhibited the effectiveness of the proposed method.

The proposed learning algorithm can solve a class of optimal control problems for physical systems as demonstrated in the present paper. It is expected that it will be applicable to optimal gait generation problems for more complicated walking robots.

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