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Multi-ion-species effects on low-frequency electromagnetic fluctuations and energy transport

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Abstract. Low-frequency perpendicular electromagnetic fluctuations and energy transport in multi-ion-species plasmas are studied theoretically and numerically. In thermal equilibrium, the amplitudes of heavy-ion cutoff modes, whose frequencies are slightly higher than the heavy-ion cyclotron frequencies, can be comparable to that of the magnetosonic mode. The heavy-ion cutoff modes can cause damping of the magnetic fluctuations. Long time evolution of a macroscopic magnetic disturbance is also investigated with particle simulations. It is demonstrated that the disturbance is damped more quickly than the prediction of the linear theory. Owing to the damping, ion kinetic energy is increased.

1. Introduction

The presence of multiple ion species significantly influences wave propagation and energy transport. For example, nonlinear magnetosonic pulses are damped, even when they propagate perpendicular to a magnetic field [1]. The damping is due to the energy transfer from the pulse to heavy ions [2]. If a plasma contains many ion species, such as in space plasmas, autocorrelation functions of quasi-modes consisting of electrostatic ion Bernstein waves are practically damped [3], although each perpendicular sinusoidal wave is undamped [4]. It was also demonstrated with particle simulations that a macroscopic disturbance is damped with the energy transfer from the electric field to the ions. This work [3] was on the electrostatic waves with short wavelength $k\rho_{\rm i} \gg 1$, where $\rho_{\rm i}$ is the ion gyroradius. In this paper, we consider electromagnetic waves with long wavelengths, $k\rho_{\rm i} \ll 1$, where magnetohydrodynamic perturbations are important.

In Sec. 2, we describe power spectra of magnetic fluctuations in a thermal equilibrium state. We also theoretically study the low-frequency ($\omega \ll \omega_{\rm LH}$) evolution of a macroscopic magnetic disturbance, where $\omega_{\rm LH}$ is the lower hybrid frequency. In Sec. 3, by means of an electromagnetic particle code, we investigate the long time evolution of this disturbance and associated energy transport.

2. Theory for low-frequency electromagnetic fluctuations

We consider low-frequency ($\omega \ll \omega_{\rm LH}$), long-wavelength ($k\rho_{\rm i} \ll 1$), perpendicular electromagnetic fluctuations in a plasma containing hydrogen (H) and heavy ions

(He, C, ...). In this frequency domain, there exist three types of modes [5]; the magnetosonic mode with $\omega \approx kv_{\rm A}$, where $v_{\rm A}$ is the Alfvén speed, ion cyclotron modes with $\omega \approx n\Omega_{\rm i}$, and heavy-ion cutoff modes with

$$\omega \approx \omega_{\rm s0} \equiv \Omega_{\rm s} + (\omega_{\rm ps}^2/\omega_{\rm pH}^2)\Omega_{\rm H}\Omega_{\rm s}/(\Omega_{\rm H} - \Omega_{\rm s}). \tag{2.1}$$

Here, Ω_j and $\omega_{{\rm p}j}$ are the cyclotron and plasma frequencies for particle species j, respectively, and the subscript s denotes heavy-ion species. (In a H and He plasma, the frequency of the fundamental He cyclotron mode is $\Omega_{\rm He} < \omega < \omega_{\rm He0}$, and that of the He cutoff mode increases with k and is in the region $\omega_{\rm He0} < \omega < 2\Omega_{\rm He}$, as shown in [5, fig. 2]) In a thermal equilibrium state, the amplitudes of the ion cyclotron modes are quite small. Thus, in the single-ion-species plasma, the magnetosonic mode is dominant. The spectra of the magnetosonic and heavy-ion cutoff modes are estimated as

$$P(k, kv_{\rm A}) \sim (\pi k_{\rm B}T/2) \Bigg/ \left[1 + \sum_i (\omega_{\rm pi}^2/\Omega_{\rm i}^4)(v_{\rm A}^4/c^2)k^2\right], \tag{2.2}$$

$$P(k, \omega_{\rm s0}) \sim (\pi k_{\rm B}T/2)(\omega_{\rm ps}^2/\omega_{\rm pH}^4)(\Omega_{\rm H}^2/\Omega_{\rm s}^4)(\Omega_{\rm H} - \Omega_{\rm s})^2 c^2 k^2. \tag{2.3}$$

These indicate that $P(k, kv_{\rm A})$ can be comparable to $P(k, \omega_{\rm s0})$ if $kv_{\rm A} \sim \Omega_{\rm s}$.

We also study a one-dimensional ($\partial/\partial y = \partial/\partial z = 0$), initial value problem, where the magnetic field $B_z(x, t)$ has a sinusoidal disturbance with a wavenumber k_0,

$$\delta B_z(x, 0) \equiv B_z(x, 0) - B_0 = B_{k_0}(0)\cos(k_0 x), \tag{2.4}$$

where B_0 is the strength of the external magnetic field, $\boldsymbol{B}_0 = (0, 0, B_0)$, and $B_{k_0}(0)$ is the amplitude of the initial disturbance. Other physical quantities have no macroscopic disturbances at $t = 0$. We then find that the time variation of the Fourier component $B_{k_0}(t)$ varies with time as

$$B_{k_0}(t)/B_{k_0}(0) \approx \sum_n [P(k_0, \omega_n)/(\pi k_{\rm B}T/2)]\cos(\omega_n t), \tag{2.5}$$

for $t > 0$, where ω_n is a root of the linear dispersion relation. In a single-ion-species plasma, therefore, $B_{k_0}(t)$ oscillates with the frequency $\omega \approx k_0 v_{\rm A}$ and is undamped. In a multi-ion-species plasma, however, $B_{k_0}(t)$ is initially damped owing to the phase mixing of the magnetosonic and heavy-ion cutoff modes.

In a collisionless plasma, $B_{k_0}(t)$ may be recovered to its initial value $B_{k_0}(0)$ at the time of the least-common multiple of $2\pi/(k_0 v_{\rm A})$ and $2\pi/\omega_{\rm s0}$. In a multi-ion-species plasma, however, this time should be extremely long. On such a long time scale, $B_{k_0}(t)$ would disappear before the recurrence owing to collisions or some nonlinear effects. Practically, therefore, $B_{k_0}(t)$ would be irreversible.

3. Simulations of macroscopic disturbance

By means of a one-dimensional (one space coordinate and three velocity components), electromagnetic particle code with full ion and electron dynamics, we study the long-time evolution of a macroscopic magnetic disturbance.

We simulate single- and four-ion-species plasmas; we denote the ion species in the former by a and those in the latter by a, b, c, and d. We choose the mass ratios as $m_{\rm a}/m_{\rm e} = 50$, $m_{\rm b}/m_{\rm a} = \sqrt{3}$, $m_{\rm c}/m_{\rm a} = 2$, and $m_{\rm d}/m_{\rm a} = \sqrt{5}$. In order to see the effect of multiple ion species with a small number of ion species, we have taken the irrational ion mass rations for b and d ions. The electric charges are the same.

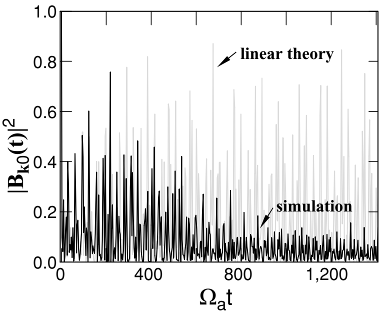

Figure 1. Time variations of $|B_{k_0}(t)|^2$ in the four-ion-species plasma. Here, $|B_{k_0}(t)|^2$ are normalized to $|B_{k_0}(0)|^2$.

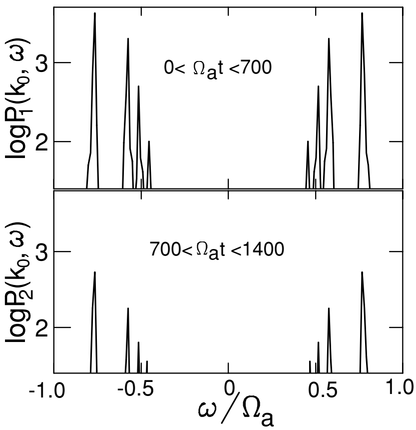

Figure 2. Power spectra of magnetic perturbations with $k = k_0$ in the four-ion-species plasma.

The ion densities are set to be $n_b = n_c = n_d = 0.2n_a$. The heavy-ion cutoff frequencies are $\omega_{b0}/\Omega_a \approx 0.69$, $\omega_{c0}/\Omega_a \approx 0.54$, and $\omega_{d0}/\Omega_a \approx 0.46$.

We use periodic boundary conditions with periodicity $L_x = 4096\Delta_g$, where Δ_g is the grid spacing. The total number of electrons is $N_e \approx 4.2 \times 10^6$. The electrons and ions initially have Maxwellian velocity distribution functions. All of the ion species have the same temperature, where the electron-to-ion temperature ratio is $T_i/T_e = 0.1$. The external magnetic field is in the z direction with $|\Omega_e|/\omega_{pe} = 4.0$.

As in the theory, the magnetic field initially has a sinusoidal disturbance given by (2.4) with an amplitude $B_{k_0}(0)/B_0 = 0.04$ and wavenumber $k_0 v_A = 0.59\Omega_a$. We observe time variation of the Fourier component $B_{k_0}(t)$ until the time $\Omega_a t = 1400$. In the single-ion-species plasma, it oscillates with frequency $\omega \approx k_0 v_A$ and is undamped. In the four-ion-species plasma, however, $B_{k_0}(t)$ does not return to the initial value, as shown in Fig. 1. In the early stage, $\Omega_a t < 100$, the simulation result (the black line) is in good agreement with the linear theory (2.5) (the gray line). However, for $\Omega_a t > 300$, $|B_{k_0}(t)|$ is damped faster than the theory predicts.

Figure 2 shows the power spectra of magnetic perturbations with $k = k_0$ in the four-ion-species plasma. The upper and lower panels show the spectrum $P_1(k_0, \omega)$ for $0 < \Omega_a t < 700$ and $P_2(k_0, \omega)$ for $700 < \Omega_a t < 1400$, respectively. We find several

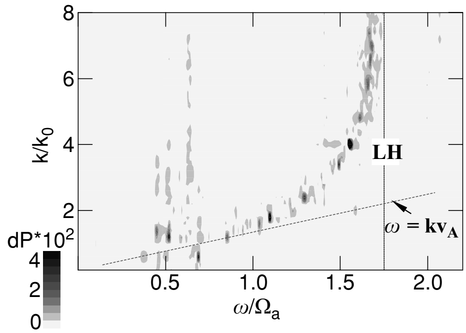

Figure 3. Contour map of the increase in P, $dP = P_2 - P_1$ in the (ω, k) plane.

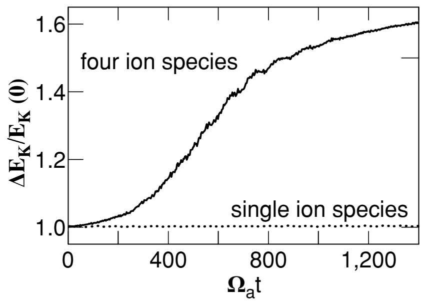

Figure 4. Time variations of ion kinetic energies in the single- and four-ion-species plasmas.

peaks at $\omega \approx \pm k_0 v_{\mathrm{A}}$ and near the heavy-ion cutoff frequencies, $\pm\omega_{\mathrm{b}0}, \pm\omega_{\mathrm{c}0}$ and $\pm\omega_{\mathrm{d}0}$. The peak values of P_2 are smaller than those of P_1.

The damping of these k_0 modes is caused by nonlinear mode couplings, which generate many waves with $k \neq k_0$. Figure 3 shows the contour map of increase in P, $dP = P_2 - P_1$, in the (ω, k) plane; we pay attention to the waves with $k \neq k_0$. The vertical line represents the lower hybrid frequency (line LH). We find that many waves are excited along the magnetosonic curve up to $\omega \sim \omega_{\mathrm{LH}}$. Also, waves are generated along the heavy-ion cutoff frequencies. In the single-ion-species plasma, the spectrum change $dP(k, \omega)$ is quite small. The presence of the multiple ion species clearly enhances nonlinear mode couplings.

Figure 4 shows time variations of change in ion kinetic energy, $\Delta E_{\mathrm{K}} = E_{\mathrm{K}}(t) - E_{\mathrm{K}}(0)$, where E_{K} is defined as

$$E_{\mathrm{K}} = \sum_i \int dx \int d\boldsymbol{v}\, m_{\mathrm{i}} f_{\mathrm{i}}(x, \boldsymbol{v})(\boldsymbol{v} - \langle \boldsymbol{v}_{\mathrm{i}}(x)\rangle)^2/2. \tag{3.1}$$

The density $n_{\mathrm{i}}(x)$ and fluid velocity $\langle \boldsymbol{v}_{\mathrm{i}}(x)\rangle$ are defined as $n_{\mathrm{i}}(x) = \int d\boldsymbol{v}\, f_{\mathrm{i}}(x, \boldsymbol{v})$, and $\langle \boldsymbol{v}_{\mathrm{i}}(x)\rangle = \int d\boldsymbol{v}\, f_{\mathrm{i}}(x, \boldsymbol{v})\boldsymbol{v}/n_{\mathrm{i}}(x)$. The ion kinetic energy in the single-ion-species plasma is nearly constant. In the four-ion-species plasma, however, the ion energy keeps increasing. The value of E_{K} increases by $\Omega_{\mathrm{a}} t \sim 800$, which is of the order of the energy lost by the magnetic field shown in Fig. 1.

4. Summary

We have studied low-frequency perpendicular electromagnetic fluctuations in multi-ion-species plasmas. In thermal equilibrium, heavy-ion cutoff modes can have amplitudes comparable to that of the magnetosonic mode and can cause the damping of the fluctuations. We have also investigated with particle simulations the evolution of a macroscopic magnetic disturbance. Simulations have demonstrated that the disturbance is damped more quickly than the linear theory predicts. This is due to nonlinear mode couplings. In association with this damping, ion kinetic energy is increased.

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