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Theory and simulations of relativistic particle motions in a magnetosonic shock wave

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Abstract. The motions of relativistic particles in a magnetosonic shock wave propagating obliquely to an external magnetic field are studied. In the zeroth-order theory, particles continue to move nearly parallel to the external magnetic field in the shock transition region, when the shock speed is close to $c\cos\theta$, where c is the speed of light and θ is the propagation angle. Perturbations to this zeroth-order motion are also analyzed for positrons and ions. The perturbation frequency of positrons is $\omega \sim \Omega_{\rm p0}\gamma^{-1}$ and that of ions is $\omega \sim \Omega_{\rm i0}\gamma^{-1/2}$, where $\Omega_{\rm p0}$ and $\Omega_{\rm i0}$ are the non-relativistic gyrofrequencies of positrons and of ions, respectively, and γ is the Lorentz factor. These theoretical predictions are confirmed with numerical simulations.

1. Introduction

Magnetosonic shock waves can accelerate thermal hydrogen ions, heavy ions, and electrons with various non-stochastic mechanisms [1]. Recently, two acceleration mechanisms of non-thermal, relativistic particles in magnetosonic shock waves have been studied with theory and simulations. One mechanism is associated with large-radius gyromotions; particles absorb energy from the electric field perpendicular to the magnetic field **B** [2]. The other is the acceleration parallel to **B** [3, 4]. Here, we investigate particle motions in the latter. In Sec. 2, we describe the zeroth-order and perturbation theories for relativistic particles. In Sec. 3, we verify them with simulations. Section 4 gives a summary of our work.

2. Theory

2.1. *Zeroth-order theory*

We analyze the motions of relativistic particles in a magnetosonic shock wave propagating in the x direction with a speed $v_{\rm sh}$ in an external magnetic field $\mathbf{B}_0 = B_0(\cos\theta, 0, \sin\theta)$. If the particle speed v is very close to the speed of light c, a slight change in the particle speed can lead to a great change in the Lorentz factor γ. Therefore, ignoring $\gamma d\mathbf{v}/dt$ compared with $\mathbf{v}d\gamma/dt$, we obtain the zeroth-order equation of motion for a particle with a mass m_j and a charge q_j (j denotes particle species, $j = $ p or i):

$$m_j \frac{d\gamma_0}{dt}\mathbf{v}_0 = q_j\left(\mathbf{E} + \frac{\mathbf{v}_0}{c}\times\mathbf{B}\right), \tag{2.1}$$

where the subscript 0 refers to the zeroth-order quantities.

We consider particles moving with the wave, i.e. $v_{x0} = v_{\rm sh}$. When $v_{\rm sh} \approx c\cos\theta$, we find

$$\frac{v_{z0}}{v_{\rm sh}} \approx \frac{B_{z0}}{B_{x0}} \tag{2.2}$$

from (2.1). It is also shown that $|v_{y0}|$ is much smaller than v_{x0} and v_{z0}; thus, the particles move nearly parallel to $\mathbf{B}_0$. Moreover, we obtain the time rate of change of γ_0 as

$$\frac{d\gamma_0}{dt} = \frac{q_j B_{x0}}{m_j v_{\rm sh}} \frac{(\mathbf{E}\cdot\mathbf{B}_0)}{(\mathbf{B}\cdot\mathbf{B}_0)}. \tag{2.3}$$

If the particle position in the wave does not change, γ_0 continues to grow linearly with time [3].

2.2. Perturbation theory

The zeroth-order theory is applicable to either positrons or ions. We do, however, need to treat positron and ion perturbations separately [4]. We assume that $d\mathbf{v}_1/dt \sim \gamma_0^{-1}\Omega_{\rm p0}\mathbf{v}_1$ for positrons and $d\mathbf{v}_1/dt \sim \gamma_0^{-1/2}\Omega_{\rm i0}\mathbf{v}_1$ for ions, where $\Omega_{\rm p0}$ and $\Omega_{\rm i0}$ are the non-relativistic positron and ion gyrofrequencies, respectively, and the subscript 1 refers to perturbed quantities. We then expand the exact equation of motion. After some algebra, the perturbation frequency ω of the positrons is obtained as

$$\omega^2 = \left(\frac{e}{m_{\rm p}c}\right)^2 \frac{(\gamma_0^2/c^2)(\mathbf{B}_0\cdot\mathbf{v}_0)^2 + B^2}{\gamma_0^4}, \tag{2.4}$$

which is obviously positive, while that of the ions is given as

$$\omega^2 = -\frac{q_{\rm i}}{m_{\rm i}\gamma_0\gamma_{\rm sh}}\left(\frac{dE_x}{d\xi_0} + \frac{v_{y0}}{c}\frac{dB_z}{d\xi_0} - \frac{v_{z0}}{c}\frac{dB_y}{d\xi_0}\right), \tag{2.5}$$

where $\gamma_{\rm sh} = [1-(v_{\rm sh}/c)^2]^{-1/2}$, $\xi = x - v_{\rm sh}t$, and ξ_0 is the center position of the perturbation; $d/d\xi_0$ designates the derivative at $\xi = \xi_0$. The ion perturbation is stable when $\omega^2 > 0$.

3. Numerical studies

We numerically investigate the motions of relativistic particles. For positrons, we use a one-dimensional, relativistic, electromagnetic particle simulation code [3]. As in the theory, waves propagate in the x direction in an external magnetic field $\mathbf{B}_0$. The field strength is $|\Omega_{\rm e0}|/\omega_{\rm pe} = 3$, where $|\Omega_{\rm e0}|$ and $\omega_{\rm pe}$ are the electron gyro and plasma frequencies, respectively. The propagation angle is taken to be $\theta = 42°$. The propagation speed of a shock wave studied here is observed to be $v_{\rm sh} = 2.4v_{\rm A}$, where $v_{\rm A}$ is the Alfvén speed. It has a typical shock profile with the width of the transition region of the order of the ion inertial length as shown, for instance, in [3]. Figure 1 shows the time variation of γ of a positron accelerated by a magnetosonic shock wave in an electron–positron–ion plasma with a positron-to-electron density ratio of 0.02 [3]. The energy increases up to $\gamma \sim 600$. We did, however, find oscillations in γ. Figure 2 displays the oscillation frequency ω as a function of γ_0 [4]. The data points represent simulation results, while the solid line shows the theoretical curve given by (2.4). The simulation results are explained by the theory.

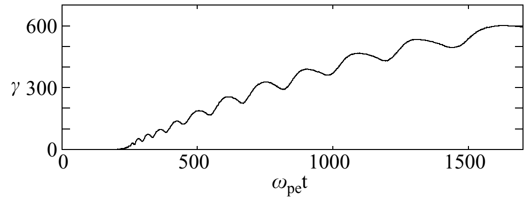

Figure 1. Time variation of γ of an accelerated positron. Here, ω_{pe} is the electron plasma frequency.

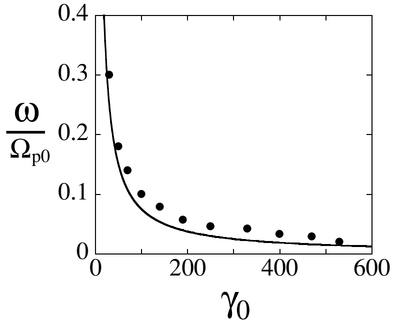

Figure 2. Positron perturbation frequency ω versus γ_0.

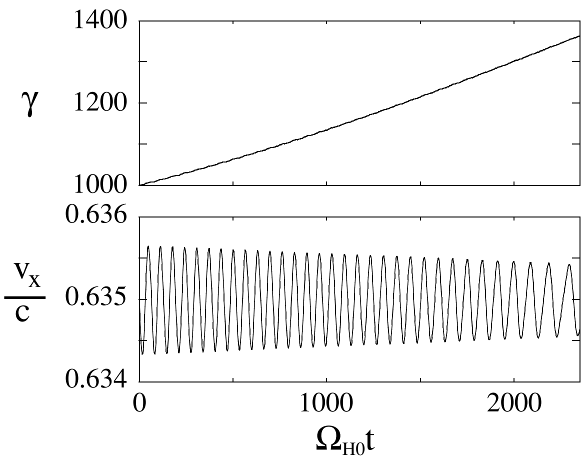

Figure 3. Time variations of γ and v_x of an accelerated hydrogen ion. Here, Ω_{H0} is the non-relativistic hydrogen gyrofrequency.

For ions, we calculate test particle orbits; we first obtain the electromagnetic fields in a shock wave from a particle simulation and then follow particle motions in the fields, assuming stationary wave propagation. Here, plasma parameters are $|\Omega_{e0}|/\omega_{pe} = 1.5$, $\theta = 50°$, and $v_{sh} = 3.2v_A$. The initial velocities of the test particles are given by the zeroth-order theory. In Fig. 3 we show the time variations of γ and v_x of an ion with an initial energy $\gamma = 1000$. The energy increases from $\gamma = 1000$

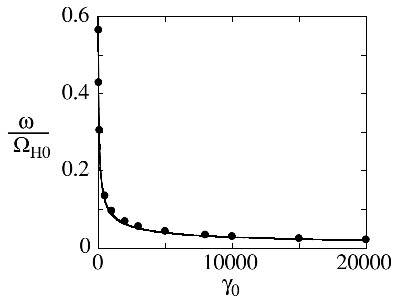

Figure 4. Ion perturbation frequency ω versus γ_0.

to approximately 1400. Also, we find small-amplitude oscillations in v_x. In Fig. 4, we display the perturbation frequency ω as a function of γ_0. The simulation values fit well to the theoretical curve obtained from (2.5)

4. Summary

The motions of relativistic particles in a magnetosonic shock wave have been studied. In the zeroth-order theory, where the relation $\gamma|d\mathbf{v}/dt| \ll v d\gamma/dt$ is assumed, particles are accelerated almost parallel to the external magnetic field, when $v_{\rm sh} \approx c\cos\theta$. This is applicable to either positrons or ions. Perturbation theories for positrons and ions have been separately investigated. The perturbation frequency of positrons is $\omega \sim \Omega_{\rm p0}\gamma_0^{-1}$ and that of ions is $\omega \sim \Omega_{\rm i0}\gamma_0^{-1/2}$. The zeroth- and first-order theories have been verified with numerical simulations.

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