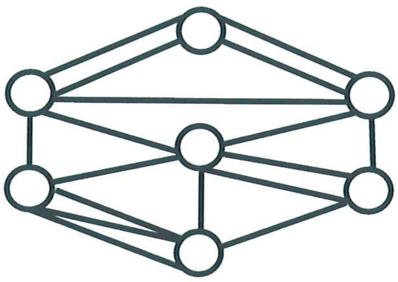
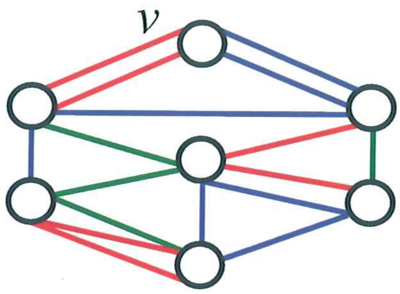
NEARLY EQUITABLE EDGE-COLORING PROBLEM

Xuzhen Xie, Takao Ono, Shin-ichi Nakao, Tomio Hirata

1. What is Nearly Equitable Edge-coloring?

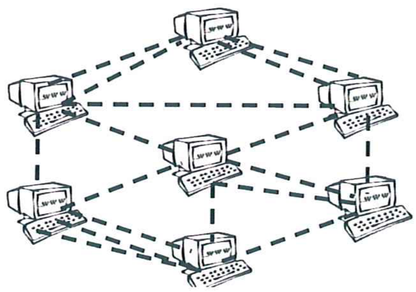


(Fig 1.1) Input: a multigraph $G=(V,E)$ and a k-color set $C=\{C1,...,Ck\}$



(Fig 1.2) Output: an edge-coloring such that for any vertex $v \in V$ and different colors $Ci, Cj \in C$, $|d(v,Ci)-d(v,Cj)| \leq 2$. Here, for $k=3$ and the vertex v: $d(v,\mathbf{R})=d(v,\mathbf{B})=2$, $d(v,\mathbf{G})=0$.

2. Application: Considering the problem of transferring files among computers in k unit times, while all computers have the nearly equitable transfer tasks (Suppose that each file can be transferred in 1 unit time.). This scheduling problem can be changed into a nearly equitable edge-coloring problem.



(Fig 2.1) Transfer files among computers: Let computers be vertices and files be edges response, we obtain a multigraph as Fig 1.2. The same colored edges in Fig 1.2 refer to the files transferred at the same time.

3. Results for Nearly Equitable Edge-coloring Problem

1) Running Time

Hilton and Werra	$O(k\|E\|^2)$
Nakao, Suzuki and Nishizeki	$O(\|E\|^2/k+\|E\|\|V\|)$
Our Result	$O(\|E\|^2/k)$

Moreover, our coloring satisfies "balanced constraint": $|e(Ci)-e(Cj)| \leq 1$ for any colors Ci and Cj, where $e(Ci)$ is the number of edges colored with Ci.

2) Details

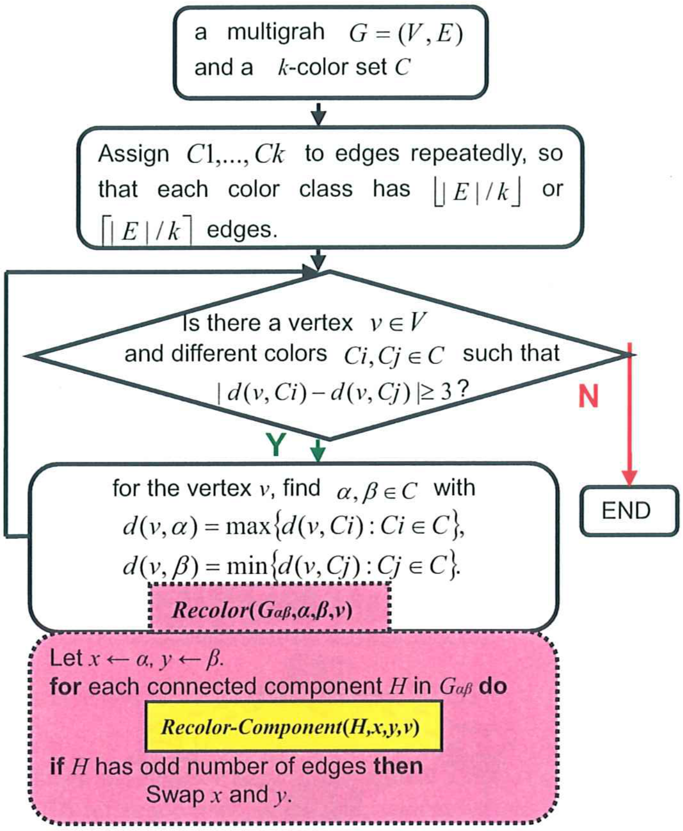
For a vertex $s \in V$, let $\bar{d}(s)=\lfloor d(s)/k \rfloor$ ($d(s)$ denotes the degree of s), we define the cost of s as:

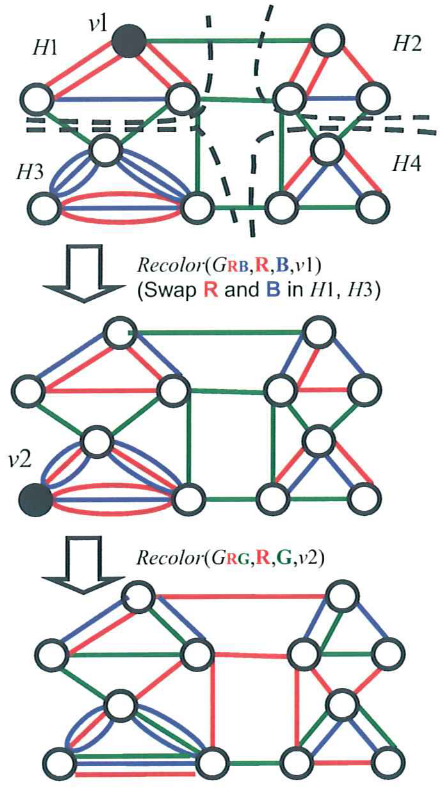
$\Phi(s)=\sum_{Ci \in C}\{|d(s,Ci)-(\bar{d}(s)+1/2)|-1/2\}$

, which is obviously an integer. Then, the cost Φ of the coloring is bounded as follows: $\Phi=\sum_{s \in V}\Phi(s) \leq 4|E|$.

We can prove that Φ must decrease by at least 1 when we invoke ***Recolor***. The running time of ***Recolor*** is $O(|E|/k)$, which implies that the whole running time is $O(|E|^2/k)$.

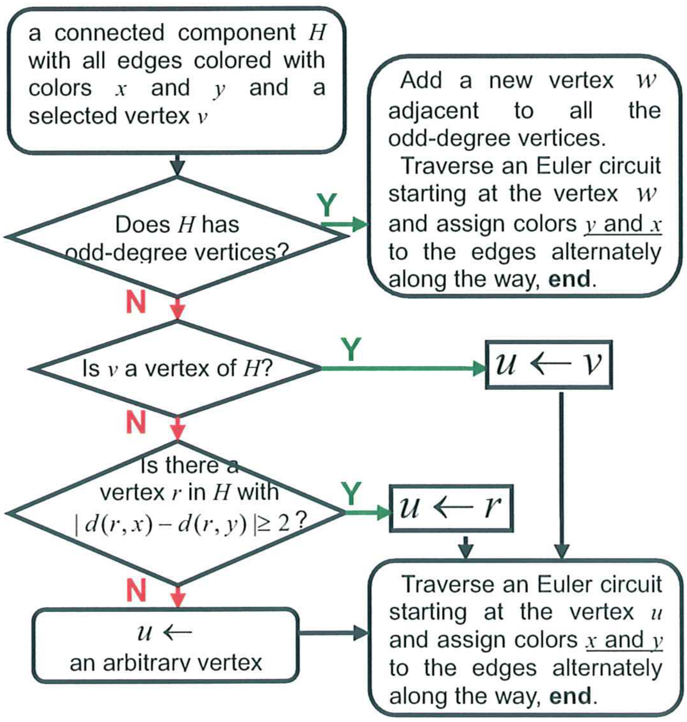
Algorithm(G,C)

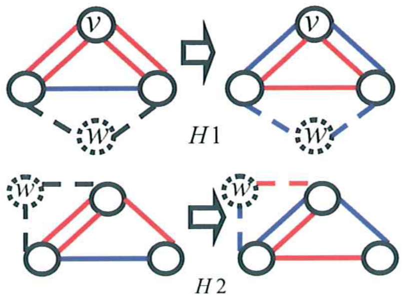




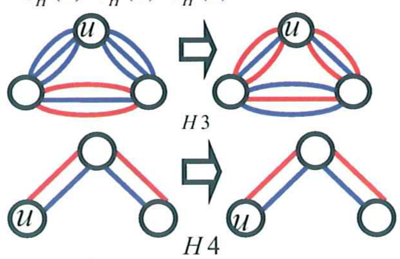
Nearly equitable edge-coloring
$e(\mathbf{R}) = e(\mathbf{B}) = 11$ and $e(\mathbf{G}) = 10$
(Fig 3.1) *Algorithm(G,C)*

Recolor-Component(H,x,y,v)





Result 1: $|d(s,\mathbf{R})-d(s,\mathbf{B})|=1$ for any odd-degree vertex s and $d(s,\mathbf{R})=d(s,\mathbf{B})$ for any even-degree vertex s, and $e_H(\mathbf{B}) \leq e_H(\mathbf{R}) \leq e_H(\mathbf{B})+1$.



Result 2: $d(u,\mathbf{R})=d(u,\mathbf{B})+2$ for the start vertex u in $H3$ with odd number of edges and $d(s,\mathbf{R})=d(s,\mathbf{B})$ for any other vertex s, and $e_H(\mathbf{B}) \leq e_H(\mathbf{R}) \leq e_H(\mathbf{B})+1$.

(Fig 3.2) *Recolor-Component(H,x,y,v)*