

# SUPER-EXPONENTIAL ALGORITHMS AND EIGENVECTOR ALGORITHMS FOR BLIND SOURCE SEPARATION

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## ABSTRACT

In this paper we deal with the blind source separation (BSS) problem in the frequency domain. To solve this problem we adopt the idea of super-exponential methods (SEM) and eigenvector algorithms (EVA) with reference signals. SEMs have the attractive property that they are computationally efficient and converge to the desired solutions at a super-exponential rate. Conventional SEMs have the problem that they are sensitive to Gaussian noise. To overcome this issue, we propose the robust SEM, which utilizes only the higher-order cumulants. SEMs, however, have the drawback that they fail to extract all the source signals due to the failure of the deflation process. To compensate for this drawback, we propose the EVA with reference signals, which makes it possible to extract all the sources.

## 1. INTRODUCTION

This paper deals with the blind source separation problem for a multiple-input multiple-output (MIMO) static system driven by independent source signals. To solve this problem, we draw on the two ideas of super-exponential methods (SEM)[1, 2, 3] and eigenvector algorithms (EVA) with reference signals[4, 5, 6].

To date several researchers have proposed SEMs for Independent Component Analysis (ICA) and Blind Source Separation (BSS). One of the attractive properties of SEMs is that they are computationally efficient and converge to the desired solution at a super-exponential rate. However, conventional SEMs suffer the drawback that they are very sensitive to Gaussian noise, because SEMs utilize the second-order and the higher-order cumulants of the observations. Therefore, in this paper we propose an SEM robust against Gaussian noise, that utilizes only the higher-order statistics of the observations. However, when we extract independent sources one by one using SEMs, a deflation process is needed after the separation process to make the output signals independent of each other. The deflation process, however, sometimes fails.

To avoid the deflation process, we propose an eigenvector algorithm (EVA) with reference signals. The EVA de-

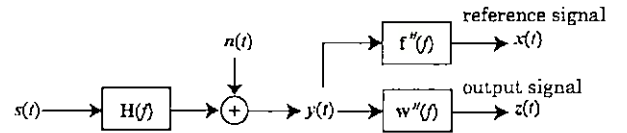


Fig. 1. The composite system of an unknown system and a filter, and reference system

rived from a criterion with reference signals is used for solving the BSS problem of MIMO static systems. The proposed EVA has the attractive feature that all source signals are separated simultaneously from their mixtures, whereas the other methods using deflation process extracted signals one by one. Therefore, if the deflation process fails, none of the the signals can be separated. However, the EVA with reference signals enables us to extract all the sources without resorting any deflation methods.

We demonstrate the effectiveness of the proposed methods through computer simulations and an experiment in a real environment.

## 2. PROBLEM FORMULATION

Throughout this paper, let us consider the following MIMO static system with  $n$  inputs and  $m$  outputs, a convolutive mixture model with additive noise (See Fig. 1.);

$$\mathbf{y}(t) = \sum_k \mathbf{H}(k) \mathbf{s}(t-k) + \mathbf{n}(t), \quad (1)$$

where  $\mathbf{y}(t)$  represents an  $m$ -column output vector called the *observed signal*,  $\mathbf{s}(t)$  represents an  $n$ -column input vector called the *source signal*,  $\mathbf{n}(t)$  represents an  $m$ -column noise vector and  $\mathbf{H}(t)$  is an  $m \times n$  ( $m \geq n$ ) mixing matrix.

To achieve the blind source separation for the system (1), a convolutive mixture in the time domain is converted into instantaneous mixtures in the frequency domain with the short-time Fourier transform (STFT),

$$\mathbf{y}(f, t) = \mathbf{H}(f) \mathbf{s}(f, t) + \mathbf{n}(f, t). \quad (2)$$

The following  $n$  filters, which are  $m$ -input single-output (MISO) static systems driven by the observed signals, are used for each frequency bin:

$$z_l(f, t) = \mathbf{w}_l^H(f) \mathbf{y}(f, t), \quad l = 1, 2, \dots, n, \quad (3)$$

where superscript  $H$  denotes the conjugate transpose of a matrix or a vector  $z_l(f, t)$  is the  $l$ th output of the filter,  $\mathbf{w}_l(f) = [w_{l1}(f), w_{l2}(f), \dots, w_{lm}(f)]^H$  is an  $m$ -column vector representing the  $m$  coefficients of the filter in frequency bin  $f$ . Substituting Eq. (2) into (3), we obtain

$$\begin{aligned} z_l(f, t) &= \mathbf{w}_l^H(f) \mathbf{H}(f) \mathbf{s}(f, t) + \mathbf{w}_l^H(f) \mathbf{n}(f, t), \\ &= \mathbf{g}_l^H(f) \mathbf{s}(f, t) + \mathbf{w}_l^H(f) \mathbf{n}(f, t), \\ & \quad l = 1, 2, \dots, n, \end{aligned} \quad (4)$$

where  $\mathbf{g}_l(f) = [g_{l1}(f), g_{l2}(f), \dots, g_{ln}(f)]^H = \mathbf{H}^H(f) \mathbf{w}_l(f)$  is an  $n$ -column vector. The BSS problem considered in this paper can be formulated as follows: Find  $n$  filters  $\mathbf{w}_l(f)$ 's denoted by  $\tilde{\mathbf{w}}_l(f)$ 's satisfying the following condition, without the knowledge of  $\mathbf{H}(f)$ , even if the Gaussian noise  $\mathbf{n}(f, t)$  is added to the observed signal  $\mathbf{y}(f, t)$ ,

$$\tilde{\mathbf{g}}_l(f) = \mathbf{H}^H(f) \tilde{\mathbf{w}}_l(f) = \tilde{\delta}_l(f), \quad l = 1, 2, \dots, n, \quad (5)$$

where  $\tilde{\delta}_l(f)$  is an  $n$ -column vector whose elements  $\tilde{\delta}_{lr}(f)$  ( $r = 1, 2, \dots, n$ ) are equal to zero except for the  $\rho_l(f)$ th element.

To solve the blind separation problem, we put the following assumptions on the system and the source signals.

- A1) The matrix  $\mathbf{H}(f)$  in (2) has full column rank.
- A2) The input sequence  $\{\mathbf{s}(f, t)\}$  is a zero-mean, non-Gaussian vector whose element processes  $\{s_i(f, t)\}$ ,  $i = 1, 2, \dots, n$ , are mutually statistically independent and have nonzero variance,  $\sigma_{s_i}^2(f)$  and nonzero fourth-order cumulants,  $\gamma_i(f)$ ,  $i = 1, 2, \dots, n$ .
- A3) The noise sequence  $\{\mathbf{n}(f, t)\}$  is a stationary process vector, whose elements,  $\{n_i(f, t)\}$ ,  $i = 1, 2, \dots, m$  are zero-mean Gaussian processes.
- A4) The two vector sequences  $\{\mathbf{n}(f, t)\}$  and  $\{\mathbf{s}(f, t)\}$  are mutually independent.

Hereafter, for the sake of simple notation, we omit the frequency bin  $f$ , i.e.,  $\mathbf{s}(f, t) = \mathbf{s}(t)$ .

### 3. ROBUST SUPER-EXPONENTIAL METHOD (RSEM)

#### 3.1. RSEM for frequency-domain BSS

To find a solution  $\mathbf{w}_l$  satisfying Eq. (5), we extend the robust super-exponential method (RSEM) proposed in [7] to complex-valued signals. Using a matrix  $\mathbf{A}$ , we must find  $\mathbf{w}_l$

$$\min_{\mathbf{w}_l} (\mathbf{H}^H \mathbf{w}_l - \tilde{\delta}_l)^H \mathbf{A} (\mathbf{H}^H \mathbf{w}_l - \tilde{\delta}_l). \quad (6)$$

Solving the above equation brings us the following two-step equations:

$$\mathbf{w}_l^{[1]} = \mathbf{Q}^\dagger \mathbf{d}_l, \quad (l = 1, 2, \dots, n), \quad (7)$$

$$\mathbf{w}_l^{[2]} = \frac{\mathbf{w}_l^{[1]}}{\sqrt{\mathbf{w}_l^{[1]H} \mathbf{w}_l^{[1]}}}, \quad (l = 1, 2, \dots, n), \quad (8)$$

where  $^\dagger$  denotes the pseudo-inverse operation.  $\mathbf{d}_l = [d_{l1}, d_{l2}, \dots, d_{lm}]^T$ 's  $j$ -th element  $d_{lj}$  is calculated by

$$d_{lj} = \text{cum} \{z_l^*(t), z_l^*(t), z_l(t), y_j(t)\}, \quad (9)$$

where superscript  $*$  denotes the complex conjugate of a signal. This means that  $d_{lj}$  is given by the fourth-order cumulant of the output signal  $z_l(t)$  and the observed signal  $y_j(t)$  in the super-exponential methods, the above two-step procedure becomes one cycle of iterations. In conventional SEMs[1, 2, 3],  $\mathbf{Q}$  is calculated by the second-order cumulants of the observed signals, whereas in the RSEM, only the fourth-order cumulants of the observed signals are used. For complex-valued signals,  $\mathbf{Q}$  is calculated using

$$\mathbf{Q} = \sum_{k=1}^m \mathbf{R}_{\mathbf{y},k}, \quad (10)$$

$$[\mathbf{R}_{\mathbf{y},k}]_{i,j} = \text{cum} \{y_i(t), y_j^*(t), y_k(t), y_k^*(t)\}, \quad (11)$$

where  $[\mathbf{R}_{\mathbf{y},k}]_{i,j}$  denotes the  $(i, j)$ th element of the matrix  $\mathbf{R}_{\mathbf{y},k}$ . Because RSEM utilizes only the higher-order cumulants, it is insensitive to Gaussian noise.

#### 3.2. Adaptive Robust Super-Exponential Method (ARSEM)

We consider implementing adaptively the two-step procedure in Eqs. (7) and (8) for the case of two sources and two sensors. To this end, we must specify the dependency of each time  $t$ , and rewrite Eqs. (7) and (8) as follows:

$$\mathbf{w}_l^{[1]}(t) = \tilde{\mathbf{Q}}^{-1}(t) \tilde{\mathbf{d}}_l(t), \quad (l = 1, 2), \quad (12)$$

$$\mathbf{w}_l^{[2]}(t) = \frac{\mathbf{w}_l^{[1]}(t)}{\sqrt{\mathbf{w}_l^{[1]H}(t) \mathbf{w}_l^{[1]}(t)}}, \quad (l = 1, 2), \quad (13)$$

where  $\tilde{\mathbf{Q}}(t)$  and  $\tilde{\mathbf{d}}_l(t)$  are the estimator at time  $t$  of  $\mathbf{Q}$  and  $\mathbf{d}_l$ , respectively. We calculate  $\tilde{\mathbf{Q}}(t)$  in accordance with a moving average:

$$\begin{aligned} \tilde{\mathbf{Q}}(t) &= \alpha \tilde{\mathbf{Q}}(t-1) \\ &+ (1-\alpha) \left\{ (\mathbf{C}_1(t) - \tilde{\mathbf{C}}_1(t) - \text{tr}\{\tilde{\mathbf{C}}_1(t)\}) \mathbf{C}_1(t) \right. \\ &\quad \left. - \tilde{\mathbf{C}}_2(t) \mathbf{C}_2^*(t) \right\}, \end{aligned} \quad (14)$$

where  $\alpha$  is a forgetting factor close to but less than 1, and  $\text{tr}\{X\}$  denotes the trace of matrix  $X$ . Here  $\mathbf{C}_1(t)$  and  $\mathbf{C}_2(t)$

in Eq. (14) are defined by  $C_1(t) = \mathbf{y}(t)\mathbf{y}^H(t)$ ,  $C_2(t) = \mathbf{y}(t)\mathbf{y}^T(t)$ , respectively. Matrices  $\tilde{C}_1(t)$  and  $\tilde{C}_2(t)$  are moving averages of  $C_1(t)$  and  $C_2(t)$ , respectively, which are calculated by

$$\tilde{C}_1(t) = \beta\tilde{C}_1(t-1) + (1-\beta)C_1(t), \quad (15)$$

$$\tilde{C}_2(t) = \beta\tilde{C}_2(t-1) + (1-\beta)C_2(t), \quad (16)$$

where  $\beta$  is also a forgetting factor close to but less than 1, and  $\alpha > \beta$ .

The update of  $\tilde{d}_l(t)$  is

$$\begin{aligned} \tilde{d}_l(t) &= \alpha\tilde{d}_l(t-1) \\ &+ (1-\alpha) \{ (|z_l(t)|^2 - 2\tilde{v}_l(t))z_l^*(t)y(t) \\ &- \hat{v}_l(t)z_l(t)y(t) \}. \end{aligned} \quad (17)$$

In Eq. (17),  $\tilde{v}_l(t)$  and  $\hat{v}_l(t)$  are the moving averages of  $|z_l(t)|^2$  and  $(z_l^*(t))^2$ , respectively, and are calculated by

$$\tilde{v}_l(t) = \beta\tilde{v}_l(t-1) + (1-\beta)|z_l(t)|^2, \quad (18)$$

$$\hat{v}_l(t) = \beta\hat{v}_l(t-1) + (1-\beta)(z_l^*(t))^2. \quad (19)$$

### 3.3. Deflation

Since the source signals are extracted one by one from the observed signals, the filters  $\mathbf{w}_l$ 's must be constructed such that different sources are extracted. To achieve this goal, Inouye et al. and Kawamoto et al. subtracted the components contributed by extracted sources from the observed signals.[3, 8]. However, the estimation error of the separated signals results in severe miscalculation. Therefore, we decorrelate the output signals  $z_l(t)$  in the sense of fourth-order statistics using a Gram-Schmidt-like deflation:

$$\mathbf{w}_2(t) = \mathbf{w}_2(t) - \frac{\mathbf{w}_2^H(t)\tilde{\mathbf{Q}}(t)\mathbf{w}_1(t)}{\mathbf{w}_1^H(t)\tilde{\mathbf{Q}}(t)\mathbf{w}_1(t)}\mathbf{w}_1(t). \quad (20)$$

Nevertheless, this deflation process sometimes fails. To avoid the deflation, we propose the eigenvector algorithm with reference signals.

## 4. EIGENVECTOR ALGORITHM (EVA)

### 4.1. Analysis of EVAs with reference signals for MIMO static systems

In this subsection we assume that there is no noise  $\mathbf{n}(t)$  in the output  $\mathbf{y}(t)$ . Next we propose the eigenvector algorithm with reference signals. To solve the BSS problem, the following cross-cumulant between  $z_l(t)$  and the reference signal  $\mathbf{x}(t)$  is defined:

$$C_{zx} = \text{cum}\{z_l(t), z_l^*(t), \mathbf{x}(t), \mathbf{x}^*(t)\}, \quad (21)$$

where  $*$  denotes the complex conjugate and the reference signal  $\mathbf{x}(t)$  is given by  $\mathbf{f}^H\mathbf{y}(t) = \mathbf{f}^H\mathbf{H}\mathbf{s}(t) = \mathbf{a}^H\mathbf{s}(t)$  ( $\mathbf{a}^H =$

$\mathbf{f}^H\mathbf{H}$  is a vector whose elements are  $a_1, a_2, \dots, a_n$ ), using an appropriate filter  $\mathbf{f}$ . The filter  $\mathbf{f}$  is called a *reference system*. Moreover, we define the constrain  $\sigma_{z_l}^2 = \sigma_{s_{p_l}}^2$ , where  $\sigma_{z_l}^2$  and  $\sigma_{s_{p_l}}^2$  denote the variance of the output  $z_l(t)$  and a source signal  $s_{p_l}(t)$ , respectively. In the case of SISO systems, Jelonnek et al. [5, 6] have shown that the maximization of  $|C_{zx}|$  under  $\sigma_{z_l}^2 = \sigma_{s_{p_l}}^2$  leads to a closed-form expression as shown by the following generalized eigenvector problem:

$$C_{yx}\mathbf{w}_l = \lambda\mathbf{R}\mathbf{w}_l. \quad (22)$$

Then they utilized the facts that  $C_{zx}$  and  $\sigma_{z_l}^2$  can be expressed in terms of the vector  $\mathbf{w}_l$  as, respectively,

$$C_{zx} = \mathbf{w}_l^H C_{yx} \mathbf{w}_l, \quad (23)$$

$$\sigma_{z_l}^2 = \mathbf{w}_l^H \mathbf{R} \mathbf{w}_l, \quad (24)$$

where  $C_{yx}$  is a matrix whose  $(i, j)$ th element is calculated by  $\text{cum}\{y_i(t), y_j^*(t), x(t), x^*(t)\}$ ,  $\mathbf{R} = E[\mathbf{y}(t)\mathbf{y}^H(t)]$  is the covariance matrix of  $m$ -column vector  $\mathbf{y}(t)$ , and  $\lambda$  is an eigenvalue of  $\mathbf{R}^\dagger C_{yx}$ . Furthermore they have shown that the eigenvector corresponding to the maximum eigenvalue of  $\mathbf{R}^\dagger C_{yx}$  becomes the solution of the blind equalization problem in [5, 6], which is referred to as an *eigenvector algorithm* (EVA). However, the algorithm proposed by Jelonnek et al. is for SISO or SIMO infinite impulse response channel. Therefore, we want to show how the eigenvector algorithm Eq. (22) works for the BSS in the case of the MIMO static system in the frequency domain. To this end, we use the following equalities:

$$\mathbf{R} = \mathbf{H}\mathbf{\Sigma}\mathbf{H}^H, \quad (25)$$

$$C_{yx} = \mathbf{H}\mathbf{\Lambda}\mathbf{H}^H, \quad (26)$$

where  $\mathbf{\Sigma}$  is a diagonal matrix whose elements are  $\sigma_{s_i}^2$ ,  $i = 1, 2, \dots, n$  and  $\mathbf{\Lambda}$  is a diagonal matrix whose elements are  $|a_i|^2\gamma_i$ ,  $i = 1, 2, \dots, n$ . We then obtain the following theorem.

**Theorem 1.** Suppose the values  $|a_i|^2\gamma_i/\sigma_{s_i}^2$ ,  $i = 1, 2, \dots, n$  are all nonzero and distinct. If the noise  $\mathbf{n}(t)$  is absent in Eq. (2), the  $n$  eigenvectors corresponding to  $n$  nonzero eigenvalues of  $\mathbf{R}^\dagger C_{yx}$  become the vectors  $\tilde{\mathbf{w}}_l$ 's satisfying Eq. (5).

*Proof.* Based on Eq. (22), we consider the following eigenvector problem:

$$\mathbf{R}^\dagger C_{yx} \mathbf{w}_l = \lambda \mathbf{w}_l. \quad (27)$$

Then, substituting Eqs. (25) and (26) into (27), we obtain

$$\mathbf{H}^{H\dagger} \mathbf{\Sigma}^{-1} \mathbf{H}^\dagger \mathbf{H} \mathbf{\Lambda} \mathbf{H}^H \mathbf{w}_l = \lambda \mathbf{w}_l. \quad (28)$$

Since  $\mathbf{H}$  has full column rank, using a property of the pseudo-inverse operation ([9], p.433),

$$\mathbf{H}^{H\dagger} \mathbf{\Sigma}^{-1} \mathbf{\Lambda} \mathbf{H}^H \mathbf{w}_l = \lambda \mathbf{w}_l. \quad (29)$$

Multiplying Eq. (29) by  $\mathbf{H}^H$  from the left-hand side and using a property of the pseudo-inverse operation again, Eq. (29) becomes

$$\Sigma^{-1}\mathbf{A}\mathbf{H}^H\mathbf{w}_l = \lambda\mathbf{H}^H\mathbf{w}_l. \quad (30)$$

By noting that  $\Sigma^{-1}\mathbf{A}$  is a diagonal matrix whose elements,  $|a_i|^2\gamma_i/\sigma_{s_i}^2$ ,  $i = 1, 2, \dots, n$  are all nonzero and distinct, if  $\mathbf{g}_l = \mathbf{H}^H\mathbf{w}_l \neq \mathbf{0}$ , then the eigenvector  $\mathbf{g}_l$  obtained from Eq. (30) becomes the vector  $\tilde{\mathbf{g}}_l$  satisfying Eq. (5). Namely, the  $n$  eigenvectors  $\mathbf{w}_l$  corresponding to  $n$  nonzero eigenvalues of  $\mathbf{R}^\dagger\mathbf{C}_{yx}$  obtained from Eq. (27) become the vectors  $\tilde{\mathbf{w}}_l$  satisfying Eq. (5).  $\square$

## 4.2. Robust eigenvector algorithm (REVA)

In the previous subsection, we assumed that there are no noises in the output signals. In this subsection, we will show such an eigenvector algorithm in which the solutions (Eq. (5)) can be obtained, even if the noise  $n(t)$  is present in the output  $\mathbf{y}(t)$ . We then replace the covariance matrix  $\mathbf{R}$  by  $\mathbf{Q}$  defined in Eq. (10), which utilizes only the higher-order statistics. It is shown by a simple calculation (see [8]) that Eq. (10) becomes

$$\mathbf{Q} = \mathbf{H}\tilde{\mathbf{A}}\mathbf{H}^H, \quad (31)$$

where  $\tilde{\mathbf{A}}$  is a diagonal matrix defined by

$$\tilde{\mathbf{A}} = \text{diag}\{\gamma_1\tilde{a}_1, \dots, \gamma_n\tilde{a}_n\}, \quad (32)$$

$$\tilde{a}_r = \sum_{i=1}^m h_{ir}h_{ir}^*, \quad r = 1, 2, \dots, n, \quad (33)$$

and  $\text{diag}\{\dots\}$  denotes a diagonal matrix with the diagonal elements built from its arguments;  $h_{ir}$  is the  $(i, r)$ th element of  $\mathbf{H}$ .

Here, as a constraint, we take the following value:

$$\begin{aligned} |C_{zy}| &= \left| \sum_{i=1}^m \text{cum}\{z_i(t), z_i^*(t), y_i(t), y_i^*(t)\} \right| \\ &= |\mathbf{w}_l^H \mathbf{Q} \mathbf{w}_l| \\ &= \left| \sum_{i=1}^m \tilde{a}_i \gamma_i \mathbf{g}_l \mathbf{g}_l^* \right|. \end{aligned} \quad (34)$$

Then, we consider solving the problem whereby the fourth-order cumulants  $|C_{zx}|$  is maximized under the condition that  $|C_{zy}| = |\tilde{a}_{\rho_l} \gamma_{\rho_l}|$ . Then, according to the Lagrangian method, the following generalized eigenvector problem is derived from the problem:

$$\mathbf{C}_{yx}\mathbf{w}_l = \tilde{\lambda}\mathbf{Q}\mathbf{w}_l. \quad (35)$$

From the following theorem, by solving the eigenvector problem of the matrix  $\mathbf{Q}^\dagger\mathbf{C}_{yx}$ , the  $n$  eigenvectors  $\mathbf{w}_l$  ( $l = 1, 2, \dots, n$ ) correspond to the vectors  $\tilde{\mathbf{w}}_l$  ( $l = 1, 2, \dots, n$ ) in Eq. (5).

**Theorem 2.** Suppose the values  $|a_i|^2/\tilde{a}_i$ ,  $i = 1, 2, \dots, n$  are all nonzero and distinct. The  $n$  eigenvectors corresponding to  $n$  nonzero eigenvalues of  $\mathbf{Q}^\dagger\mathbf{C}_{yx}$  become the vectors  $\tilde{\mathbf{w}}_l$ 's satisfying Eq. (5).

*Proof.* We omit the proof because it is easily proved, just like Theorem 1.  $\square$

**Remark 1.** Since the matrix  $\mathbf{Q}^\dagger\mathbf{C}_{yx}$  consists of only the fourth-order cumulants, the eigenvector derived from the matrix can be obtained with as little influence from Gaussian noise as possible, which is referred to as a robust eigenvector algorithm (REVA).

## 4.3. Adaptive version of REVA

REVA can be implemented adaptively. To this end we must specify the dependency of each time  $t$ . Here we show the update procedure in the case of a two-input two-output static system.

$\tilde{\mathbf{Q}}(t)$ , which is the estimator of  $\mathbf{Q}$  at time  $t$  is also calculated by using Eq.(14).

$\tilde{\mathbf{C}}_{yx}(t)$ , which is the estimator of  $\mathbf{C}_{yx}$  at time  $t$  is calculated by

$$\begin{aligned} \tilde{\mathbf{C}}_{yx}(t) &= \alpha\tilde{\mathbf{C}}_{yx}(t-1) \\ &\quad + (1-\alpha)\{\mathbf{y}(t)\mathbf{y}^H(t)\mathbf{x}(t)\mathbf{x}^*(t) \\ &\quad - \mathbf{y}(t)\mathbf{y}^H(t)\tilde{\mathbf{v}}_x(t) \\ &\quad - \mathbf{y}(t)\mathbf{x}(t)\tilde{\mathbf{v}}_{y_1}(t) - \mathbf{y}(t)\mathbf{x}^*(t)\tilde{\mathbf{v}}_{y_2}(t)\}, \end{aligned} \quad (36)$$

where  $\tilde{\mathbf{v}}_x(t)$  and  $\tilde{\mathbf{v}}_{y_i}(t)$ ,  $i = 1, 2$  are the moving averages of  $\mathbf{v}_x(t)$  and  $\mathbf{v}_{y_i}(t)$  defined by

$$\tilde{\mathbf{v}}_x(t) = \beta\tilde{\mathbf{v}}_x(t-1) + (1-\beta)\mathbf{v}_x(t), \quad (37)$$

$$\tilde{\mathbf{v}}_{y_i}(t) = \beta\tilde{\mathbf{v}}_{y_i}(t-1) + (1-\beta)\mathbf{v}_{y_i}(t), \quad i = 1, 2, \quad (38)$$

where  $\mathbf{v}_x(t) = \mathbf{x}(t)\mathbf{x}^*(t)$ ,  $\mathbf{v}_{y_1}(t) = \mathbf{y}^H(t)\mathbf{x}^*(t)$  and  $\mathbf{v}_{y_2}(t) = \mathbf{y}^H(t)\mathbf{x}(t)$ .

The separator  $\mathbf{w}_l(t)$  is then calculated by solving eigenvector problem Eq. (35).

## 5. EXPERIMENTS

### 5.1. Simulation

We conducted a simulation experiment.  $\mathbf{H}(z)$ , which is  $z$ -transform of the mixing matrix  $\mathbf{H}(t)$ , is defined as:

$$\mathbf{H}(z) = \begin{pmatrix} 1 - 0.4z^{-1} & 0.5z^{-1} - 0.2z^{-2} \\ 0.5z^{-1} - 0.2z^{-2} & 1 - 0.4z^{-1} \end{pmatrix}. \quad (39)$$

The BSS problem is solved by adaptive REVA. To measure the separation performance, multichannel intersymbol-

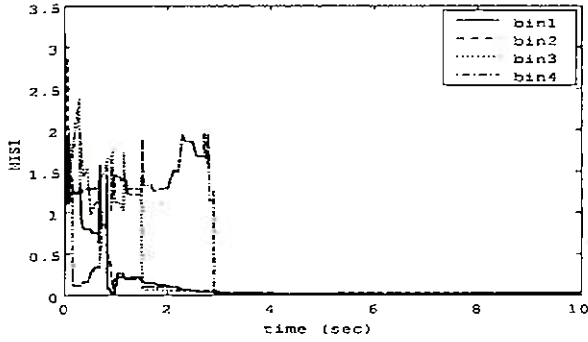


Fig. 2. MISIs of REVA

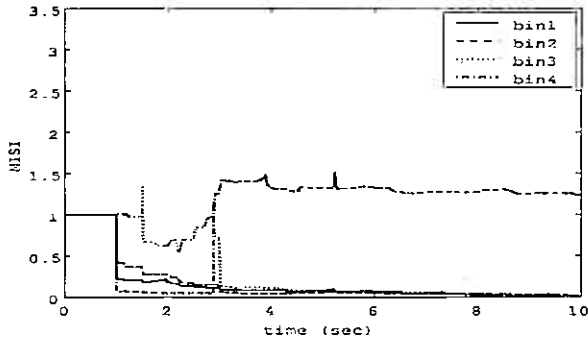


Fig. 3. MISIs of RSEM

interference (MISI) was used, which is defined as

$$\text{MISI} = \sum_{i=1}^2 \left( \frac{\sum_{j=1}^2 |g_{ij}|^2}{\max_j |g_{ij}|^2} - 1 \right) + \sum_{j=1}^2 \left( \frac{\sum_{i=1}^2 |g_{ij}|^2}{\max_i |g_{ij}|^2} - 1 \right). \quad (40)$$

The MISI becomes zero if  $\hat{g}_i$ 's satisfying Eq. (5) are obtained, and the smaller the MISI value, the closer the obtained solution is to the desired one. Figure 2 shows the MISIs of some frequency bins using EVA with the reference signal and Fig. 3 shows those of SEM, which uses the deflation process. Obviously using SEM caused the deflation process to fail in a frequency bin, whereas EVA with the reference signal could converge to the desired solution in all frequency bins.

**Remark 2.** REVA utilizes the fourth-order cumulants. Since estimating the fourth-order cumulants accurately generally requires a large number of samples, it takes a rather long time for convergence to occur when using REVA.

## 5.2. Real environment

We conducted separation experiments using REVA in an office room, with microphones and loudspeakers placed as

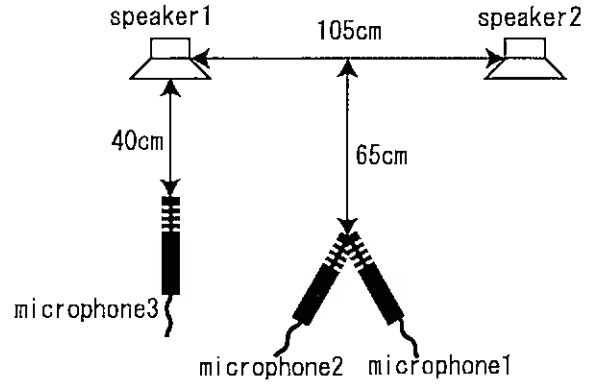


Fig. 4. Layout of the experiment

shown in Fig. 4. Because a reference signal is needed, the number of microphones is three while the number of sources is two; the observed signal of microphone 3 is used as a reference signal. Artificially, 5-dB Gaussian noises are added to the observed signals to demonstrate that the proposed REVA works in a noisy environment. Figure 5 shows a set of waveforms of the source signals, the separated signals and the enhanced signals which were provided by the ES 202 050 software [10]. In the enhanced signals, additive Gaussian noises were reduced, indicating that REVA can extract independent but distorted source signals.

## 6. CONCLUSION

In this paper we described the BSS problem in the frequency domain. We proposed and the super-exponential method (SEM) and the eigenvector algorithm (EVA) with reference signals. EVA's advantage is that all source signals are extracted simultaneously without the deflation process. Therefore, EVA can be robust to Gaussian noises using only the higher-order cumulants (REVA). In addition, an adaptive version of REVA was presented.

The computer simulations and an experiment in a real environment clarified the validity of the proposed methods.

## 7. REFERENCES

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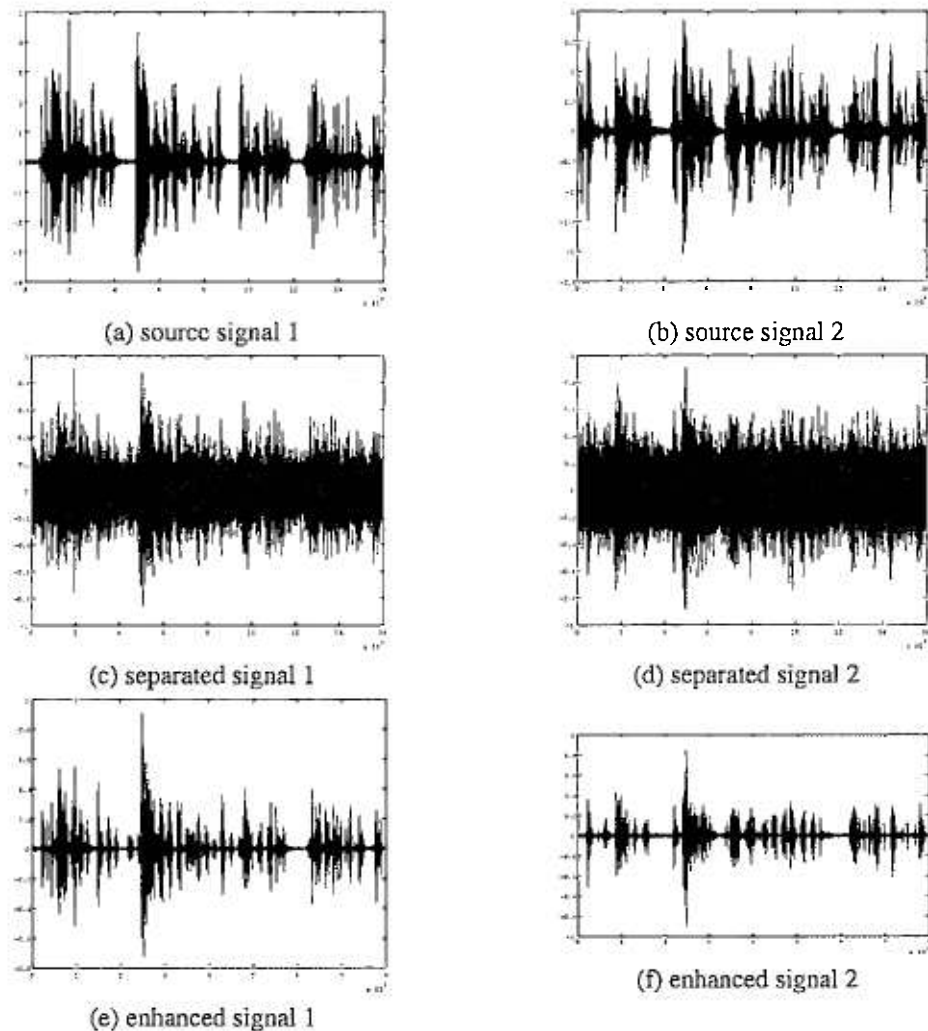


Fig. 5. Waveforms of source, separated and enhanced signals

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