# A Theory of Firm-Specific Training in Labor Markets*

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This paper reinterprets firm-specific training and examines the firm's optimal training policy. The notable feature of our model is that the extent of usefulness of skills is determined by the relationship between firms. That is, the worker's productivity depends not only on how the current firm requires specific skills but also on how the other does. Specificity matching between two firms plays an important role in the determination of productivity. We analyzed what conditions are necessary for the firms that require firm-specific skills to exist in a competitive market. In our model, worker's risk aversion is shown to be essential for the existence of these firms.

## 1 Introduction

This paper will make an attempt to tackle a simple question on firm-specific skills. That is, in the competitive product market where all the firms are producing the same goods making use of firm-specific skills, is it possible that they coexist at the market equilibrium? It might be a disadvantageous factor for firms to adopt firm-specific technology since it requires firm-specific skills and reduces the market value of human capital. Many researchers have shown the characteristics of training policies under the existance of firm-specific skills, but have not examined satisfactorily how the firms' profits differs depending on skill specificity. This paper will deal with the problem of what conditions are required to make it possible that the firms adopting different firm-specific technologies coexist at the market equilibrium.

The dichotomy of skills between firm-specific and general ones has been recognized since the seminal study by Becker (1964).[1] General skills are defined as being useful in the other firms as well as in the current, while specific skills being productive only in the current firm. According to Becker, firms never pay for the investments in general training since they can not recoup it. But the recent study by Acemoglu and Pischke (1999a, 1999b) showed that firms will pay for general training when a market is imperfect. This is because market imperfection makes workers recoup lower return from training than the one realized in a competitive market, that is, general skills are rewarded as if they are partly specific.

There has been several examinations, which elaborate Becker's dichotomy, as

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well as Acemoglu and Pischke's papers. McLaughlin (1994) regarded a worker as a collection of skills and constructed a model taking into consideration that the valuation of skills must be firm-specific on the ground that some skills are more valuable in one firm than in another. Stevens (1994) introduced the notion of transferable training, which is not equally effective for all firms, but is useful for some firms. Marimon and Zilibotti (1999) constructed a model that specifies the two-sided heterogeneity of firms and workers. That is, workers and firms are assumed to be distributed along a circle and the angle between the matched parties determines their productivity.

The purpose of this paper is to introduce intermediate skills, which are classified between firm-specific and general polar cases, to examine the effects on the training policies and to explore for the conditions under which firms requiring firm-specific skills can exist in a competitive market. The notable feature of the model in this paper is that the extent of how useful the skills that are acquired in the current firm are in the other depends on the relationship between two firms. That is, the worker's productivity depends not only on how the current firm requires specific skills but also on how the other does. If both firms require more specific skills respectively, then the skills acquired in the current are less useful in the other. Therefore the specificity matching between two firms plays an important role in the determination of productivity. This point has been neglected in the models where skills are dichotomized between general skills and specific ones, because their analysis are limited to the partial equilibrium.

The problem whether or not the specific firms can exist in a competitive market is of theoretical interest and our paper makes an attempt to tackle this problem. An important conclusion is that worker's risk aversion is essential to for firms requiring specific skills to coexist in a competitive market. But interestingly, if workers are risk-neutral, all the firms in the same product market will choose the technology which requires only general skills. At the equilibrium, it holds that the higher the skill specificity is, the more the optimal amount of training is.

In Section 2, we introduce a basic framework to examine a long-term contract. Distinguishing highly specific firms and less specific firms, we examine the characteristics of both firms. We conclude this paper in Section 3 by suggesting the areas where further researches are required.

## 2 The Model

### 2.1 The Assumptions

The model contains many identical workers and a continuum of firms. All workers are risk averse and do not discount future income. When a worker receives a wage $w$, he gains the utility $U(w)$ with $U'(w)>0$ and $U''(w)<0$. The firms are risk neutral and expected profit maximizers, facing a competitive product market. Workers live for two periods.

In this paper, we focus on the characteristics of long-term contracts[2] and consider the following two-period model.[3] The firm are assumed to offer wage packages that specifies two-period wages at the beginning of period 1.[4] In the first period, after employment decision was made, the worker receives a training program that enhances firm-specific skills. The skills affect his productivity only when he is assigned to a difficult job at period 2. At the beginning of period 2, the worker may move to the firm where only a part of the acquired skills is useful. For simplicity, we assume that such a worker receives a wage that is equal to his or her productivity evaluated at a new employer.

Each firm has two types of jobs, denoted by job E and job D respectively. As simple tasks constitute job E, all the workers who are engaged in job E are equally productive at any firm. The productivity of workers at job E is normalized to unity. The amount of training received at period 1 is denoted by $t$ and the firm bears its costs $c(t)=t$. The skills acquired through training at period 1 increases the productivity of a worker at job D, which is denoted by $x(t)$ with the properties $x(0)=1$, $x'(0)>1$, $x'(t)>0$, and $x''(t)<0$.[5]

The existance of skill specificity lowers productivity at job D in the other firms, but do not affect productivity at job E. Let $\alpha$ and $\hat{\alpha}$ denote the index of skill specificity of the current firm and the others respectively, taking values from 0 to 1, i. e. $\alpha$, $\hat{\alpha}\in[0,1]$. As $\alpha$ increases, skills are more firm-specific; $\alpha=1$ means that skills are perfectly firm-specific. The same is true for $\hat{\alpha}$. The type-$\alpha$ firm refers to the firm that enhances skills with specificity $\alpha$. Thus it is formalized that, when a worker-trained at the type-$\alpha$ firm is hired as a job-D worker at the type-$\hat{\alpha}$ firm, his output level will be $m(\alpha,\hat{\alpha})x(t)$,[6] where $m(\alpha,\hat{\alpha})$ refers to the match quality. We assume the function $m(\alpha,\hat{\alpha})$ will have the following properties: (1) $m(\alpha,\hat{\alpha})=m(\hat{\alpha},\alpha)$, (2) $\frac{\partial m(\alpha,\hat{\alpha})}{\partial\alpha}<0$, $\frac{\partial m(\alpha,\hat{\alpha})}{\partial\hat{\alpha}}<0$, and (3) $m(1,\hat{\alpha})=m(\alpha,1)=0$ for all $\alpha$, $\hat{\alpha}$.

These critical assumptions reflects that skills are less useful at the other firms where more specific skills are required to work at job D. The traditional theories which dealt with firm-specific human capital did not assume productivity depreciation in this manner. That is, they assume in a traditional way, when a worker leaves the current firm, a certain level of depreciation occurs equally at the others.[7] In other words, the model in this paper assumes that the skill specificity affects productivity both when a worker separates from the current firm and when an outside worker joins the firm. This two-sided aspects of the skill specificity is a key concept of this paper.[8)]

After acquiring the skills, the worker's job dissatisfaction becomes known to him. Let $u$ denote the job dissatisfaction measured in terms of money. If the expected utility of employment continuation, that is consisted of the job dissatisfaction and the next period's wage, exceeds that of separation, he decides to quit the current relationship. We assume that $u$ is a random variable which has a density distribution, $g(u)$.[9)] In our model, firms do not sort workers *ex-ante* by specifying personal requirements. This implies that the worker who quits the firm is assumed to face a random match with a new employer and receive a wage that is equal to the value of his productivity. As the firm is assumed to know the old worker's productivity after the employment decision, he is assigned to job E or job D depending on his productivity. More formally, if a worker trained at the type-$\alpha$ works at the type-$\hat{\alpha}$, then he will be assigned to job D if $m(\alpha, \hat{\alpha})x(t) \geq 1$ holds, or to job E otherwise.[10)] At this point, let $\alpha_1$ denote the critical level of the new employer's index of skill specificity that determines whether or not a worker is assigned to job-D. Then, $\alpha_1$ refers to a solution of the equation $m(\alpha, \hat{\alpha})x(t)=1$ with respect to $\hat{\alpha}$ and $\alpha_1=\alpha_1(\alpha, t)$ with its derivatives $\frac{\partial\alpha_1}{\partial\alpha}<0$ and $\frac{\partial\alpha_1}{\partial t}>0$.[11)]

Consider a worker who is trained at the type-$\alpha$ firm quits. Under the condition $m(\alpha, 0)x(t) \geq 1$ or equivalently $\alpha_1(\alpha, t) \geq 0$ holds, the type-$\hat{\alpha}$ firms, where $\hat{\alpha} \in [0, \alpha_1]$, will assign him to job-D and the other firms will assign him to job E. Let $\bar{\alpha}$ a solution to $m(\alpha, 0)x(t)=1$ with respect to $\alpha$.[12)] A worker trained at the type-$\alpha$ firm which $\alpha \in [0, \bar{\alpha}]$, have a chance to be employed as a Job-D worker at the type-$\hat{\alpha}$ firms, where $\hat{\alpha} \in [0, \alpha_1]$, after separation. On the other hand, there is a case that he will never be assigned to job-D after separation because his skills are too specific to utilize at the other firms. This case happens when $m(\alpha, 0)x(t)<1$, that is $\alpha>\bar{\alpha}$.

Following the assumptions above, we

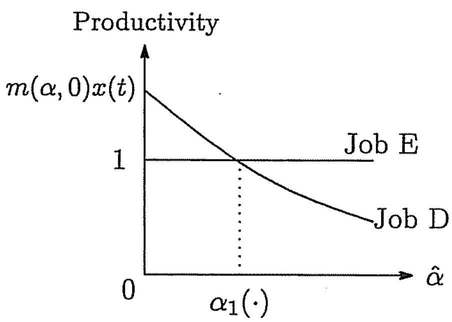


Figure 1. Productivity after separation ($\alpha \in [0, \bar{\alpha}]$)

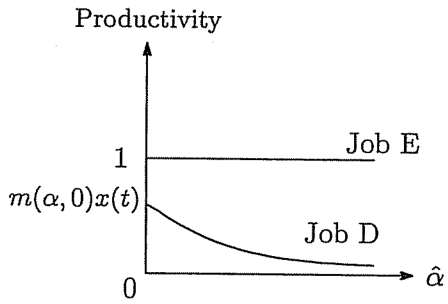


Figure 2. Productivity after separation ($\alpha \in [\bar{\alpha}, 1]$)

can show the expected utility he receives outside after quit at period 2, $EU_{quit}$, in the following two cases.

$$EU_{quit}[p, \alpha, x(t)] = \int_{-\infty}^{\infty}\int_{0}^{\alpha_1(\alpha,t)} U[p\cdot m(\alpha, \hat{\alpha})\cdot x(t)-u] f(\hat{\alpha})d\hat{\alpha}g(u)du + \int_{-\infty}^{\infty}\int_{\alpha_1(\alpha,t)}^{1} U(p-u)f(\hat{\alpha})d\hat{\alpha}g(u)du, \quad \text{for } \alpha\in[0, \bar{\alpha}] \tag{1}$$

$$EU_{quit}[p, \alpha, x(\mathrm{t})] = \int_{-\infty}^{\infty} U(p-u)g(u)du = U[\bar{x}(p)], \quad \text{for } \alpha\in[\bar{\alpha}, 1] \tag{2}$$

Here we utilize the certainty equivalent of $EU_{quit}$, which is denoted by $\hat{x}(p, \alpha, t)$, that is

$$U[\hat{x}(p, \alpha, t)]\equiv EU_{quit}[p, \alpha, x(t)].$$

Note that if $\alpha\in[\bar{\alpha}, 1]$, the certainty equivalent is independent of $\alpha$ , we differentiate the case by letting $\hat{x}=\bar{x}(p)$ as is shown in the last part of equation (2). If otherwise, we obtain its derivatives $\frac{\partial\hat{x}(\cdot)}{\partial t}>0$ and $\frac{\partial\hat{x}(\cdot)}{\partial\alpha}<0$, which play an important role in our analysis, and their derivation are shown in the Appendix. As a worker will quit if $U(w_2-u)<U(\hat{x})$, or $w_2-\hat{x}<u$, we can show that quit probability of a worker, $q$, is given by

$$q=\int_{w_2-\hat{x}}^{\infty} g(u)du=q(w_2-\hat{x}), \tag{3}$$

and the conditional expected utility of job when he continues to work at the current firm, $EU_{stay}$, is[13)]

$$EU_{stay}=\frac{\int_{-\infty}^{w_2-\hat{x}} U(w_2-u)g(u)du}{\int_{\infty}^{w_2-\hat{x}} g(u)du} = \frac{\int_{-\infty}^{w_2-\hat{x}} U(w_2-u)g(u)du}{1-q} \tag{4}$$

The firm selects $w_1$, $w_2$ and $t$ to maximize the expected profit, $\pi$, subject to the worker's supply condition. As equations (1) and (2) say that the worker's expected productivity after separation has two cases, depending on the equilibrium amount of training, it is convenient to distinguish the two cases.

## 2.2 The characteristics of highly specific firm's behavior

In this subsection, we examine the case that the condition $\alpha>\bar{\alpha}$ holds at the equilibrium. It means that the skills embodied in the current firm have no value at others and that the expected productivity is unity at any firm. In this case, the certainty equivalent of the expected utility after quit is independent of $t$, that is $\hat{x}=\bar{x}(p)$. We name such firms highly specific firms. We might say that this case deals with the firm-specific skills in Becker's sense, because the skills embodied here is utterly useless at the other firms. We can expect that the condition holds for the firms which the value of $\alpha$ is sufficiently high as will be shown below. Noting that no gain is derived from a worker employed from the outside at period 2, the expected profit to such a firm is given by

$$\pi = p - (w_1 + t) + (1-q)[px(t) - w_2], \tag{5}$$

where $q = \int_{w_2 - \bar{x}(p)}^{\infty} g(u)\,du = q(w_2 - \bar{x}(p))$.

The worker's supply condition is given by

$$U(w_1) + [1 - q(w_2 - \bar{x}(p))] \frac{\int_{-\infty}^{w_2 - \bar{x}(p)} U(w_2 - u) g(u)\,du}{1-q} + q(w_2 - \bar{x}(p))\,U(\bar{x}(p)) = \bar{U}, \tag{6}$$

where $\bar{U}$ refers to the exogenous level of expected life-time utility.

Here we define the following Lagrangian function,

$$L = p - (w_1 + t) + (1-q)[px(t) - w_2] + \lambda\left\{U(w_1) + \int_{-\infty}^{w_2 - \bar{x}(p)} U(w_2 - u) g(u)\,du + q(w_2 - \bar{x}(p))\,U(\bar{x}(p)) - \bar{U}\right\},$$

where $\lambda$ is a nonnegative multiplier. Then, the first-order necessary conditions are the following equations.

$$\frac{\partial L}{\partial \lambda} = U(w_1) + \int_{-\infty}^{w_2 - \bar{x}(p)} U(w_2 - u) g(u)\,du + q(w_2 - \bar{x}(p))\,U(\bar{x}(p)) - \bar{U} = 0. \tag{7}$$

$$\frac{\partial L}{\partial w_1} = -1 + \lambda U'(w_1) = 0, \tag{8}$$

$$\frac{\partial L}{\partial w_2} = -[1 - q(w_2 - \bar{x}(p))] - q'(\cdot)[px(t) - w_2] + \lambda \int_{-\infty}^{w_2 - \bar{x}(p)} U'(w_2 - u) g(u)\,du + \lambda\{g(w_2 - \bar{x}(p))\,U(\bar{x}(p)) + q'(w_2 - \bar{x}(p))\,U(\bar{x}(p))\} = 0, \tag{9}$$

$$\frac{\partial L}{\partial t} = -1 + [1 - q(w_2 - \bar{x}(p))]px'(t) = 0. \tag{10}$$

The system described by the above conditions can be simplified in the following way. From (3), we know that $q'(w_2 - \bar{x}(p)) = -g(w_2 - \bar{x}(p)) < 0$. Using this relationship, the last two terms in (9) reduced to zero and we have the modified version of (9) as follows.

$$-[1 - q(w_2 - \bar{x}(p))] - q'(\cdot)[px(t) - w_2] + \lambda \int_{-\infty}^{w_2 - \bar{x}(p)} U'(w_2 - u) g(u)\,du = 0. \tag{11}$$

Let $t^*$ denote the solution of the system described by the equations from (7) to (10) and $\alpha^*$ denote that of the equation $m(\alpha, 0)x(t^*) = 1$ with respect to $\alpha$, that is, $m(\alpha^*, 0)x(t^*) = 1$. We can say that $\alpha^*$ is the equilibrium level of $\bar{\alpha}$. Then, we establish the following proposition.

**Proposition 1** *For $\alpha \geq \alpha^*$, all the type-$\alpha$ firm's optimal amount of training is of the same. In this case, if $w_1 < \bar{x}(p)$, the period-2 wage is less than the productivity value.*

**Proof** As none of the equations from (7) to (10) depend on the value of $\alpha$, it is easy to see all the type-$\alpha$ firms where $\alpha \geq \alpha^*$ have the same strategy. Substituting (8) into (11) and rearranging it give us the following.[14)]

$$\begin{aligned} px(t) - w_2 &= \frac{1}{-q'(w_2 - p)}\left\{1 - q(w_2 - p) - \lambda \int_{-\infty}^{w_2 - \bar{x}(p)} U'(w_2 - u) g(u)\,du\right\} \\ &= \frac{1}{-q'(\cdot)}\left\{\int_{-\infty}^{w_2 - \bar{x}(p)} [1 - \lambda U'(w_2 - u)] g(u)\,du\right\} \\ &= \frac{1}{-q'(\cdot)} \int_{-\infty}^{w_2 - \bar{x}(p)} \left\{1 - \frac{U'(w_2 - u)}{U'(w_1)}\right\} g(u)\,du > 0. \end{aligned}$$

The sign of last equation is positive if $w_1 < \bar{x}(p)$ holds. This is because the concavity of $U$ guarantees that $U'(w_1) > U'(\bar{x}(\mathrm{p})) > U'(w_2 - u)$ for all $u < w_2 - \bar{x}(p)$.

Therefore, the bracket in the integral at the last equation always takes a positive value. As we know that $-q'(\cdot)>0$, we can show the sign of the last equation is positive, that is $px(t)>w_2$. (Q. E. D.)

As a worker who acquired highly specific skills can not utilize his skills outside the current firm, the quit probability is reduced to the minimum level. The fact that the period-2 wage is suppressed below the productivity value parallels the results of Azariadis' implicit contract theory (Azariadis (1975)). In a model incorporating worker's disutility at period 2, as Carmichael (1983) notes, firms will induce workers to quit by offering as much wages as possible. That leads to the property that $px(t)-w_2=0$ holds if a worker is risk-neutral. The residual between $px(t)$ and $w_2$ can be said to reflect the risk premium the firm can gain owing to a worker's risk aversion. With worker's risk-neutrality and competitive product market, as Ohashi (1988) has also shown, the wage profile has a positive slope, that is $w_1=p-t$ and $w_2=px(t)$. Worker's risk aversion makes the wage profile flatter as is shown in Proposition 1. If the firm earns zero profit at the equilibrium, we can see that $w_1>p-t$ and $w_2<px(t)$ hold. The profit of highly specific firms, $\pi$, is independent of the level of skill specificity, that is $\frac{\partial \pi}{\partial \alpha}=0$ for $\alpha \geq \alpha^*$.

## 2.3 The characteristics of less specific firm's behavior

Next let us focus on the case that the condition $m(\alpha,0)x(t^*)>1$ holds. In this case we will analyze the type-$\alpha$ firms where $\alpha \leq \alpha^*$, and we name them less specific firms. In this case, there is a possibility that a worker can utilize the skills embodied in the current firm to the others. The expected profit is given by the same function as the equation (5), but the quit probability is given by $q=\int_{w_2-\hat{x}}^{\infty} g(u)du=q(w_2-\hat{x})$, where $\hat{x}$ refers to the certainty equivalent of the expected utility after quit at the period 2. Note that the value of $\hat{x}$ is a function of the two variables; the amount of training, $t$, and the skill specificity, $\alpha$, with the derivatives $\frac{\partial \hat{x}}{\partial t}>0$ and $\frac{\partial \hat{x}}{\partial \alpha}<0$.[15] The worker's supply condition is also modified to the following form.

$$U(w_1)+[1-q(w_2-\hat{x})]\frac{\int_{-\infty}^{w_2-\hat{x}} U(w_2-u)g(u)du}{1-q}+q(w_2-\hat{x})U(\hat{x})=\bar{U}.$$

We modify the Lagrangian function as follows.

$$L'=p-(w_1+t)+[1-q(w_2-\hat{x})][px(t)-w_2]+\lambda\left\{U(w_1)+\int_{-\infty}^{w_2-\hat{x}} U(w_2-u)g(u)du+q(w_2-\hat{x})U(\hat{x})-\bar{U}\right\}$$

We obtain the following first-order necessary conditions through the same procedure that we have proceeded at the case of highly specific firms.

$$\frac{\partial L'}{\partial \lambda}=U(w_1)+\int_{-\infty}^{w_2-\hat{x}}U(w_2-u)g(u)du + q(w_2-\hat{x})U(\hat{x})-\bar{U}=0. \tag{12}$$

$$\frac{\partial L'}{\partial w_1}=-1+\lambda U'(w_1)=0 \tag{13}$$

$$\frac{\partial L'}{\partial w_2}=-[1-q(w_2-\hat{x})] -q'(w_2-\hat{x})[px(t)-w_2] +\lambda\int_{-\infty}^{w_2-\hat{x}}U'(w_2-u)g(u)du=0, \tag{14}$$

$$\frac{\partial L'}{\partial t}=-1+(1-q)px'(t) +\left(-\frac{\partial \hat{x}}{\partial t}\right)\{-q'(\cdot)[px(t)-w_2] -\lambda q(w_2-\hat{x})U'(\hat{x})\}=0. \tag{15}$$

Rearranging (14) with the same operation at the analysis of highly specific firms, we obtain the following condition.

$$px(t)-w_2=\frac{1}{-q'(\cdot)}\int_{-\infty}^{w_2-\hat{x}}\left\{1-\frac{U'(w_2-u)}{U'(w_1)}\right\} g(u)du. \tag{16}$$

At this point, given $p$ and $\bar{U}$, less specific firms will choose the optimal level of $w_1$, $w_2$ and $t$ according to the equations from (12) to (15) and achieve the maximized profit.

There is, however, a possibility that the equilibrium level of maximized profit varies depending on skill specificity, $\alpha$. If it occurs, the firm will gain more profit by varying the level of skill specificity.[16] To examine whether there is such a possibility or not, we obtain the following equation from the envelope theorem.

$$\frac{\partial \pi(\alpha)}{\partial \alpha}=\frac{\partial L'}{\partial \alpha}=\left(-\frac{\partial \hat{x}}{\partial \alpha}\right)\{-q'(w_2 -\hat{x})[px(t)-w_2] -\lambda q(w_2-\hat{x})U'(\hat{x})\} \tag{17}$$

where $\pi(\alpha)$ refers to the equilibrium level of expected profit ofthe type-$\alpha$ firm. If $\frac{\partial \pi(\alpha)}{\partial \alpha}<0$ holds, all the firms will prefer to the least specific firm, that is $\alpha=0$, and if $\frac{\partial \pi(\alpha)}{\partial \alpha}>0$ holds, all the firms will adopt more specific technology, that is, $\alpha \geq \alpha^*$. As we know that $-\frac{\partial \hat{x}}{\partial \alpha}>0$ and $-q'(w_2-\hat{x})>0$, we can see that $\frac{\partial \pi(\alpha)}{\partial \alpha}=0$ holds at the equilibrium if the condition $px(t)-w_2>0$ is satisfied. As our model relys on the continuous distribution of firms, it is a crucial requirement.

Let us interpret what equation (17) stands for. The first term in the second bracket of (17) refers to the marginal returns from adopting more specific technology. Skill specificity makes quit probability smaller because it lowers the worker's productivity outside. If there is a gap between productivity and wage, firms will obtain marginal returns from the increased stay probability. On the other hand, the second term of the bracket means that specific technology will simply become cost to workers. Less usefulness of skills to the others gives a negative effect on a worker's participation constraint, and firms need to compensate it.

As we know that $\frac{\partial \hat{x}}{\partial \alpha}\neq 0$ always holds, using (17), the condition $\frac{\partial \pi(\alpha)}{\partial \alpha}=0$ can be written as follows and let us call it the survival condition of less specific firms.

$$\begin{aligned} &-q'(w_2-\hat{x})[px(t)-w_2] \\ &\quad -\lambda q(w_2-\hat{x})U'(\hat{x})=0. \end{aligned} \tag{18}$$

If the condition (18) is satisfied for $\alpha \in [0, \alpha^*]$,[17] the type-$\alpha$ firms will exist at the equilibrium.

**Proposition 2** *When the less specific firms exists at the equilibrium, the optimal wages must satisfy the condition $px(t)>w_2$. If $w_1<\hat{x}$ holds, the condition is always satisfied.*

**Proof** The condition (18) requires that $px(t)-w_2>0$ holds. If it does not hold, the left hand of equation (18) is negative. It is sufficient to hold that $w_1<\hat{x}$. The proof follows the same procedure given in that of Proposition 1. (Q. E. D.)

Proposition 2 tells us why the specific firms can exist at the equilibrium. If the skills obtained after training are less useful to other firms, the worker must face a wage decrease because his productivity is lower at other firms than at the current. The wage decrease makes quit probability low or equivalently stay probability high. As the current firm will pay a wage less than his productivity, it is possible to increase the expected profit from increased stay probability. That is the reason for the specific firms to exist. Though Black and Loewenstein (1997) have also shown there may be a gap between productivity and wage, but the gap is an essential element in our model. Note that, if workers were to be risk-neutral, the condition (18) is never satisfied. Here we establish the following proposition.

**Proposition 3** *If workers are risk-neutral, any specific firms, that is, all the type-$\alpha$ firms except $\alpha=0$ will never exist at the equilibrium.*

**Proof** From the risk-neutrality we obtain $px(t)-w_2=0$ from (16).[18] In this case the right hand of (17) is strictly negative. It follows that the profit of the type-$\alpha$ firms, where $\alpha \leq \alpha^*$, is decreasing function of skill specificity. As that of highly specific firms, where $\alpha \geq \alpha^*$, is the same, the firm' s profit are maximized at $\alpha=0$. (Q. E. D.)

Let us interpret the meaning of Proposition 3. If workers were to be risk-neutral, as the firm's optimization program maximizes the social welfare because it can be easily checked from that $\lambda=1$. Then, the firm-specific training is socially unfavorable because of uselessness to other firms. Uselessness of skills lowers their expected income after the separation and it is simply disadvantageous for workers. The risk-neutrality of workers gives no opportunity for the firms to gain the gap between productivity and wage. Though skill specificity contributes to raise the stay probability, the firm will receive no gain from it. In our model, we found the rationale that specific firms can exist at the equilibrium in worker's risk-averse behavior. To avoid wage depreciation, workers will stay even if the firms assign firm-specific training. On the other hand, the firm will gain profits from such a behavior of workers.

Inserting the condition (18) into equations (14) and (15), these first-order necessary conditions are modified as follows.

$$\frac{\partial L'}{\partial w_2} = -[1-q(w_2-\hat{x})]+\lambda q(w_2-\hat{x})U'(\hat{x}) + \lambda\int_{-\infty}^{w_2-\hat{x}} U'(w_2-u)g(u)du=0, \tag{19}$$

$$\frac{\partial L'}{\partial t} = -1+[1-q(w_2-\hat{x})]px'(t)=0. \tag{20}$$

Now we can see the characteristics of the less specific firms described by the equations (12), (13), (19) and (20). We can obtain the following interpretation from the equation (20). When the less specific firms determines the optimal amount of training, it is determined by equating the expected marginal value of employment continuation to marginal cost of training.

As the previous condition (15) is determined without considering the survival condition, it indicates a cooperative condition in a sense that permits skill enhancement to utilize at the other employers. That is, marginal cost of training equals expected marginal productivity plus marginal effects on firm and worker owing to the usefulness of skills to the other firms. Under the partial analysis, an optimal amount of training is determined by considering two cases; both employment continuation and separation.[19]

Our model, however, is not confined to the partial analysis, because it considers the market condition, that is, the equilibrium level of maximized profit. Under the setting, the condition of optimal training is modified to the one that evaluates employment continuation only. Even though the skills obtained via training can be utilized to the other firms after separation, the firm will choose the optimal level of trainingwithout considering its usefulness to the other.

Since $t$ is endogenous, we conduct a comparative static analysis on the system consisting of the four equations. Through the analysis, we can obtain the following results.

**Proposition 4** *When the less specific firms determines the amount of training by the optimal policy, an increase of skill specificity leads to an increase in the amount of training under the less restricted conditions.*

**Proof** Taking the derivatives of the four equations with respect to $\alpha$, we can obtain $\frac{dt}{d\alpha}>0$ under the conditions: (1) $|U''(\hat{x})|$ is sufficiently small and (2) $U'(\hat{x})\geq 1-U''(\hat{x})$. For details, see the Appendix. (Q. E. D.)

If a worker quit the relationship with the specific firm, he must face a wage depreciation due to the skill specificity. To compensate the disadvantage, firms can offer high wages in exchange for harder training, which is a source of wage growth. As firms make risk-averse workers better-off by offering flatter wage profile, they receive the opportunity to receive gains from a gap between productivity and wage. Skill specificity itself prevents a worker from quitting so that the firms can assign a large amount of training with an increased probability of recouping their investments. The agreement of the both sides' needs can be achieved under the less restricted conditions described above. Though Ohashi (1988) have proved that more specific firms will assign a harder training, this property holds when the analysis are extended to consider the market condition.

### 2.4 Further Analysis

In this model, facing a competitive market, all the firms earn zero profit at the equilibrium. To calculate the expected profit of the highly specific firm, which is denoted by $\pi_1$, we obtain the following equation from (5).

$$\pi_1[p, t^*(p), \bar{U}]=p-w_1^*-t^* +[1-q(w_2^*-\bar{x}(\mathrm{p}))] [px(t^*)-w_2^*], \quad (21)$$

where $w_1^*$, $w_2^*$ and $t^*$ denote the solution of the equations from (7) to (10).

Note that the value of $\pi_1$ depends on $p$ and $\bar{U}$, but is determined irrespective of the distribution of firms with respect to skill-specificity, i. e. $f(\hat{\alpha})$. Once $p$ and $\bar{U}$ are determined, the value of $\pi_1$ is also determined. If the value is positive, all the firms might choose the highly specific technology, and if it is negative, no firm will adopt it. Were the level of $p$ and $\bar{U}$ are correctly determined, the highly specific firms earn zero profit.

Let us examine the case of less specific firms. The expected profit of less specific firms, denoted by $\pi_2$, is

$$\pi_2[\alpha, p, t(\alpha, p), \bar{U}\,|\,f(\hat{\alpha})] =\mathrm{p}-(w_1+t)+[1-q(w_2-\hat{x})][px(t)-w_2]. \quad (22)$$

where $w_1$, $w_2$ and $t$ denote the solution of the equations (12), (13), (19) and (20). Facing the competitive market, firms earn zero profit at the equilibrium. As our model adopts the continuous distribution of firms, the following two conditions must hold at the equilibrium.

$$\pi_2[\alpha, p, t(\alpha, p), \bar{U}\,|\,f(\hat{\alpha})]=0, \quad (23)$$

$$\frac{\partial \pi_2[\alpha, p, t(\alpha, p), \bar{U}\,|\,f(\hat{\alpha})]}{\partial \alpha}=0. \quad (24)$$

Note that the condition (24) is satisfied if equation (18) holds. Though $p$ and $\bar{U}$ are exogenous in our model and we have not specified the dynamics that describes how the distribution of firms with respect to skill specificity changes, we can expect that the model closes in the following way. Depending on the functions $q(\cdot)$ and $x(t)$, the combination of $p$ and $\bar{U}$ are determined to satisfy $\pi_1[p, t^*(p), \bar{U}]=0$. From the survival condition (18) and the zero profit condition (23), the shape of distribution $f(\hat{\alpha})$ are determined. Unless we specify the functions, we can not predict how the equilibrium distribution of firms becomes. Unfortunately our model becomes complicated when we proceed to a calculation of equilibrium profits. We utilized the condition (17) or equivalently equation (24) as a necessary condition.

## 3 Conclusions

The present paper has developed the analysis of firm-specific training with considering market conditions. One of the main results of this paper is that worker's risk aversion is essential for the firms requiring highly specific skills to coexist in a competitive market. That is, if workers are risk-neutral, all the firm in the same product market will choose the technology requiring only general skills.

Under less restricted conditions, higher skill specificity leads to an increase in the optimal amount of training. If a worker quit the relationship with such a firm, he must face wage depreciation due to skill specificity. To compensate the disadvantage, firms can offer high wages in exchange for harder training, which is a source of wage growth. As firms make risk-averse workers better off by offering flatter wage profile, they receive the opportunity to receive gains from a gap between productivity and wage. Furthermore, skill specificity itself prevents a worker from quitting so that the firms can assign a large amount of training with an increased probability of recouping their investments. Worker's risk-neutrality, however, gives no room for such an opportunity and simply leaves the disadvantages of adopting firm-specific technology.

The interesting finding of this paper is that, though there is usefulness to the other firms, the amount of training is determined without considering its usefulness to the other. That is, regardless of positive externalities from usefulness to the other, the firms will choose training intensity neglecting such externalities.

By suggesting that skill usefulness is determined by the relationship between firms, we can see how the firm-specific training is characterized in the labor markets. We have emphasized that the worker's productivity depends not only on how the current firm requires specific skills but also on how the other does. The determination of the expected productivity outside after separation, however, is just given from the assumption and seems to lack the theoretical background at this point in our model. Yet it might be safe to say that, even if some skills have usefulness to the other firms or, in other words, have generality, the extent of contribution to production may vary across the firms.

As our study just cast a light on the case of long-term contracts, the analysis of short-term contracts with firm-specific training should be examined. In this case, the bargaining between firms and workers or the possibility of renegotiation will arise. As our model has complicated-structure, it may be necessary to modify the model drastically for the analysis of short-term contracts. The problem is left to future works.

## 4 Appendix

### 4.1 Derivation of the function $\hat{x}(p, \alpha, t)$ and its derivatives

Under the assumption that each firm has two jobs, i. e. job-D and job-E, there is a critical level of the new employer's index of skill specificity, which decides whether a newly matched worker is assigned to job-D or not. Let $\alpha_1$ denote the critical value. In other words, $\alpha_1$ denotes a solution of the equation $m(\alpha, \hat{\alpha})x(t)=1$ with respect to $\hat{\alpha}$ and when a worker separates from the type-$\alpha$ firm, he can be assigned to job-D at type-$\hat{\alpha}$ firm if $\hat{\alpha} \geq \alpha_1$ holds. We can write $\alpha_1 = \alpha_1(\alpha, t)$ with $\frac{\partial \alpha_1}{\partial \alpha}<0$ and $\frac{\partial \alpha_1}{\partial t}>0$. Let $EU_{quit}$ denote the expected utility that a worker can receive outside after quit at period 2. If $m(\alpha, 0)x(t)>1$ or equivalently $\alpha_1(\alpha, t)>0$, some firms will assign a worker trained at the type-$\alpha$ to job-D and $EU_{quit}$ can be written as follows.

$$
\begin{aligned}
&EU_{quit}[p, \alpha, x(t)] \\
&\quad = \int_{-\infty}^{\infty}\int_{0}^{\alpha_1(\alpha,t)} U[p \cdot m(\alpha, \hat{\alpha}) \cdot x(t) \\
&\qquad -u]f(\hat{\alpha})d\hat{\alpha}g(u)du \\
&\qquad + \int_{-\infty}^{\infty}\int_{\alpha_1(\alpha,t)}^{1} U(p-u)f(\hat{\alpha})d\hat{\alpha}g(u)du.
\end{aligned} \tag{A1}
$$

Here, using the function $\hat{x}(p, \alpha, t)$, we define the certainty equivalent of $EU_{quit}$ in the following way.

$$U[\hat{x}(p, \alpha, t)] \equiv EU_{quit}[p, \alpha, x(t)]$$

From (A1), we can show the derivatives of the function $EU_{quit}[\alpha, x(t)]$ as follows.

$$
\begin{aligned}
&\frac{\partial EU_{quit}[p, \alpha, x(t)]}{\partial t} \\
&= \int_{-\infty}^{\infty}\int_{0}^{\alpha_1(\alpha,t)} U'[pm(\alpha, \hat{\alpha})x(t)-u] \\
&\quad \cdot pm(\alpha, \hat{\alpha})x'(t)f(\hat{\alpha})d\hat{\alpha}g(u)du \\
&\quad + \int_{-\infty}^{\infty}[U[pm(\alpha, \alpha_1)x(t)-u] \\
&\quad - U(p-u)]f(\alpha_1)\frac{\partial \alpha_1}{\partial t}g(u)du \\
&= px'(t)\int_{-\infty}^{\infty}\int_{0}^{\alpha_1(\alpha,t)} m(\alpha, \hat{\alpha})U' \\
&\quad [pm(\alpha, \hat{\alpha})x(t)-u]f(\hat{\alpha})d\hat{\alpha}g(u)du>0.
\end{aligned} \tag{A2}
$$

$$
\begin{aligned}
&\frac{\partial EU_{quit}[p, \alpha, x(t)]}{\partial \alpha} \\
&= \int_{-\infty}^{\infty}\int_{0}^{\alpha_1(\alpha,t)} U'[pm(\alpha, \hat{\alpha})x(t)-u] \\
&\quad \cdot p\frac{\partial m(\alpha, \hat{\alpha})}{\partial \alpha}x'(t)f(\hat{\alpha})d\hat{\alpha}g(u)du \\
&= px'(t)\int_{-\infty}^{\infty}\int_{0}^{\alpha_1(\alpha,t)} \frac{\partial m(\alpha, \hat{\alpha})}{\partial \alpha} \\
&\quad \cdot U'[pm(\alpha, \hat{\alpha})x(t)-u]f(\hat{\alpha})d\hat{\alpha}g(u)du<0.
\end{aligned} \tag{A3}
$$

The last inequity of (A3) is obtained from the assumption $\frac{\partial m(\alpha, \hat{\alpha})}{\partial \alpha}<0$. As we know that $U(\cdot)$ is monotonically increasing function, we obtain the following properties of the function $\hat{x}(\cdot)$.

$$\frac{\partial \hat{x}(p, \alpha, t)}{\partial t}>0, \tag{A4}$$

$$\frac{\partial \hat{x}(p, \alpha, t)}{\partial \alpha}<0. \tag{A5}$$

In addition, if $m(\alpha, 0)x(t)<1$ or equivalently $\alpha_1(\alpha, t, h)<0$, no firm will assign a worker who is trained at the type-$\alpha$ to job-D, then the certainty equivalent of expected utility depends only on $p$. This means that it can be written as $\hat{x}(p, \alpha, t)=\bar{x}(p)$. It follows from

$$EU_{quit}[p, \alpha, x(t)]$$
$$= \int_{-\infty}^{\infty}\int_0^1 U(p-u)f(\hat{\alpha})d\hat{\alpha}g(u)du$$
$$= \int_{-\infty}^{\infty} U(p-u)\int_0^1 f(\hat{\alpha})d\hat{\alpha}g(u)du$$
$$= \int_{-\infty}^{\infty} U(p-u)g(u)du = U[\bar{x}(p)].$$

Differentiating by $p$, we obtain

$$\frac{\partial \bar{x}(p)}{\partial p} > 0.$$

It follows from

$$\frac{\partial EU_{quit}[p, \alpha, x(t)]}{\partial p}$$
$$= \int_{-\infty}^{\infty} U'(p-u)g(u)du > 0.$$

## 4.2 Proof of Proposition 4

For notational simplicity, let the subscripts 1, 2, 3 and $\alpha$ accompanied to $L$ refer to the derivatives of $L$ with respect to $w_1$, $w_2$, $t$ and $\alpha$ respectively. The same rule applies to twice differentials. For example, $L_{11}$ means $\frac{\partial^2 L}{\partial w_1^2}$. Let us denote the worker's supply constraint by $s(\cdot)=0$. Taking the derivatives of the equations (12), (13), (19) and (20) with respect to $\alpha$, we obtain the following equation.[20] Note that we omitted the prime accompanied to $L$.

$$\begin{bmatrix} 0 & s_1 & s_2 & s_3 \\ s_1 & L_{11} & 0 & 0 \\ s_2 & 0 & L_{22} & L_{23} \\ s_3 & 0 & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} d\lambda \\ dw_1 \\ dw_2 \\ dt \end{bmatrix} = - \begin{bmatrix} s\alpha \\ 0 \\ L_2\alpha \\ L_3\alpha \end{bmatrix} d\alpha. \tag{A6}$$

with the following notations:

$$s_1 = U'(w_1) > 0,$$
$$s_2 = \int_{-\infty}^{w_2-\hat{x}} U'(w_2-u)g(u)du > 0,$$
$$s_3 = pq(\cdot)U'(\hat{x})\frac{\partial \hat{x}}{\partial t} > 0,$$
$$s\alpha = q(w_2-\hat{x})U'(\hat{x})\frac{\partial \hat{x}}{\partial \alpha} < 0,$$
$$L_{11} = \lambda U''(w_1) < 0,$$
$$L_{23} = L_{32} = -q'(\cdot)px'(t) > 0,$$
$$L_{22} = q'(\cdot) + \lambda\int_{-\infty}^{w_2-\hat{x}} U''(w_2-u)g(u)du < 0,$$
$$L_2\alpha = q'(\cdot)\left(-\frac{\partial \hat{x}}{\partial \alpha}\right)[1-\lambda U'(p\hat{x})]$$
$$+\lambda q'(\cdot)\left(-\frac{\partial \hat{x}}{\partial \alpha}\right) + \lambda q(\cdot)U''(\hat{x})\frac{\partial \hat{x}}{\partial \alpha}$$

If $w_1 < \hat{x}$, then $1-\lambda U'(\hat{x}) > 0$ holds and, in addition, if $|U''(\hat{x})|$ is sufficiently small, we have $L_2\alpha < 0$.

$$L_3\alpha = -q'(\cdot)\left(-\frac{\partial \hat{x}}{\partial \alpha}\right)px'(t) > 0.$$

Therefore, by Cramer's formula, we obtain the following equations.

$$\frac{dt}{d\alpha} = \frac{-1}{|D|}\begin{vmatrix} 0 & s_1 & s_2 & s_\alpha \\ s_1 & L_{11} & 0 & 0 \\ s_2 & 0 & L_{22} & L_2 \\ s_3 & 0 & L_{32\alpha} & L_{3\alpha} \end{vmatrix} \tag{A7}$$

where $|D|$ means the determinant of the coefficient matrix. We have $|D| < 0$ from the second-order condition. Then, we obtain

$$\frac{dt}{d\alpha} = \frac{-1}{|D|} \times$$
$$\{s_\alpha L_{11}[s_2L_{32} - s_3L_{22}] + s_2s_3L_{11}L_{2\alpha}$$
$$-s_2^2L_{3\alpha}\,L_{11} + s_1^2[L_{2\alpha}\,L_{32} - L_{3\alpha}L_{22}]\} \tag{A8}$$

In the bracket of the right hand, the sign of the term $s_1^2[L_{2\alpha}L_{32} - L_{3\alpha}L_{22}]$ can not be easily identified, while all the other terms are positive

$$L_{2\alpha}L_{32}-L_{3\alpha}L_{22}$$
$$=px'(t)q'(\cdot)\frac{\partial\hat{x}}{\partial\alpha}\times$$
$$\left\{-\lambda q'(\cdot)\cdot[U'(\hat{x})-1+U''(\hat{x})]\right.$$
$$\left.-\lambda\int_{-\infty}^{w_2-\hat{x}}U''(w_2-u)g(u)du\right\}$$

If $U'(\hat{x})-1+U''(\hat{x})>0$ holds, the term mentioned above have a positive sign, therefore the sign of $\frac{dt}{d\alpha}$ will be positive. To summarize, under the conditions (1) $w_1<\hat{x}$, (2) $|U''(\hat{x})|$ is sufficiently small, and (3) $U'(\hat{x})\geq 1-U''(\hat{x})$ holds, the sign of $\frac{dt}{d\alpha}$ will be positive. (Q. E. D.)

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## Notes

1 ) We can see a good survey on skills and training in Smits and Stromback (2001).

2 ) The exclusive usefulness of firm-specific training or relation-specific investments causes underinvestment problem that is called hold-up problem (MacLeod and Malcomson (1993a, 1993b), Malcomson (1997)). To examine the problem, short-term contract models serve effective tools.

3 ) On constructing the basic structure in this paper, we are motivated from the model of Ohashi (1988).

4 ) The framework of our model is similar to that of the full-competition regime in Acemoglu and Pischke (1999b). Their model, however, assume that training is general.

5 ) Though we introduce the heterogeneity of firms later, the function that determines efficiency of training, i. e. $x(t)$, is assumed to be common across the firms. This assumption avoids an easy conclusion that the firm has high efficiency of training when skill specificity is high.

6 ) When the output realization is assumed in this manner, the element of training can not be decomposed into the general/specific training in Becker's sense. The assumption formalize a situation in which firms have differentiated skill requirements and skill specificity is determined by the relationships between firms.

7 ) It seems to me that traditonal theories have defined the firm-specific skill in the relationship between only two types of firm or the current firm and the competitor ; general skill enhances the productivity equally at both, while firm-specific skill only at the current firm. In this paper, as the heterogeneity of firms is introduced, the relationship between the currentand the competitor is modified.

8 ) When both workers and firms are heterogeneous, the matching of the two is crucial to determine productivity. Crawford and Knoer (1981) represented such an idea with the one-to-one function. Marimon and Zilibotti (1999) utilized a circle, where both workers and firms are distributed along and the angle of the two determines productivity.

9 ) When a worker is hired at the new employer after separation, he is assumed to face a new draw of job dissatisfaction from the same distribution.

10) If, at the eqilibrium, $1>m(\alpha,0)x(t)$ holds, no

firm will assign the worker who quitted the type-$\alpha$ to job D.

11) To obtain these properties, differenciate $m(\alpha, \hat{\alpha})x(t)=1$.

12) From the definition, $\alpha_1(0, t)=\bar{\alpha}(t)$.

13) In the case where $\hat{x}=\bar{x}$, the definition of $q$ and $EU_{stay}$ given by (3) and (4) are applied by replacing $\hat{x}$ by $\bar{x}$.

14) We assume that $-q'(w_2-p)=g(w_2-p)\neq 0$.

15) We can see how the derivatives of the function $\hat{x}(\cdot)$ are obtained in the Appendix.

16) Here, we have made the following assumptions implicitly. At the very initial stage, a firm choose its production technology or specificity of required skill randomly. Furthermore, as a firm adjust its skill specificity smoothly, it matters the derivatives of expected profit with respect to skill specificity.

17) Here we assume that by adjusting the distribution of firms with respect to skill specificity, $f(\hat{\alpha})$, properly, the condition (18) holds for all $\alpha$ that belongs to the domain $K$.

18) As the risk neutrality means $U'(\cdot)=0$, we have $\dfrac{U'(w_2-u)}{U'(w_1)}=1$.

19) We can find similar results, for example, in equation (18) of Ohashi (1988) or Proposition 5 of Acemoglu and Pischke (1999b).

20) One of helpful readings on static optimization or comparative statics is Lēonard and Long (1992).

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