The Effects of Knowledge Accumulation on Intergenerational Allocation of an Exhaustible Resource with Amenity Value*

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This paper analyzes equilibrium intergenerational allocation of an exhaustible resource with amenity value in an overlapping generations (OLG) economy. The paper first reviews two allocation schemes and concludes the 'trust fund' scheme, which is deemed to be 'intergenerationally democratic', is preferred to the 'grandfathering' scheme, which represents the business-as-usual practice of resource use. It then introduces knowledge accumulation into the model by imposing a public R&D sector that creates new knowledge. It concludes that augmented knowledge stock leads the economy to higher stationary consumption levels and larger stocks of the resource to be preserved forever in the steady states under both of the above schemes. This is because the introduction of a public R&D sector addresses externalities concerning the creation of new knowledge. Furthermore, the paper also concludes that through the manipulation of the new knowledge output over time, it is possible to achieve certain 'democratic' allocation effects under the business-as-usual practice without reallocating property rights among generations.

1. Introduction

There is a large amount of literature that addresses the allocation of exhaustible resources among generations and its implications for economic growth and intergenerational equity. For example, Olson and Knapp (1997) analyze competitive allocations of an exhaustible resource in an OLG model and conclude the behavior of equilibrium extractions is endogenously determined. Gerlagh and Keyzer (2001) study an OLG model of an exhaustible resource with amenity value and compare several allocation scenarios. It is clear to see that recognizing the resource amenity leads the economy to a more optimal path.

However, knowledge accumulation, being an important dynamic element and a key index that distinguishes the present from the future, is largely missed in the ongoing discourse. The projections based on the implicit assumption that the stock level of knowledge remains to be constant over time are not well grounded and often unrealistic. On the other hand, outcomes obtained from allocation schemes that treat the effects of knowledge accumulation 'too optimistically' generally lead to scenarios in which later generations might be deprived of the rights to sustain their livelihood even at subsistence levels. Therefore, it is highly urgent, and has policy relevance in abundance to construct

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a more comprehensive framework so that the following fundamental questions can be addressed: Will knowledge accumulation lead to more sustainable utilization of the resource? How will its evolvement influence various intergenerational allocation schemes? And subsequently, can each generation benefit from it and how?

The current paper expands the literature by explicitly introducing knowledge accumulation into the model of Gerlagh and Keyzer (2001) and examining its effects on the intergenerational allocation of an exhaustible resource with amenity value. Since the main focus is to examine the consequences of exhaustibility, we only examine the accumulation of reactionary knowledge,[1] which reacts to the scarcity of resources by developing technologies that utilize the inputs more efficiently.

Examples of exhaustible resources with amenity values[2] represent the focuses of concern in the recent literature on pollution control and global warming.[3] Clearly fossil fuel should not be classified into this category: It has no value other than as inputs to production. However, as suggested by Heal (1998), the atmosphere,[4] most of the other crucial environmental resources, and even the environment itself fit this category to a first approximation.[5] Recently negotiated intergovernmental treaties aiming at restricting the amount of the resource that the current generation is entitled to extract, such as the Kyoto Protocol on climate change,[6] are based on the conclusions of the above literature.[7] These treaties call for new orders of exploiting exhaustible resources, and consequently, the establishment of international allocation agencies to supervise the redistribution process. However, the controversies surrounding the ratification of the Kyoto Protocol suggest this concept, as represented by the 'trust fund' scheme, though idealistic, may not be favored in reality. Therefore, alternative practices that offer similar allocation effects without altering the business-as-usual distribution practice are highly needed in the discussion.

In this paper, we first review the model of Gerlagh and Keyzer (2001) and conclude that the 'trust fund' scheme is preferred to the 'grandfathering' scheme. Next, we incorporate knowledge accumulation into the model. Our conclusion is that knowledge accumulation leads to higher stationary consumption levels and larger stocks of the resource to be preserved forever in the steady states under both of the schemes. We conclude that this is because the introduction of a public R&D sector corrects the externalities concerning the new knowledge. Furthermore, we also examine the possibility of achieving certain 'democratic' allocation effects under the grandfathering scheme with a public R&D sector.

The paper is organized as follows. Section 2 contains a formal description of the model. Section 3 introduces the two allocation schemes that entitle each generation to a nonnegative amount of claim and compares the resultant steady states. In Section 4, we explicitly introduce knowledge accumulation and examine the accompanying differences, and Section 5 concludes.

2. Model specification

The model to be examined in this paper is one with an exhaustible resource that has amenity value. The economy contains an allocation agency, a production sector, and an infinite sequence of overlapping generations that consists of finite-lived consumers. Time is discrete, with $t \in T = \{1, \cdots, \infty\}$. It is assumed that there are two generations in each period, a young and an old generation, each consists of a representative agent. The consumers live two periods and are indexed by their date of birth, with the initial old and young generations designated as generations 0 and 1, respectively. The production sector consists of competitive firms and produces one good. The allocation agency, which is endowed with the whole initial value of the resource at the beginning of period 1, allocates its assets to generations over time.[8)]

Let x_t be the resource stock level of the resource at the beginning of period t, from which r_t units are to be extracted in period t.[9)] The transition equation for the resource stock is as follows:

$$x_{t+1} = x_t - r_t. \tag{2.1}$$

The stock x_t is assumed to yield the amenity value within the same period.[10)]

Let p_t^r be the price of the resource as input, and p_t^x the price of the resource's amenity value, respectively. According to this definition, the total value derived from the resource in each period is $p_t^r r_t + p_t^x x_t$. In period t, φ_t is the given Lindahl price for the consumption of the resource amenity. Because the amenity value to be derived from the resource is non-rivalry in nature, it is straightforward to notice that agents living in the same period face an identical level of amenity value,

$$x_t^1 = x_t^2 = x_t, \tag{2.2}$$

and, according to the definition of Lindahl prices, we have:

$$p_t^x = \varphi_t^2 + \varphi_t^1. \tag{2.3}$$

For generation t, c_t^1 and x_t are the consumption of the consumption goods and the consumption of the resource amenity when young, respectively; while c_{t+1}^2 and x_{t+1} denote the corresponding values when old. Each generation maximizes its lifetime utility derived from the above two sources, namely, the consumption goods and the amenity value of the resource. Thus, the consumers' utility function is $U(c_t^1, x_t, c_{t+1}^2, x_{t+1})$, which is to be maximized subject to the constraint that expenditures on consumption in each period are less than or equal to the available income. Furthermore, the utility function is postulated to be non-negative, differentiable, strictly concave and increasing in all four arguments, and satisfies the Inada conditions for c_t^1, c_{t+1}^2, x_t, and x_{t+1}. In addition, we postulate that c_1^2 and x_1 of the first old agent to be strictly positive and exogenously determined. In this economy, the price of the consumption goods is p_t^c in each period.

Young agents are endowed with 1 unit of labor, to be supplied inelastically. A profit-maximizing firm hires labor and purchases the exhaustible resource as inputs to produce consumption goods. The price of the labor input is w_t, and the firm's production function, $F(r, l)$, is assumed to be homogeneous of degree one and exhibits constant returns to scale, with its reduced form denoted by $y_t=f(r_t)$, with $F(r, 1)\equiv f(r)$, where y_t is the output of the consumption goods in period t. We assume f is continuous, increasing, differentiable and strictly concave, with $f(0)>0$, and $\infty>f'(0)>0$, i.e., the resource input is assumed to be important, but not essential for production. Together with exhaustibility, the last assumption implies the production technology permits consumption goods to churn out even when r_t is zero.

In the economy, there are three separate markets to determine the distribution of the resource stocks and production output within each period. The first, a resource market, consists of the allocation agency, whose objective is to maximize the value of resource output over time, both as a production input and as a source of amenity value; the firms, who purchase the resource stock as production inputs under the motivation of maximizing its profits; and consumers, who pay for the resource amenity values and aim to maximize their utility levels. In the resource stock market, the allocation agency is the supply side, while the demand side consists of the firms and consumers. Their interaction determines the equilibrium prices and the extraction levels within each period. The second market, a consumer goods market, consists of firms and consumers. Together they determine the equilibrium prices and quantities of the consumer goods in each period. The third is a labor market, consisting of workers (young agents) and firms, in which the equilibrium size of the work force and the wage level are determined.

The allocation agency facilitates value transformation over periods in the form of private saving through the issuance and the support of the trading of certificates that represent the claims to the resource. In each period, the allocation agency aggregates the demands for the resource amenity as reported by the young and the old consumers, x_t, so as to implement the Lindahl price mechanism.

Firms

Firms in this economy act competitively under the motivation of maximizing their profits. In equilibrium, they purchase each input to the point where its price equals its marginal product so that

$$p_t^c f'(r_t)=p_t^r, \tag{2.4}$$

and

$$p_t^c f(r_t)-p_t^c f'(r_t)r_t=w_t. \tag{2.5}$$

Using (2.4), (2.5) can be restated as (2.5′), which is the firm's zero-profit condition

$$p_t^c y_t = w_t + p_t^r r_t. \tag{2.5'}$$

In equilibrium, the asset price, ψ_{t+1}, and extractive (input) prices for the resource, $p_t^c f'(r_t)$, should be the same, $p_t^c f'(r_t) = \psi_{t+1}$. Here ψ_t denotes the price of the resource stock at the beginning of the period t. Combined with equation (2.1) and (2.5′), the above analysis implies that over time

$$\psi_t = p_t^c f'(r_t) + p_t^x, \tag{2.6}$$

which amounts to say that the introduction of the amenity value increases the price of the resource by the price of its amenity value.

Consumers

From the allocation agency, each agent in generation t receives an income claim to the natural resource of the amount H_t^1 when young, and H_{t+1}^2, when old. An agent's life-cycle budget constraint facing generation t is as follows:

$$\begin{aligned} &p_t^c c_t^1 + p_{t+1}^c c_{t+1}^2 + \varphi_t^1 x_t \\ &\quad + \varphi_{t+1}^2 x_{t+1} \leq w_t + H_t^1 + H_{t+1}^2. \end{aligned} \tag{2.7}$$

The first generation maximizes her utility subject to a one-period budget constraint

$$\begin{aligned} &\max_{c_1^2, x_1,} U(c_1^2, x_1), \\ &\text{subject to}: H_1^2 \geq p_1^c c_1^2 + \varphi_1^2 x_1. \end{aligned} \tag{2.8}$$

Notice each agent votes on the resource stock level in the two periods in which she lives, the consumer optimization problem for an agent born at time $t \geq 1$ is

$$\begin{aligned} &\max_{c_t^1, c_{t+1}^2, x_t, x_{t+1}} U(c_t^1, x_t, c_{t+1}^2, x_{t+1}), \\ &\quad \text{subject to (2.1) and (2.7).} \end{aligned} \tag{2.9}$$

Under the assumptions of the utility function, there is a unique interior solution to this problem. The first order conditions are given by the following equations:

$$U_{c_t^1}/U_{c_{t+1}^2} = p_{t+1}^c/p_t^c, \tag{2.10}$$

$$U_{c_{t+1}^2}/U_{x_{t+1}} = \varphi_{t+1}^2/p_{t+1}^c, \tag{2.11}$$

$$U_{c_t^1}/U_{x_t} = \varphi_t^1/p_t^c, \tag{2.12}$$

$$U_{x_t}/U_{x_{t+1}} = \varphi_{t+1}^2/\varphi_t^1. \tag{2.13}$$

Here $U_{c_t^1}$, $U_{c_{t+1}^2}$, U_{x_t}, and $U_{x_{t+1}}$, denote the derivatives of utility with respect to consumption of the consumption goods and the consumption of resource amenity when young and old, respectively. Equations (2.10)–(2.13) imply when agents optimize their consumption, they equate the marginal rate of substitution to the ratio of the corresponding prices, with all the prices in their present values.

Allocation agency

The need to establish an international institution to allocate resource intergenerationally has been widely debated in the last several decades. Kennan (1970) initiates the discussion, and argues such an 'International Environmental Agency' is capable of alleviating various forms of stresses resulting from biodegradation. The main objective for such an institution is to establish a channel through which future generations' willingness to pay for environmental resources can be properly expressed and recognized. Put it in other words, this is a built-in mechanism to ensure that the current generation takes future generations' interests into account and chooses on behalf of them, thus establishing some form of "intergenerational equity".[11] The essence of this idea goes as follows: a resource allocation agency, to which all the resource values are attributed, allocates each individuals to come a share of the resource, and charges any additional use beyond what is allocated.

The objective of the allocation agency is to maximize the aggregated output over time, subject to the production technology to be specified in the following sections

$$\operatorname*{Max}_{x_t, r_t} \sum_{t=1}^{\infty} (p_t^c y_t + p_t^x x_t).^{12)} \qquad (2.14)$$

The value of total assets held by the allocation agency at period t, F_t, is the sum of the value of the resource to be allocated to the subsequent generations at the ending of the same period

$$F_t = H_{t+1}^2 + \sum_{\tau=t,\cdots,\infty} (H_{\tau+1}^1 + H_{\tau+2}^2). \qquad (2.15)$$

Along the same lines, the value of the resource the allocation agency holds at the beginning of the first period equals

$$\psi_1 x_1 = \sum_{t=1,\cdots,\infty} (H_t^1 + H_t^2). \qquad (2.16)$$

It is straightforward to notice the stock values evolve over time as follows

$$\psi_t x_t = p_t^r r_t + p_t^x x_t + \psi_{t+1} x_{t+1}. \qquad (2.17)$$

Equilibrium

DEFINITION 1. An *OLG competitive equilibrium* is an intertemporal resource allocation defined as $\{p_t^r, p_t^x, p_t^c, \varphi_t^1, \varphi_t^2, \psi_t, r_t, x_t, w_t, c_t^1, c_t^2\}$ that solves the maximization problems of the allocation agency, consumers and producers and clears all markets for $t=1, \cdots, \infty$.

In Section 3, we set to study the equilibrium path of the OLG economy. Normally, in an OLG economy with an exhaustible resource, in which the resource is essential to production and capital does not exist, the steady state occurs only when the resource is exhausted and the extraction is zero (Olson and Knapp, 1997). However, as indicated in the next section, together with the assumption that resource is not essential to the production process, the introduction of amenity value may lead to significantly different conclusions.

3. Allocation schemes and steady states

In order to investigate the properties that are pertinent to our focus in a more accurate manner, next we specify the utility function of consumers and the production function of individual firms. This elaboration presents the outcomes in a much clearer fashion and enables explicit comparison with outcomes obtained after the introduction of a public R&D sector in Section 4.[13]

ASSUMPTION 1. Each generation has a separable, log utility function and a Cobb Douglas branch for the substitution between the amenity value and the consumer goods

$$U(c_t^1, x_t, c_{t+1}^2, x_{t+1}) = \ln((c_t^1)^{1-v}(x_t)^v) + \beta \ln((c_{t+1}^2)^{1-v}(x_{t+1})^v), \qquad (3.1)$$

where β is the subjective discount factor. The above assumption also implies that each agent spends a constant share $0<v<1$ of their income on the non-rival consumption of the resource amenity, while the rest, $(1-v)$ is spent on the consumption goods. Production function is specified as follows:

$$f(r)=[\alpha l^{-\sigma}+(1-\alpha)r^{-\sigma}]^{-1/\sigma}, \quad 0<\alpha<1, \text{ and } -1<\sigma<0. \tag{3.2}$$

Following the exhaustibility of the resource, it is assumed that the initial stock, $\psi_1 x_1$, is finite.[14] Each generation's consumer optimization problem yields their desired net investment, s_t, which is the difference between the earning and consumption when young

$$s_t=p^c_t f(r_t)-p^c_t f'(r_t)r_t + H^1_t - p^c_t c^1_t - \varphi^1_t x_t. \tag{3.3}$$

Each generation maximizes her utility, as depicted in (3.1), subject to the budget constraint described in the previous section. The first order condition indicates that in equilibrium, the consumption when young for generation $t\geq 1$ is

$$p^c_t c^1_t=[p^c_t f(r_t)-p^c_t f'(r_t)r_t+H^1_t + H^2_{t+1}](1+v)/(1+\beta), \tag{3.4}$$

while the life-cycle saving is

$$\begin{aligned} s_t &= p^c_t f(r_t)-p^c_t f'(r_t)r_t \\ &\quad + H^1_t-[p^c_t f(r_t)-p^c_t f'(r_t)r_t \\ &\quad + H^1_t+H^2_{t+1}]/(1+\beta) \\ &= p^c_t f'(r_t)x_{t+1}-F_{t+1}. \end{aligned} \tag{3.5}$$

The far right-hand side of equation (3.5) depicts the nature of the private life-cycle savings. The young generation consumes some of their income and purchases resources with the rest at the market price p^r_t. However, the existence of the allocation agency implements restrictions on the amount that each young agent can purchase, as the allocation agency also relies on its possession of the resource, if not of all that is left, to finance its allocation scheme.

Next we set to describe alternative schemes that the allocation agency can adopt. These schemes stand as specific rules for resource extraction, and for allocating the resource value, while meeting their respective intertemporal budget constraint.

3.1 *Grandfathering scheme*

The grandfathering scheme entitles the first old generation to own the initial resource exclusively, $H^2_1=\psi_1 k_1$, and for generations $t\geq 1$, $H^1_t=H^2_{t+1}=0$. The old then exchanges her claims with her successors to finance her retirement years.[15] Such a transaction takes place in the resource market, and instead of the allocation agency, the old is the only supplier. Under this scheme, the life-cycle budget constraint of an agent born since period 1 is

$$p^c_t c^1_t+p^c_{t+1}c^2_{t+1}+\varphi^1_t x_t+\varphi^2_{t+1}x_{t+1}\leq w_{gt}.\text{[16]} \tag{3.6}$$

Under the grandfathering scheme,[17] optimal consumption level of the consumption goods when young, c^1_{gt}, and desired life-cycle saving, s_{gt}, are

$$c^1_{gt}=[f(r_{gt})-f'(r_{gt})r_{gt}](1-v)/(\beta+1), \tag{3.7}$$

and

$$\begin{aligned} s_{gt} &= p^c_{gt}[f(r_{gt})-f'(r_{gt})r_{gt}]\beta/(\beta+1) \\ &= p^c_{gt}f'(r_{gt})(x_{gt}-r_{gt})>0. \end{aligned} \tag{3.8}$$

It is straightforward to deduce that when $r_{gt}>0$, the equilibrium extraction rule is given by

$$x_{gt}=\left[\frac{\alpha\beta}{(1+\beta)(1-\alpha)}+r_{gt}^{-\sigma}\right]/r_{gt}^{-\sigma-1}. \quad (3.9)$$

3.2 *Trust fund scheme*

The second allocation scheme, a trust fund,[18] entitles every generation to an income claim to one unit of the resource amenity, that is, for $t=0,\cdots,\infty$, $H_t^1=\varphi_t^1 x_1$, and $H_t^2=\varphi_t^2 x_1$. Under this scheme, the life-cycle budget constraint for generation $t\geq 1$ is

$$p_t^c c_t^1+p_{t+1}^c c_{t+1}^2+\varphi_t^1 x_t+\varphi_{t+1}^2 x_{t+1}\leq w_{ft}+\varphi_t^1 x_1+\varphi_{t+1}^2 x_1.^{19)} \quad (3.10)$$

It is easy to deduce that the income claims to resources that the agents are entitled to are : $H_t^1=\varphi_t^1 x_1=vp_t^c c_t^1 x_1/[(1-v)x_t]$, and $H_{t+1}^2=\varphi_{t+1}^2 x_1=\beta vp_t^c c_t^1 x_1/[(1-v)x_{t+1}]$, respectively. Under the trust fund scheme,[20] the optimal consumption level when young, c_{ft}^1, is

$$c_{ft}^1=[f(r_{ft})-f'(r_{ft})r_{ft}](1-v)/[(\beta+1)-vx_1/x_{ft}-\beta vx_1/x_{f(t+1)}], \quad (3.11)$$

while the life-cycle saving of each agent, s_{ft}, is

$$s_{ft}=p_{ft}^c[f(r_{ft})-f'(r_{ft})r_{ft}]\left\{1+\frac{vx_1-x_{ft}}{x_{ft}\left[(\beta+1)-\frac{vx_1}{x_{ft}}-\frac{\beta vx_1}{x_{ft}-r_{ft}}\right]}\right\}=p_{ft}^c f'(r_{ft})(x_{ft}-r_{ft})-F_{f(t+1)}. \quad (3.12)$$

The far right-hand side of the above equation follows (3.5). From (2.14) and (2.15), it is straightforward to induce that over time, $F_t=\psi_t x_1$, which is sufficient for the trust fund to meet its commitment. Hence, (3.12) changes into the following one, which implicitly defines the equilibrium extraction rule under the trust fund scheme

$$s_{ft}=p_{ft}^c[f(r_{ft})-f'(r_{ft})r_{ft}]\left\{1+\frac{vx_1-x_{ft}}{x_{ft}\left[(\beta+1)-\frac{vx_1}{x_{ft}}-\frac{\beta vx_1}{x_{ft}-r_{ft}}\right]}\right\}=p_{ft}^c f'(r_{ft})(x_{ft}-r_{ft}-x_1). \quad (3.12')$$

3.3 *Steady states*

The first order conditions of the consumer's optimization problem indicate that when extraction is zero, the economy enters a steady state and the consumption level of the consumption good when young can be restated as follows[21]

$$c^{1*}=\left(a^{-1/\sigma}+\frac{H^{1*}}{p^{c*}}+\frac{H^{2*}}{p^{c*}}\right)(1-v)/(1+\beta). \quad (3.13)$$

While the stock when the extraction is zero is

$$x^*=\left[a^{-1/\sigma}-\left(a^{-1/\sigma}+\frac{H^{1*}}{p^{c*}}+\frac{H^{2*}}{p^{c*}}\right)/(1+\beta)+\frac{H^{1*}}{p^{c*}}+\frac{F^*}{p^{c*}}\right]/f'(0). \quad (3.14)$$

PROPOSITION 1. Under Assumption 1, a strictly positive amount of the resource is preserved forever.

PROOF OF PROPOSITION 1. Assume on the contrary that in period T, the entire resource is exhausted, then from period T onwards, extraction level remains to be zero. Under Assumption 1, as long as there is labor input, production continues and the output is $\alpha^{-1/\sigma}(>0)$. Also, equation (3.14) implies when the resource stock level is zero, the numerator of the right-hand side of the equation has to be zero. We know when the stock held by the allocation agency becomes zero, the asset value held by the allocation agency, F_T, and $H_\tau(\tau \geq T)$ also becomes zero. Together with some simple manipulation, the above statement would imply that $\beta=0$.

However, it cannot be established since we assume $\beta>0$. Hence, we have reached a contradiction, and we conclude that a positive amount of the resource is preserved. □

The above proposition suggests that the extraction stops whenever $x_t \leq x^*$ and the entire remaining stock is then preserved forever. Therefore, if the initial stock $x_0 > x^*$, then over time, the stock level converges to x^*. If the initial stock $x_0 \leq x^*$, the economy directly enters the steady state, in which the extraction remains to be zero, and the entire initial stock is preserved: $x_0 = x^*$. To better suit our purpose, we assume x_0 is strictly greater than x^*.

Under the grandfathering scheme, the consumption level when young in the steady state is

$$c_g^{1*} = \alpha^{-1/\sigma}(1-v)/(1+\beta), \tag{3.15}$$

and the stock of the resource to be preserved forever is

$$x_g^* = [\alpha^{-1/\sigma}\beta/(1+\beta)]/f'(0). \tag{3.16}$$

Under the trust fund scheme, the consumption level when young in the steady state is

$$c_f^{1*} = \left(\alpha^{-1/\sigma} + \frac{H^{1*}}{p_f^{c*}} + \frac{H^{2*}}{p_f^{c*}}\right)(1-v) \\ /(1+\beta) > c_g^{1*}. \tag{3.17}$$

And the stock of the resource that will be preserved forever is

$$x_f^* = \\ \frac{\alpha^{-1/\sigma} + \frac{H^{1*}}{p_f^{c*}} - \left(\alpha^{-1/\sigma} + \frac{H^{1*}}{p_f^{c*}} + \frac{H^{2*}}{p_f^{c*}}\right)/(1+\beta)}{f'(0)} \\ + x_1 > x_g^*. \tag{3.18}$$

[22]

The consumers' optimization problem we examine at the beginning of this section indicates under Assumption 1, the consumption of the first period's amenity value, $\varphi_t^1 x_t$, the consumption of the rival-consumption goods, $p_{t+1}^c c_{t+1}^2$, and the amenity value of the second period, $\varphi_{t+1}^2 x_{t+1}$, are all proportional to the consumption of the consumption goods when young, $p_t^c c_t^1$, all t. This fact indicates that if the two schemes are applied to the same amount of the initial stock, then in the steady states, the four arguments of the utility function and the resultant utility level under the trust fund scheme are strictly higher, thus are much preferred to those under the grandfathering scheme. Obviously, these results highly support the adoption of the trust fund scheme as an intergenerational allocation instrument since it is more sustainable in nature than the grandfathering scheme. This completes our description of the model in which knowledge production does not exist. Next, we explicitly introduce knowledge accumulation into the model and examine the resulting differences.

4. Knowledge accumulation and its effects

The preceding analysis has looked, in a rather simplified fashion, at various intergenerational distribution schemes of an exhaustible resource with amenity value and their implications for each agent's utility level. However, several dynamic elements are missing from the model. For example, not the least of the above analysis and the related literature concerns the accumulation of technological knowledge. It is quite possible, however, for the accumulation of knowledge to result in a reduction in the amount of the resource needed to acquire a certain level of output over time. In reality, in the wake of recognizing the adverse effects accompanying the utilization of the atmosphere as a reservoir of GHG emissions, numerous technologies and equipments that are more efficient have been developed and applied in the past decades. This trend has led to a continuous reduction in the amount of carbon discharged from producing a given amount of energy and consumption goods, as manifested by the gradual decarbonization of energy use. It is worth noting that these technologies are quite often not the fruits of voluntary private R&D activities, but are the results of social pressures or legislation. Also, the replacement of the old and much depreciated equipments with new ones that improve the prevailing productivity are in many cases, the direct results of government-led campaigns.

In this section, we explicitly address the questions we have raised in the introduction by allowing the production function to account for knowledge accumulation and examining the resulting impacts on the extraction rules and agents' utility levels. Here by knowledge we mean only reactionary knowledge ; we simply ignore the accumulation of the progressive knowledge.[23] This deliberate choice may lead to rather pessimistic conclusions ; nevertheless, it gives us a chance to catch a glimpse of the outcomes of the cases in which human beings cannot alter the exhaustibility of an imperative resource.

Furthermore, the knowledge we consider in this paper is assumed to have strong spillover effects, meaning it is uniformly diffused over the whole economy. Like other public goods, adequate private production of new knowledge does not occur because no single firm has an economic incentive to engage in R&D activities and to capture the returns, since the benefits can be equally enjoyed by other firms at no cost.[24] To correct this flaw and to increase the efficiency of the economy, it is necessary to introduce a public R&D sector. We postulate that this newly added public R&D sector, which employs labor only, is supervised and fostered by the allocation agency.

We follow the models of R&D developed by Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992).

There are two sectors, a good-producing sector where the consumption goods are produced and an R&D sector where additions to the stock of the knowledge are made. Based on the decision of the allocation agency, fractin $0 \leq a < 1$ of the labor force is directed into the R&D sector and the rest into the goods-producing sector. We assume knowledge accumulation augments consumption goods producible from a given level of production inputs, which consist of both the resource inputs and the labor inputs. Since there is no capital in the model, the production function for output becomes

$$f(r_t)=A_t\{\alpha[(1-a_t)l]^{-\sigma}+(1-\alpha)r_t^{-\sigma}\}^{-1/\sigma}, \quad (4.1)$$

here the values of σ and α follow the description in the previous section. We assume the value of A_t is unity when no addition to the knowledge stock is made over time.

The production function for new knowledge A_t is

$$A_{t+1}=B(a_t l)^{\gamma}A_t^{\theta}+A_t, \quad \text{with } B>0,\ 0<\gamma<1,\text{[25]} \quad (4.2)$$

here B is a shift parameter, γ and θ are technological parameters and A_1 is assumed to be unity. We follow the assumption on the labor force stated in the previous sections, i. e., in each period, young agents supply 1 unit of labor inelastically. Notice $l=1$ further simplifies our definition of the production function. Notice also the utility function in Assumption 1 still applies in the following analysis.

In the following analysis, we label the two schemes we examined in the previous section, in which the production of new knowledge does not take place, and the knowledge stock is constant over time, as 'grandfathering scheme without an R&D sector' and 'trust fund scheme without an R&D sector', respectively. Along the same lines, following the introduction of a public R&D sector, these two schemes change into 'grandfathering scheme with a public R&D sector' and 'trust fund scheme with a public R&D sector', respectively.

Next, we define the output of the production before the introduction of knowledge accumulation, y_t^b,[26] to be $f(r_t^b)$, then the resultant wage is

$$w_t^b=p_t^{bc}\alpha[\alpha+(1-\alpha)r_t^{-\sigma}]^{-(1+\sigma)/\sigma}. \quad (4.3)$$

Here, it is assumed that the allocation agency imposes a tax on the wage income of the workers who engage in the consumer goods production sector, and then distributes this tax revenue as a lump-sum transfer to those who work in the knowledge production sector so as to equalize the latter's wage income to that of the former. It is straightforward to deduce that the needed tax rate τ equals a_t in each period. Output and wage after the introduction of the R&D sector change into y_t^a, $A_t[\alpha(1-a_t)^{-\sigma}+(1-\alpha)r_t^{-\sigma}]^{-1/\sigma}$ and the accompanying wage, w_t^a, becomes

$$w_t^a=p_t^{ac}A_t\alpha(1-a_t)^{-\sigma}[\alpha(1-a_t)^{-\sigma}+(1-\alpha)r_t^{-\sigma}]^{-(1+\sigma)/\sigma}. \quad (4.4)$$

We pause here for a moment to study θ, which reflects the effects of the existing stock of knowledge on the production of new knowledge. θ can actually take any value; with a positive θ denoting the fact that past discoveries make future discoveries easier, while negative ones denote the fact it is harder for new discoveries to be made once the existing knowledge stock is considerably large. However, it is straightforward to notice the transition equation of A_t implies that even an arbitrary selection of θ results in an increment of the value of A_t, albeit it does affect how fast A_t can increase.

In addition, we assume the value of a changes with the passage of time, which means the economic structure is not rigid. We assume further that it is possible for the allocation agency to choose freely the values of a_t that vary over time. We consider this assumption to be realistic, since in reality, governments not only assume the leadership in the course of technology development actively, but also exert that responsibility frequently. Along the same lines, the benevolent allocation agency actively alters the volume of the output by choosing the fraction of the labor force to be directed into the public R&D sector in each period so to achieve certain policy objectives over time.

Here we assume the allocation agency chooses a sequence of $\{a_t\}_{t=1}^{\infty}$[27] that maximizes the aggregated output over time, as specified in (2.14), subject to the production function for consumption goods (4.1) and the production function for new knowledge (4.2). It is clear to see that the sequence of $\{a_t\}_{t=1}^{\infty}$ so chosen leads the economy to more desirable allocation effects in later periods than those obtained under schemes without an R&D sector and those with a public R&D sector but does not choose the optimal sequence of $\{a_t\}_{t=1}^{\infty}$. This conclusion is rather intuitive since one form of the externalities existing in this economy has been addressed. In the following sections, we examine the effects of knowledge accumulation on consumption levels and extraction rules by introducing a public R&D sector into different allocation schemes.

4.1 *Grandfathering scheme with a public R&D sector*

We first introduce a public R&D sector into the grandfathering scheme without an R&D sector and examine the resultant outcomes. As before, each agent maximizes her utility as described by equation (3.1). The budget constraint for all generations $t \geq 1$ becomes

$$p_t^c c_t^1 + p_{t+1}^c c_{t+1}^2 + \varphi_t^1 x_t + \varphi_{t+1}^2 x_{t+1} = w_{gpt}^a. \text{[28]} \quad (4.5)$$

Accordingly, the consumption level when young for all generations $t \geq 1$, c_{gpt}^1, becomes

$$c_{gpt}^1 = w_{gpt}^a (1-v)/[(\beta+1) p_{gpt}^c], \quad (4.6)$$

and agents' life-cycle saving, s_{gpt}, becomes[29]

$$\begin{aligned} s_{gpt} &= w_{gpt}^a \beta/(1+\beta) \\ &= p_{gpt}^c f'(r_{gpt})(x_{gpt} - r_{gpt}). \end{aligned} \quad (4.7)$$

Here it is assumed that the agents treat the sequence of $\{a_t\}_{t=1}^{\infty}$, which is determined by the allocation agency, as given. Almost certain, under the previous extraction plan and knowledge level, even an arbitrary selection of $a>0$ results in a reduction in the output in the first period regardless of the value of γ since less labor is devoted into the production process while the technology level remains to be unchanged. This decrease in output, perceived to be 'harmful to (the current domestic) economy', as suggested by a number of politicians in the ongoing negotiation concerning the ratification of the Kyoto Protocol, is one of the essential reasons why the Kyoto framework is highly unacceptable to some countries. However, since the sequence $\{a_t\}_{t=1}^{\infty}$ can always be deliberately chosen so as to limit the trade-off effect to the first few periods only,[30] this decrease not only is temporary in nature, but also has permanent economic augmentation effects that highly support its adoption as a countermeasure against pollution and global warming.

The equilibrium extraction rule after the introduction of knowledge accumulation can be stated as follows

$$x_{gpt}=\left[\frac{\alpha\beta(1-a_t)^{-\sigma}}{(\beta+1)(1-\alpha)}+r_{gpt}^{-\sigma}\right]r_{gpt}^{\sigma+1}. \qquad (4.8)$$

Under this modified extraction rule, numerical manipulation shows that the extraction needed in each period is reduced compared with the previous extraction rule under the grandfathering scheme without an R&D sector.

Next we study the properties of the steady states under this modified version of grandfathering scheme. In the steady states, the consumption level when young, c_{gp}^{1*}, and the stock level to be preserved forever, x_{gp}^{*}, are

$$\begin{aligned} c_{gp}^{1*}&=\frac{w_{gp}^{a*}(1-v)}{p_{gp}^{c*}(1+\beta)} \\ &=\frac{A^*\alpha^{-1/\sigma}(1-v)}{1+\beta}>c_g^{1*} \\ &=\frac{w_g^{b*}(1-v)}{p_g^{c*}(1+\beta)}=\frac{\alpha^{-1/\sigma}(1-v)}{1+\beta}, \end{aligned} \qquad (4.9)$$

[31]

and

$$\begin{aligned} x_{gp}^{*}&=\frac{w_{gp}^{a*}\beta}{(1+\beta)p_{gp}^{c*}f'(0)} \\ &=\frac{A^*\alpha^{-1/\sigma}\beta}{(1+\beta)f'(0)}>x_g^{*} \\ &=\frac{w_g^{b*}\beta}{(1+\beta)p_g^{c*}f'(0)} \\ &=\frac{\alpha^{-1/\sigma}\beta}{(1+\beta)f'(0)}, \end{aligned} \qquad (4.10)$$

respectively.[32] Notice the above strict inequalities hold since with the passage of time, starting from the second period, the knowledge stock level A_t is strictly greater than unity if $a_1>0$. It is then obvious that the introduction of a public R&D sector leads the economy into much desirable outcomes in the steady states. In the next section, we show that introducing a public R&D sector into the trust fund scheme without an R&D sector leads to similar outcomes.

4.2 *Trust fund scheme with a public R&D sector*

As stated above, introducing a public R&D sector into the economy where the allocation scheme is denoted by the trust fund scheme corrects the externalities existing in this economy: The absence of new knowledge. When a public R&D sector is introduced into the economy, the consumption level when young of the generations $t \geq 1$, c^1_{fpt},[33] changes into

$$c^1_{fpt} = \frac{A_t \alpha (1-a_t)^{-\sigma} [\alpha (1-a_t)^{-\sigma} + (1-\alpha) r_{fpt}^{-\sigma}]^{-\frac{1+\sigma}{\sigma}} (1-v)}{1+\beta - \frac{v x_1}{x_{fpt}} - \frac{\beta v x_1}{x_{fpt} - r_{fpt}}}, \tag{4.11}$$

As shown above, the introduction of knowledge accumulation alters the wage income of the agents, and accordingly, their consumption levels of the consumption goods and demands toward the level of the resource amenity. Under the present setting, the modified extraction rule can be implicitly described by the following equation, which states the lifecycle saving of the young

$$s_{fpt} = p^c_{fpt} \alpha (1-a_t)^{-\sigma} \left[1 + \frac{v x_1 - x_{fpt}}{x_{fpt} \left(\beta + 1 - \frac{v x_1}{x_{fpt}} - \frac{\beta v x_1}{x_{fpt} - r_{fpt}} \right)} \right] = p^c_{fpt} (1-\alpha) r_{fpt}^{-\sigma-1} (x_{fpt} - r_{fpt} - x_1), \tag{4.12}$$

Numerical manipulation shows this revised extraction rule leads to lower extraction levels in each period.

In the steady states, no extraction takes place, and r^* remains to be zero. The stationary consumption level of the consumption goods when young is

$$c^{1*}_{fp} = \frac{\left[A^* \alpha^{-1/\sigma} + \frac{H^{1*}}{p^{c*}_{fp}} + \frac{H^{2*}}{p^{c*}_{fp}} \right] (1-v)}{1+\beta} \tag{4.13}$$

$> c^{1*}_{gp} > c^{1*}_f > c^{1*}_g$.[34]

Also, the stock level of the resource to be preserved forever changes into

$$x^*_{fp} = \frac{A^* \alpha^{-1/\sigma} + \frac{H^{1*}}{p^{c*}_{fp}} - \left(A^* \alpha^{-1/\sigma} + \frac{H^{1*}}{p^{c*}_{fp}} + \frac{H^{2*}}{p^{c*}_{fp}} \right) / (1+\beta)}{f'(0)} + x_1 > x^*_{gp} > x^*_f > x^*_g. \tag{4.14}$$

It is obvious to see that in the steady states, the introduction of knowledge accumulation leads to higher stationary consumption level and larger stock of resource to be preserved forever as compared with the trust fund scheme without an R&D sector. We conclude that introducing a public R&D sector into the trust fund scheme leads the economy to higher stationary consumption levels and larger stocks of resource to be preserved forever in the steady states under both of two schemes.

4.3 *Achieving 'democratic' allocation effects under the grandfathering scheme with a public R&D sector*

As we have stated in the previous section, in reality, the trust fund scheme may not be an applicable political option. Alternatives to this practice that lead to similar 'democratic' allocating effects are urgently needed in the forming of a framework designed to address the intergenerational equity issues concerning the distribution of an exhaustible resource. We know that although both of the two schemes without an R&D sector ensure intertemporal efficiency in terms of resource allocation, neither of them addresses the externalities concerning the spillover effect of new knowledge. The aim of the following analysis, therefore, is to examine under the grandfathering scheme with a public R&D sector, whether the 'democratic' allocation effects that 'protect welfare of all generations', as exhibited under the trust fund scheme without an R&D sector, can be achieved. That is, we examine the possibility of using the sequence $\{a_t\}_{t=1}^{\infty}$ as a hypothetical policy leverage to achieve certain allocation effects without redistributing resources among generations. Put in other words, we examine whether or not addressing externalities concerning the production of the public good, knowledge, can be as effective as allocating income claims for the resource among generations.

We know the allocation effects obtained under the trust fund scheme without an R&D sector is characterized by the sequence $\{c_{ft}^1, c_{ft}^2, x_{ft}, r_{ft}\}_{t=1}^{\infty}$, and the effects under the grandfathering scheme with a public R&D sector is characterized by the sequence $\{c_{gpt}^1, c_{gpt}^2, x_{gpt}, r_{gpt}\}_{t=1}^{\infty}$. Our objective then necessitates the examination of the existence of a sequence $\{a_t\}_{t=1}^{\infty}$ such that the sequence obtained under the grandfathering scheme with a public R&D sector can exhibit similar allocation effects as represented by the sequence $\{c_{ft}^1, c_{ft}^2, x_{ft}, r_{ft}\}_{t=1}^{\infty}$.

In order to replicate the allocation effects as represented by the sequence $\{c_{ft}^1, c_{ft}^2, x_{ft}, r_{ft}\}_{t=1}^{\infty}$, we need to relate the optimal consumption levels of the consumption goods when young and the desired life-cycle savings obtained under the two schemes. The relationships between the two allocation schemes can be established and expressed by equation (4.15) and (4.17). The first equates the consumption level of the young agent over time:

$$c_{ft}^1 = w_{ft}^b(1-v)/\{[(1+\beta) - vx_1/x_{ft} - \beta v x_1/(x_{ft}-r_{ft})]p_{ft}^c\} \quad (4.15)$$
$$= w_{gt}^a(1-v)/[(1+\beta)/p_{gt}^c].$$

Note the above equation can be rearranged in the following form[35]:

$$a_t^c = g(A_t, r_{ft}, x_{ft}),$$
x_1 and A_1 are exogenously given. (4.16)

It is straightforward to see that a_t^c so determined equates the consumption level of each agent over time. Meanwhile, numerical manipulation shows the sequence $\{r_{gt}\}$so determined closely resembles the trend of the extraction rule under the 'trust fund scheme without an R&D sector'. Compare with the previous one under the 'grandfathering scheme without an R&D sector', the new rule exhibits an increase in the extraction in early periods, and a decrease in later periods.

The second equation equates the desired savings, which determines the consumption level when old, and implicitly, extraction level in each period:

$$s_{ft} = w_{ft}^b\left\{1 + \frac{vx_1 - x_{ft}}{x_{ft}\left[(\beta+1) - \frac{vx_1}{x_{ft}} - \frac{\beta v x_1}{x_{ft}-r_{ft}}\right]}\right\} - p_{ft}^c f'(r_{ft})x_1 = w_{gt}^a \beta/(\beta+1). \quad (4.17)$$

The above equation can be rearranged as

$$a_t^s = h(A_t, r_{ft}, x_{ft}),$$
$$x_1 \text{ and } A_1 \text{ are exogenously given.} \tag{4.18}$$

Along the same lines, a_t^s so determined equals the extraction level in each period. In case $a_t^c = a_t^s$, all t, then the system would have the property as to allow the grandfathering scheme with a public R&D sector to mimic the allocation effects of the trust fund scheme without an R&D sector. However, numerical manipulation shows that generally, $a_t^c \neq a_t^s$, which implies only parts of the distributional properties can be mimicked. Therefore, one needs to distinguish between alternative policy objectives. The sequence $\{a_t^c\}_{t=1}^{\infty}$ can be selected to equalize the consumption levels between the grandfathering scheme with a public R&D sector and the trust fund scheme without an R&D sector; while the sequence $\{a_t^s\}_{t=1}^{\infty}$ can be chosen so that the extraction rules obtained under the trust fund scheme with a public R&D sector can be mimicked under the grandfathering scheme with a public R&D sector.

In short, the practice presented as above enables the allocation agency to achieve certain 'democratic' allocation effects under business-as-usual practices by implementing the chosen $\{a_t\}_{t=1}^{\infty}$ over time. Therefore, the establishment of a public R&D sector can be considered as an alternative to the redistribution of the property rights as specified by the trust fund scheme. It is in this meaning, we regard the introduction of a public R&D sector to be a highly effective option to policy makers when the political climate does not favor the redistribution of property rights over generations and when the business-as-usual allocation scheme has to be retained.

5. Concluding remarks

The objective of this paper has been to interpret observations on the allocation of an exhaustible resource among generations in terms of motivations and constraints of economic agents and to predict the consequences of alternative hypothetical policies when there is a public R&D sector. First, the paper has examined the resource extraction rules and the steady states under the two allocation schemes suggested by Gerlagh and Keyzer (2001), and concluded the trust fund scheme leads to higher stationary utility levels and larger stock of resource to be preserved forever, and therefore is preferred to the grandfathering scheme. However, the paper considered the outcomes obtained under these two allocation schemes to be inefficient since in this economy, there exists a public good, knowledge, which has strong spillover effects and remains to be addressed under both of the two schemes.

The paper has then explicitly addressed the externalities concerning knowledge by introducing a public R&D sector into the two schemes. It has concluded that this maneuver corrects the flaw concerning new knowledge and leads the economy to higher stationary consumption levels and larger stocks of the resource to be preserved forever in the steady states under both of the schemes. The paper has also shown the possibilities of manipulating knowledge stocks over time to achieve under the business-as-usual practice, as represented by the grandfathering scheme, certain 'democratic' allocation effects characterized by the trust fund scheme, without redistributing properties rights among generations. These results strongly justify the necessity of reinforcing the roles which have been actively played by international institutions in the past decades, as manifested by the United Nations Environmental Program (UNEP), in leading and coordinating governmentally sponsored R&D activities aiming at fighting climate change and global warming.[36] It also presents clear evidence to support the rationale behind accumulating new knowledge aimed at alleviating pollution and global warming at a faster pace, and the urgency of reconstructing the national public R&D framework within each individual country so that more public funds can be devoted to finance the needed R&D activities.[37]

The most natural way to extend the analysis would be to incorporate into the model the accumulation of progressive knowledge that leads to a decrease of dependence on the exhaustible resource. It seems clear that the conclusions of this paper regarding the properties of the steady states will change significantly in such a new model. In addition, it is obvious that the substitution of a reproducible capital stock for the exhaustible resource should also be considered.

Furthermore, there are some equity implications left to be explored in the setting suggested by this paper. Different policy objectives may favor different allocation schemes, and qualitative analysis is needed to clarify the trade-off relationships among them. Extending the model to include the accumulation of progressive knowledge and capital would provide a framework within which these and other important issues concerning sustainability can be satisfactorily examined.

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Notes

1) As Ayres (2001) points out, when a society sets to address the scarcity of resources through the development of technologies, generally there are two options available, namely,

the reactionary option and the progressive option. Here in this paper, we denote the result of the former to be the reactionary knowledge ; and that of the latter, the progressive knowledge. Reactionary knowledge refines the dominant, proven stock of existing knowledge and leads to more efficient utilization of production inputs (resources). On the other hand, the progressive knowledge, which consists of alternative technologies that utilize new forms of inputs (resources), leads to a decrease of dependence on the resources currently being used, and eventually, the abandonment of the using of the resource as a production input. However, since our focus is on exhaustibility, hereinafter, we ignore the existence of progressive knowledge and center our attention on the reactionary knowledge.

2) As suggested by Heal (1998), biodiversity also fits this category. Although biodiversity is endowed with built-in abilities to reproduce itself and hence is 'renewable', our economic activities are driving species to extinction at an unprecedented rate. The resultant loss is irreversible and definitive, at least on a time scale relevant to humanity. At the same time, the presence of biodiversity is indispensable to our existence and contains key determinants of the quality of life. Expressed otherwise, we are willing to pay both for exploiting biodiversity as a production input, and also for keeping its stock intact : i. e., for its amenity value.

3) The concept that individuals derive their utility not only from the consumption goods produced from the resource, but also from the amenity value of the resource dates from Krutilla (1967), and can also be found in Krautkraemer (1985, 1986).

4) Although the atmosphere incorporates built-in mechanism to assimilate carbon dioxide, the annual carbon emissions resulting from human economic activities are beyond the near-equilibrium of natural carbon fluxes and have the potentiality for intriguing large scale climate change. Expressed otherwise, we are exploiting atmosphere by utilizing it as a finite-sized reservoir for greenhouse gases (GHG) emissions and many other forms pollutants, while the unused stock is enjoyed as clean air, and accordingly, has amenity value.

5) Undeniably, the activities of *homo sapiens* always accompany irrevocable destruction of the surrounding environment. Engels F. once wrote "The people who, in Mesopotamia, Greece, Asia Minor and elsewhere, destroyed the forests to obtain cultivable land, never dreamed that by removing along with the forests the collecting centers and reservoirs of moisture they were laying the basis for the present forlorn state of those countries" ("Part played by labor in transition from ape to man", In (1987) *Karl Marx • Fraderick Engels : Collected works*, Vol. 25, Progress Publishers, p. 461). Weather being conscious or unconscious of the outcomes, it is clear that these activities generally lead to irreversible results. Hereinafter, we characterize this observation as the utilization of an exhaustible resource with amenity value. By amenity value we mean all forms of the immediate and more remote natural and social benefits lost due to the depletion of an exhaustible resource.

6) Kyoto Protocol on climate change commits industrial countries and former Eastern bloc nations to bring GHG emissions to 5 percent below 1990 levels by 2008-12.

7) Though not complete, the Kyoto Protocol does incorporate the Polluters Pay Principle (PPP), as opposed to the business-as-usual practice represented by the 'grandfathering' scheme, which can be viewed as a variation based on the Victims Pay Principle (VPP).

8) Throughout this paper, we use the superscripts 1 and 2 for the young and old respectively for a given generation and a time subscript for the time period to which the generation applies.

9) In order to focus attention on the consequences of the exhaustibility of the resource, in the following analysis it is assumed that the direct costs of extraction are zero.

10) Though much simplified, the amenity value defined as above closely resembles those proposed by Krautkraemer (1985) and Gerlagh and Keyzer (2001); here the initial stock level is assumed to be strictly positive.

11) Stern (1997) and Weiss (1989) examine the mechanism of such an institution in detail, while the idea is further advanced by Gerlagh and Keyzer (2001), who specify the possible sequence of steps that might be needed to activate such an institution.

12) Here as in Gerlagh and Keyzer (2001), we assume the initial value of the stock is finite, $\psi_1 x_1 < \infty$ and $\lim_{t \to \infty} \psi_t k_t = 0$. Following the proof of Lemma 2 in Gerlagh and Keyzer (2001), it is easy to see that the total value of the consumption goods produced over time is finite, $\sum_{t=1}^{\infty} p_t^c y_t < \infty$. According to the firm's zero profit condition, (2.5′), it is clear to see that over time, $\sum_{t=1}^{\infty} p_t^c y_t = \sum_{t=1}^{\infty} (w_t + p_t^r r_t)$, so the aggregated wage that agents get over time is also finite, $\sum_{t=1}^{\infty} w_t < \infty$. Notice equation (2.17) can be written out as $\psi_t x_t = \sum_{\tau = t, \cdots, \infty} (p_\tau^r r_\tau + p_\tau^x x_\tau)$, which implies for the entire time horizon, $\psi_1 x_1 = \sum_{t=1}^{\infty} (p_t^r r_t + p_t^x x_t)$. Therefore, $\sum_{t=1}^{\infty} (p_t^c y_t + p_t^x x_t) < \infty$. Notice also over the infinite horizon, resource outputs balance with expenditures, $\sum_{t=1}^{\infty} (w_t) + \psi_1 x_1 = \sum_{t=1}^{\infty} (p_t^c y_t + p_t^x x_t)$, and as shown in the proof Lemma 1 of Gerlagh and Keyzer (2001), the resultant equilibrium is dynamically efficient.

13) The following analysis differs from that of Gerlagh and Keyzer (2001) not only on the selection of the utility function of the individual agent, the production function of the consumption goods producing firm, but also on the characterization of the paths converging to the steady states under both of the two policy schemes (Gerlagh and Keyzer (2001) characterizes mainly the steady states). Moreover, it also proves that under Assumption 1, a strictly positive amount of the resource is preserved forever (Proposition 1). Notice this endeavor offers an alternative way to prove similar results obtained in Gerlagh and Keyzer (2001).

14) Together with our selection of v following Lemma 1 and Lemma 2 in Gerlagh and Keyzer (2001), it is straightforward to deduce that revenues balance with expenditures over the infinite horizon, and the resultant equilibria are dynamically efficient.

15) Refer to Mourmouras (1993), Krautkraemer and Batina (1999) and Gerlagh and Keyzer (2001).

16) Following the arguments provided by Gerlagh and Keyzer (Proposition 2 and its proof, 2001), it is easy to prove that the grandfathering scheme leads to a dynamically efficient equilibrium.

17) A 'g' is added before the time subscript 't' to identify variables under the 'grandfathering' scheme. In the following analysis, superscript* is used to identify variables in the steady states.

18) The concept of establishing a trust fund to allocate a resource among generations was initiated by Kennan (1970), and developed by Gerlagh and Keyzer (2001).

19) The trust fund scheme leads to a dynamically efficient equilibrium. Since it lies beyond

the scope of the present study, for the proof of the efficiency and existence, referred to Gerlagh (1998) and Gerlagh and Keyzer (2001).

20) A 'f' is added before the time subscript 't' to identify variables under the 'trust fund' scheme.

21) Notice in the steady states, the income claims to resources that each agent is entitled to under the trust fund scheme are: $H^{1*}=\frac{vx_1p^cc_f^{1*}}{(1-v)x_f^*}$, and $H^{2*}=\frac{\beta vx_1p^cc_f^{1*}}{(1-v)x_f^*}$.

22) Following from the fact that the value the trust fund holds over time equals to $F_t=\psi_tx_1$, equation (3.5) can be restated as follows to describe the steady states:

$$w_f^*+H^{1*}-\frac{w_f^*+H^{1*}+H^{2*}}{1+\beta}=p_f^{c*}f'(0)(x^*-x_1), \tag{3.5'}$$

Since $\beta>0$ and $F^*=p_f^{c*}f'(0)x_1$, we have, $H^{1*}>\frac{H^{1*}}{1+\beta}$, $F^*>\frac{H^{2*}}{1+\beta}$, therefore, $x_f^*>x_g^*$.

23) As predicted by many, being the catalyst of another industrial revolution, the accumulation of progressive knowledge has the potential of 'freeing energy from carbon' by completely shifting the energy sources to forms of virtually limitless flows of renewable resources.

24) In fact, in an economy that consists of countless competitive firms, there will be no production of new knowledge at all.

25) Note our production function for new knowledge is slightly different from those of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). In those models, R&D activities have a production augmentation effects within the same period, however, in our model, there exists an one period lag and the fruits of present R&D activities can only be enjoyed by future generations.

26) In the following analysis, the superscript 'a' is used to identify variables after the introduction of a public R&D sector, while the superscript 'b' is used to identify those variables before the introduction of a public R&D sector.

27) A 'gp' is added before the time subscript 't' to identify variables under the grandfathering scheme with a public R&D sector.

28) It is clear to show that even an arbitrarily selected sequence $\{a_t\}_{t=1}^{\infty}$ leads to dynamic efficiency over time.

29) After the introduction of knowledge accumulation, the price for resource input changes into $p_{gpt}^cf'(r_{gpt})$.

30) The sequence $\{a_t\}_{t=1}^{\infty}$ that limits the trade-off relationship to the first t periods only is chosen so that starting from period $t+1$ onwards (possibly from the second period on), the sum of the gains from knowledge accumulation must be large enough to offset the production loss due to a reduction of the labor force directed to the production sector for consumption goods. Notice a sequence $\{a_t\}_{t=1}^{\infty}$ so chosen may not be an optimal one. On the other hand, if the production function for new knowledge strictly follows those of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), then a_1 can be chosen so that the gain from the knowledge accumulation is equal to or larger than the loss due to a reduction of the labor force directed to the production sector for consumption goods. If such an a_1 is applicable, then under both of the two schemes we have examined in Section 3, the introduction of a public R&D sector leads the economy to lower extraction levels and higher consumption levels over time along the path converging to the steady states, higher stationary consumption levels, and larger stocks of the resource to be preserved in the steady states.

31) We assume that when the extraction level reaches zero, i. e., when economy enters the steady states, the allocation agency allocate no labor force into the R&D sector and the production for new knowledge terminates. Thus,

in the steady states, the knowledge stock level remains to be constant, and the value of a is zero over time.

32) As pointed out by a referee, the paths of x_t and c_t as described by (4.9) and (4.10) for the grandfathering scheme, and (4.13) and (4.14) for the trust fund scheme, can also be replicated within a general two sector (a consumption goods producing sector and an R&D sector) growth model framework that describes the tradeoff relation between investing the labor to produce the consumption good and to accumulate knowledge to augment the future productive potentiality.

33) A '*fp*' is added before the time subscript '*t*' to identify variables under the trust fund scheme with a public R&D sector.

34) This relationship and that stated in (4.14) establish since $H^{1*} \geq 0$, $H^{2*} \geq 0$, $\beta > 0$ and $A^* > 1$. Notice the establishment of $x^*_{gp} > x^*_f$ also requires an ad hoc condition $(A^* - 1)\beta\alpha^{-1/\sigma} > \beta H^{1*}/p^{c*}_f + F^*/p^{c*}_f - H^{2*}/p^{c*}_f$.

35) In the following analysis, rather than depending on the interaction of the three markets we have discussed, the allocation agency is allowed to determine the extraction levels in each period directly.

36) In "Rio declaration on environment and development", adopted at the United Nations conference on environment and development, the importance of the international cooperation on the creation of new knowledge is emphasized. Principle 9 of the same document reads "States should cooperate to strengthen endogenous capacity-building for sustainable development by improving scientific understanding through exchanges of scientific and technological knowledge, and by enhancing the development, adaptation, diffusion and transfer of technologies, including new and innovative technologies".

37) "Good initiatives and practices" to date include the British government's package of initiatives relating to the quality of life and sustainable development titled "A better quality of life—A strategy for sustainable development in the United Kingdom" and the Japanese government's polices aiming at providing funding and incentives for adopting more efficient energy consumption patterns.

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