# Costs on a Child, Length of Education, and Accumulation of Human Capital*

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Today, there is a bipolar situation in the fertility rate: a high fertilit yrate in developing countries and a low fertility rate in advanced countries. This paper presents a simple model to explain the reason. Parents choose the number of children they have, and the length and quality of their education. This paper shows that dependency of the cost of having a child on human capital plays a critical role in explaining the reason. If the cost is independent of human capital, there is only a positive relationship between the fertility rate and human capital per capita. If the cost is dependent on human capital, there are several relations between the fertility rate and human capital per capita. Cycles in human capital per capita may happen in the latter case.

Keywords: Human Capital; Length of Education; Fertility; Costs on a Child

## 1 Introduction

Today, the world population continues to grow at the 1.3% a year. The growth of population causes food and exhaustible resource shortages, and the loss of rainforest. The reason for world population growth is the high fertility rate in developing countries whose population makes up 80% of the world. Now this rate is showing a declining tendency but remains at a high rate (Atou, 2000). On the other hand, advanced countries reached the stage of low fertility rate and aging population in the 20th century. This bipolar situation in the fertility rate can not be explained by the first Malthusian theories. However, Becker and Barro (1988), Barro and Becker (1989), Becker, Murphy, and Tamura (1990) and Morand (1999) have explained the bipolar situation by using the relation between the number of children and the accumulation of human capital. The mechanism is as follows.

Human capital differs from physical capital in that the rate of return from human capital is assumed to increase as it is accumulated. Thus, accumulation of human capital in itself has the property that the more it is accumulated, the higher the investment becomes. In advanced countries, parents like to decrease the number of children and spend more time on children's education in expectation of a high return from their investment. These decisions are made in advanced countries where the initial stock of human capital is high. On the other hand, parents do not

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spend time on children's education in developing countries because human capital is low and they can not expect high rates of return. As a result, human capital is not accumulated. Because parents do not make much educational investment, the cost of giving a birth to a child becomes lower and they tend to give birth to more children. The relation between the initial stock of human capital and the number of children explains the bipolar situation in the fertility rate.

In these models, human capital is accumulated in proportion to the time spent by parents in teaching. Thus, there is a trade-off between the time spent by parents in producing consumer goods and that spent by parents in teaching children. That trade-off relation in the time spent by parents plays a very important role in the bipolar situation in the fertility rate. We should, however, pay attention not only to the time distribution of parents but also to the time distribution of children. Children also spend time on education to accumulate human capital and in working to help his family. It is supposed that the income of the young becomes a relatively important source of help in families in developing countries. Thus, we should take into consideration the trade-off between the time spent by children in accumulating human capital and that in producing consumer goods.

This trade-off problem in the time spent by children is considered in Glomm and Ravikumar (1992) and Glomm (1997). In the former study, children themselves decide how much time they spend studying or working. In the latter, parents decide how much time children spend studying or working. In the both cases, human capital is accumulated not only by the time spent on education but also by the amount of the educational investment parents make. Parents invest in education to support their children and children divide their time between education and other things like leisure or part-time work. These decisions describe a more real society. They show the importance of public education. However, they did not consider the problem of endogenous fertility. So this paper presents a simple model which considers the trade-off problem in the time spent by children with endogenous fertility.

This model is based on the assumption that parents decide how much time children spend studying or working. In Glomm (1997), he says ＋

> *In Glomm and Ravikumar (1992), for example, each individual when young chooses the privately optimal amount of time to allocate to schooling. While such an assumption might be appropriate for the decision to attend university, it does not seem appropriate for the decision to attend second or third grade. Few eight or nine years olds can be expected to make such decisions rationally. Perhaps that*

*model then has to be thought of as a model of the decision of how much post-secondary education to obtain.*

*In many developing countries the number of years school is attended is on average very low. If a child attends school for two or three years one might assume that the relevant educational choices are made by the parent and not by the child. If a parent decides that a child attend school for only two years the child just has to live with that decision and it might be difficult (costly) for the child to continue its education when it is sufficiently old to make such a decision independently.*

I agree parental choice is important as far as the length of education but this assumption is not necessarily appropriate for only developing countries. In advanced countries, the length of compulsory education is on average long. It seems that there is no room for parental choice in education. When we take into consideration after-school cram schools like *jyuku* in Japan, parental choice as to the length of education is thought to be very important. Parental choice on education at the time when their children are very young generally has a great influence on the way children think afterward. There is, then, no problem in applying this assumption to all countries, I suppose.

Glomm (1997) shows that the working time in youth is decreased as human capital per capita is accumulated. If human capital is over the threshold, the young-age working time falls to zero. It is thought that this explains the high average academic background in advanced countries. In this paper, I present two different types of society in terms of the cost of having a child. One is a society where the cost of having a child is independent of human capital per capita. The other is a society where the cost depends on human capital per capita.

Before presenting the model, I should take a notice that the cost of having a child is assumed to consist of two parts in this model. One is the cost of rearing a child and the other is the cost related to education. I propose the former as purchases of goods for a child to live and the latter as the cost to support children in educational pursuits, for items like books, educational material, or the fee for after-school cram school. We can think of it as the quality of education that encourages children to study. In the literature, this is often assumed constant. However, especially in advanced countries, it seems to have risen along with accumulation of human capital. Thus, I assumed that both the costs of a child are dependent on human capital. To show this assumption plays an important role in the relation between the economic growth and the fertility rate, I first present a model where both the costs are not dependent on human capital. After that, a model where both these costs of having a child depend on human capital is presented.

In the former model, it is shown that the time spent by children working, the quality of education, and the number of children are constant. The number of individuals with the same human capital increases. Thus, the stock of human capital grows but human capital per capita is not increased.

In the latter model, the same situation concerning endogenous fertility is pointed up in the conclusion, as in Glomm (1997). The youth working time decreases as human capital is accumulated. Because the cost of having a child depends on human capital in this type of society, the number of children is decided according to the elasticity of the cost of rearing a child with respect to human capital. The quality of education is decided according to the relation between the elasticity of the cost of rearing a child and the cost of improving the quality of education.

This paper is organized as follows. A general model is presented in the section 2. In the section 3, the model premised on the assumption that both types of cost on a child are constant is analyzed. In the section 4, the model premised on the assumption that they depend on human capital is analyzed. I summarize the discussion and offer a few concluding comments in the section 5.

## 2 The Model

I consider an overlapping generation framework in which individuals live across two periods: young and old. All individuals are identical within each generation and I assume that they choose the number of children and the length and quality of the children's education. I define agents who are born at time t-1 and become old at time t as members of the t-th generation.

Each individual's utility depends on his old-age consumption[1], the number of children, and human capital of their children. The utility function representing these preferences is

$$U_t \equiv ln\ c_t + \alpha ln\ n_t + \beta ln\ h_{t+1},\ \alpha > 0 \text{ and } \beta > 0. \tag{1}$$

where $U_t$ is the utility of the $t$-th generation, $c_t$ is old-age consumption, $n_t$ is the number of children and $\alpha$ and $\beta$ are the degree of paternalistic altruism toward the number of children and human capital of children, $h_{t+1}$ at time $t$.

Individuals, when they are young, are endowed with one unit of time which can be allocated to education or to labor activities. When old, individuals are endowed with one unit of time which is supplied inelastically to labor activities. Earnings for the young are determined by the production function

$$w_t^y = \lambda l_t. \tag{2}$$

where $w_t^y$ denotes earnings of the young who are born at time $t$, $\lambda$ the productivity of the young, and $l_t$ the time allocated to work. We can understand that this production function describes the help of children in a family evaluated by consumption goods. Earnings for the old are determined by the production function

$$w_t^o = qh_t. \tag{3}$$

where $w_t^o$ denotes earnings of adults who are born at time $t-1$, $q$ is the productivity of human capital and $h_t$ is the stock of human capital in old age at time $t$. Total family income at time $t$, $m_t$, is given by

$$m_t = n_t w_t^y + w_t^o. \tag{4}$$

Human capital is accumulated according to the learning function

$$h_{t+1} = \theta(1-l_t)e_t^\gamma h_t^\delta \quad \theta>0, \text{ and } \delta,\ \gamma\in(0,1). \tag{5}$$

where$(1-l_t)$ is the time allocated to education and $e_t$ is the quality of education.

I define the cost function of rearing a child as $\phi(h_t)$, the cost function of the quality of education as $\pi(h_t)$. These depend on the human capital of parents when $\phi'(h_t)>0$ and $\pi'(h_t)>0$ for any $h_t\geq 0$. The family budget constraint at time $t$ becomes

$$c_t + n_t(\phi(h_t) + \pi(h_t)e_t) = m_t. \tag{6}$$

The population of the $t$-th generation grows according to the following function,

$$N_{t+1} = n_t N_t \tag{7}$$

where $N_t$ is the number of the $t$-th generation. The following parameter restrictions are assumed throughout the paper:

Assumption 1. $\lambda \leq \phi(h_t)$ for any $h_t \geq 0$,

Assumption 2. $\alpha > \beta(1+\gamma)$.

Assumption 1 is that children are net financial burdens to parents although they work full time in youth[2]. Assumption 2 is that the altruistic value of the number of children is more attractive than the altruistic value from accumulation of human capital by decreasing the number of children. It ensures interior solutions of the number of children and the quality of education.

Each of the $t$-th generation parents solves the problem,

$$\max_{c_t, n_t, l_t, e_t} \ln c_t + \alpha \ln n_t + \beta \ln h_{t+1} \tag{8}$$

$$\begin{aligned} \text{s. t. } & c_t + n_t(\phi(h_t) + \pi(h_t)e_t) = m_t, \\ & m_t = n_t \lambda l_t + qh_t, \\ & h_{t+1} = \theta(1-l_t)e_t^\gamma h_t^\delta, \qquad l_t \in [0,\ 1]. \end{aligned}$$

## 3 Constant Costs of a Child

I assume that costs of a child are constant, $\phi'(h_t)=0$ and $\pi'(h_t)=0$ for any $h_t$. I define the cost of rearing a child and the cost of the quality of education as $\phi$ and $\pi$, respectively. Substituting the constraints and the cost for a child into the objective function rewrites the maximization problem as

$$\max_{n_t, l_t, e_t} \ln\{n_t\lambda l_t + qh_t - n_t(\phi + \pi e_t)\} + \alpha \ln n_t + \beta \ln \theta(1-l_t)e_t^\gamma h_t^\delta. \tag{9}$$

An equilibrium for the constant costs of a child is a collection of sequences $\{e_t,\ n_t,\ l_t,\ h_{t+1}\}_{t=0}^\infty$ such that given $h_t$, $\{e_t,\ n_t,\ l_t\}$ solve the maximization problem.

The first-order conditions for interior solutions are

$$\frac{-\lambda l_t + \phi + \pi e_t}{c_t} = \frac{\alpha}{n_t}, \tag{10a}$$

$$\frac{n_t \pi}{c_t} = \frac{\beta\gamma}{e_t}, \tag{10b}$$

$$\frac{\lambda n_t}{c_t} = \frac{\beta}{1-l_t}. \tag{10c}$$

The optimal solutions become,

$$n_t^A=\frac{\{\alpha-\beta(1+\gamma)\}qh_t}{(1+\alpha)(\phi-\lambda)}, \tag{11a}$$

$$e^A=\frac{(\phi-\lambda)\beta\gamma}{\{\alpha-\beta(1+\gamma)\}\pi}, \tag{11b}$$

$$l^A=\frac{\lambda(\alpha-\beta\gamma)-\beta\phi}{\lambda\{\alpha-\beta(1+\gamma)\}}. \tag{11c}$$

I define D as

$$\begin{bmatrix} \frac{-\alpha(1+\alpha)^3(\phi-\lambda)^2}{(\alpha-\beta(1+\gamma)qh_t)^2} & \frac{-(1+\alpha)^2\pi}{qh_t} & \frac{(1+\alpha)^2\lambda}{qh_t} \\ \frac{-(1+\alpha)^2\pi}{qh_t} & \frac{-(1+\beta\gamma)\pi^2(\alpha-\beta(1+\gamma))^2}{\beta\gamma(\phi-\lambda)^2} & \frac{\lambda\pi(\alpha-\beta(1+\gamma))}{(\phi-\lambda)^2} \\ \frac{(1+\alpha)^2\lambda}{qh_t} & \frac{\lambda\pi(\alpha-\beta(1+\gamma))}{(\phi-\lambda)^2} & \frac{-(1+\beta)\lambda^2(\alpha-\beta(1+\gamma))^2}{\beta(\phi-\lambda)^2} \end{bmatrix}$$

which is the Hessian matrix. It satisfies

$$\frac{-\alpha(1+\alpha)^3(\phi-\lambda)^2}{(\alpha-\beta(1+\gamma)qh_t)^2}<0, \tag{12a}$$

$$\begin{vmatrix} \frac{-\alpha(1+\alpha)^3(\phi-\lambda)^2}{(\alpha-\beta(1+\gamma)qh_t)^2} & \frac{-(1+\alpha)^2\pi}{qh_t} \\ \frac{-(1+\alpha)^2\pi}{qh_t} & \frac{-(1+\beta\gamma)\pi^2(\alpha-\beta(1+\gamma))^2}{\beta\gamma(\phi-\lambda)^2} \end{vmatrix} =\frac{(\alpha-\beta\gamma)(1+\alpha)^3\pi^2}{\gamma\beta(qh_t)^2}>0, \tag{12b}$$

$$\begin{vmatrix} \frac{-\alpha(1+\alpha)^3(\phi-\lambda)^2}{(\alpha-\beta(1+\gamma)qh_t)^2} & \frac{-(1+\alpha)^2\pi}{qh_t} & \frac{(1+\alpha)^2\lambda}{qh_t} \\ \frac{-(1+\alpha)^2\pi}{qh_t} & \frac{-(1+\beta\gamma)\pi^2(\alpha-\beta(1+\gamma))^2}{\beta\gamma(\phi-\lambda)^2} & \frac{\lambda\pi(\alpha-\beta(1+\gamma))}{(\phi-\lambda)^2} \\ \frac{(1+\alpha)^2\lambda}{qh_t} & \frac{\lambda\pi(\alpha-\beta(1+\gamma))}{(\phi-\lambda)^2} & \frac{-(1+\beta)\lambda^2(\alpha-\beta(1+\gamma))^2}{\beta(\phi-\lambda)^2} \end{vmatrix}$$

$$=-\frac{(1+\alpha)^3(\alpha-\beta(1+\gamma))^3(\pi\lambda)^2}{\gamma(\beta(\phi-\lambda)qh_t)^2}<0. \tag{12c}$$

The second order conditions are always satisfied.

Equations (11) denotes that there is a positive relation between the number of children and human capital as long as Assumption 1 and 2 are satisfied. The number of children has nothing to do with the cost related to education. It implies that parents only care about the cost of rearing a child when they decide how many children they give birth to. The quality of education and young-age working hours are independent of the productivity and human capital. This means that accumulation of human capital raises the fertility rate but has no effect on the level of education in time and investment.

Proposition 3.1. *If costs for a child are* *assumed constant, accumulation of human capital increases the fertility rate and has no effect on education in time and investment. The length and quality of education are constant over the accumulation of human capital.*

This also shows that the degrees of paternalistic altruism toward the number of children($\alpha$) and toward human capital per child($\beta$) have opposing effects on $n_t$, $e^A$ and $l^A$. Greater altruism toward the number of children($\alpha$) raises the fertility rate and reduces the length and quality of education. Greater altruism toward human capital per child($\beta$) reduces the fertility rate and raises the length and quality of education.

### 3.1 The Steady State with Constant Costs of a Child

We define a steady state as a situation where the various quantities grow at constant rates. The steady state corresponds to $h_{t+1}=h_t=h_A^*$,

$$h_A^*=\{\theta(1-l^A)(e^A)^\gamma\}^{\frac{1}{1-\delta}}. \tag{13}$$

which is the stable steady value as long as $\delta<1$. Since $h_A^*$ is constant in the steady state, $n$, $c$, $e$ are also constant. The corresponding fertility rate becomes

$$n_A^*=\frac{\{a-b(1+\gamma)\}q}{(1+a)(\phi-\lambda)}h_A^*. \tag{14}$$

Proposition 3.2. *In the case of the constancy of costs of a child, human capital per capita becomes constant in the long run so the fertility rate also becomes constant. The per capita quantities h, n, c, e, and l do not grow in the steady state.*

The constancy of the per capita magnitudes means that the levels of variables grow in the steady state at the fertility rate, n.

These corresponding relations might describe societies in developing countries. The stock of human capital grows at a constant rate. The number of children also grows at a constant rate but the per capita stock of human capital becomes constant in the long run. The furthering of human capital per capita does not occur.

## 4 Increasing Costs of a Child

In this section, I assume that costs of a child depend on parental human capital, $\phi'(h_t)>0$ and $\pi'(h_t)>0$ for any $h_t>0$. In the last section, the costs of a child are constant. It means that having children needs only the constant amount of goods like food and that the cost related to education keeps constant even if human capital is accumulated. Does the assumption describe real societies in advanced countries? Not only the cost of goods a child consumes but also the cost of educational investment has become more expensive, as the quality of the living environment has increased. It is supposed to be appropriate to think the costs of a child are effected by accumulation of human capital. In this section, I present a model where the costs of a child are assumed to depend on human capital, which represents the level of the living environment in a sense.

According to the equation (8) and the assumption of the proportionality of the costs of a child to human capital, $\phi'(h_t)>0$ and $\pi'(h_t)>0$ for any $h_t>0$, the maximization problem becomes

$$\max_{n_t, l_t, e_t} \ln\{n_t\lambda l_t + qh_t - n_t(\phi+\pi e_t)\} + \alpha \ln n_t + \beta \ln \theta(1-l_t)e_t^{\gamma}h_t^{\delta}. \quad (15)$$

The first-order conditions are

$$\frac{-\lambda l_t+\phi(h_t)+\pi(h_t)e_t}{c_t}=\frac{\alpha}{n_t}, \quad (16a)$$

$$\frac{n_t\pi(h_t)}{c_t}=\frac{\beta\gamma}{e_t}, \quad (16b)$$

$$\frac{\lambda n_t}{c_t}=\frac{\beta}{1-l_t}. \quad (16c)$$

An equilibrium for the constant costs of a child is a collection of sequences $\{e_t, n_t, l_t, h_{t+1}\}_{t=0}^{\infty}$ such that the given $h_t$, $\{e_t, n_t, l_t\}$ solve the maximization problem.

Substituting equations (16a) and (16b) into equation (16c) leads to the optimal length of young-age working time,

$$l_t=\frac{\lambda(\alpha-\beta\gamma)-\beta\phi(h_t)}{\lambda\{\alpha-\beta(1+\gamma)\}}. \quad (17)$$

Equation (17) implies that the optimal length of young-age working time, $l_t$, is a decreasing function of human capital. The optimal length of education, $(1-l_t)$, is, on the other hand, an increasing function of parental human capital. This means that there is a threshold level, $h^L \equiv \phi^{-1}\left(\frac{\lambda(\alpha-\beta\gamma)}{\beta}\right)$ so that $1-l_t=1$ if $h_t>h^L$. $\phi^{-1}(\cdot)$ is the inverse function. If the productivity of the young is relatively low, the length of young-age working time becomes zero at the lower stock of human capital.

This relation between the length of young-age working time and human capital is different from the one in the case of the constant costs on a child. Because the costs that depend on human capital and on the productivity of young-age work are constant, the burden of having a child can not be constant as human capital is accumulated. It shows the possibility that parents prefer improving the quality of education. Thus in growing economies, the length of young-age working time gradually decreases. This is not the case with constant costs of a child. This negative relation between the length of youngage working and human capital is the same as in Glomm (1997).

Proposition 4.1. *If the cost of a child depend on human capital, the length of young-age working time is a decreasing function of human capital. It becomes zero if human capital is over the threshold.*

The solutions to the maximization problem are

$$l_t=\begin{cases}\dfrac{\lambda(\alpha-\beta\gamma)-\beta\phi(h_t)}{\lambda\{\alpha-\beta(1+\gamma)\}}\equiv l_t^{Ba} & \text{if } h_t<h^L,\\ 0 & \text{if } h_t\geq h^L,\end{cases} \quad (18a)$$

$$n_t=\begin{cases}\dfrac{\{\alpha-\beta(1+\gamma)\}qh_t}{(1+\alpha)(\phi(h_t)-\lambda)}\equiv n_t^{Ba} & \text{if } h_t<h^L,\\ \dfrac{(\alpha-\beta\gamma)qh_t}{(1+\alpha)\phi(h_t)}\equiv n_t^{Bb} & \text{if } h_t\geq h^L.\end{cases} \quad (18b)$$

and

$$e_t=\begin{cases}\dfrac{\{\phi(h_t)-\lambda\}\beta\gamma}{\{\alpha-\beta(1+\gamma)\}\pi(h_t)}\equiv e_t^{Ba} & \text{if } h_t<h^L,\\ \dfrac{\beta\gamma\phi(h_t)}{(\alpha-\beta\gamma)\pi(h_t)}\equiv e_t^{Bb} & \text{if } h_t\geq h^L,\end{cases} \tag{18c}$$

According to equation (18b), there are the following relations between the optimal number of children and human capital. I define the elasticity of $\phi$ with respect to $h_t$ as $\sigma_n\equiv\frac{\phi'(h_t)h_t}{\phi(h_t)}>0$. If human capital is less than the threshold, $\frac{dn_t}{dh_t}$ becomes

$$\frac{dn_t}{dh_t}\gtreqless 0 \text{ if } 1\gtreqless\frac{\sigma_n}{\eta}. \tag{19}$$

where $\eta\equiv\frac{\phi(h_t)-\lambda}{\phi(h_t)}$. The value of $\phi(h_t)-\lambda$ represents the net cost of rearing a child when a child works full time.

According to equation (19), the relation between the number of children and human capital becomes positive when the value of the elasticity divided by the ratio of the net cost of rearing a child to the gross cost of rearing a child, $\eta$, is over 1. The increase of the elasticity divided by the ratio of the net cost of rearing a child intuitively is the increase rate of the net cost of rearing a child caused by accumulation of human capital. When it is less than one, it represents a decrease of the net cost of rearing a child. Thus, in the case of $\frac{\sigma_n}{\eta}<1$, accumulation of human capital increases the number of children.

If human capital is over the threshold, the relation between the number of children and human capital becomes relatively simple. It becomes

$$\frac{dn_t}{dh_t}\gtreqless 0 \text{ if } \sigma_n\gtreqless 1. \tag{20}$$

$\sigma_n>1$ means that accumulation of human capital increases the cost of rearing a child. Parents decrease the number of children. $\sigma_n<1$ symbolizes the decrease of the cost of rearing a child caused by accumulation of human capital. Parents increase the number of children. If $\sigma_n=1$ holds, the cost of rearing a child is recognized as an opportunity cost for parents. In this case, the cost of rearing a child increases at the same rate as human capital is accumulated. This implies that even if human capital is accumulated, the cost does not relatively increase or decrease. Thus, the number of children is not effected by accumulation of human capital.

According to equation (18c), there are also following relations between the optimal quality of education and human capital. I define the elasticity of $\pi$ with respect to $h_t$ as $\sigma_e\equiv\frac{\pi'(h_t)h_t}{\pi(h_t)}>0$. If human capital is less than the threshold, $\frac{de_t}{dh_t}$ becomes

$$\frac{de_t}{dh_t}\gtreqless 0 \text{ if } \frac{\sigma_n}{\eta}\gtreqless\sigma_e. \tag{21}$$

This means that individuals improve the quality of education when the rise in the rate of the net cost of rearing a child is over the elasticity of the cost related to education, which implies a relative increase in the cost of rearing a child. The optimal quality of education is decided according to the relation between the elasticity of the cost of rearing a child and that of the cost related to education with respect to human capital.

If human capital is over the threshold, the relation between the optimal quality of education and human capital becomes simpler. It is represented by

$$\frac{de_t}{dh_t} \gtreqless 0 \text{ if } \sigma_n \gtreqless \sigma_e. \qquad (22)$$

In the case of $\sigma_n > \sigma_e$, it means that the increase of the cost of rearing a child is higher than that of the cost related to education, which is caused by accumulation of human capital. In that case, parents improve the quality of education. When $\sigma_n < \sigma_e$ holds, it means that accumulation of human capital increases the cost related to education at the higher rate. Parents don't improve the quality of education. When $\sigma_n = \sigma_e$ holds, accumulation of human capital increases the cost of rearing a child and the cost related to education at the same rate. This means that accumulation of human capital does not increase either the number of children or the quality of education. The quality of education is decided according to the relation between the elasticity of the costs. The relation is not effected by the accumulation of human capital when $\sigma_n = \sigma_e$ holds. Thus, there is no relation between the quality of education and human capital. According to the relation between the elasticity of the cost of rearing a child and of the cost related to education, the following proposition is concluded.

Lemma 4.1.1. *Human capital per capita does not grow unless the elasticity of the cost of rearing a child is greater than the elasticity of the cost related to education.*

According to the Table 1, we can easily

Table 1 The relations between the number of children and quality of education and the stock of human capital.

| — | $\sigma_n \geq 1$ | $\sigma_n < 1,\ \frac{\sigma_n}{\eta} \geq 1$ | $\frac{\sigma_n}{\eta} \leq 1$ |
|---|---|---|---|
| $\sigma_n \geq \sigma_e$ | (A) $\frac{dn_t}{dh_t} \leq 0$, $\frac{de_t}{dh_t} \geq 0$ | (B) $\frac{dn_t}{dh_t} \leq 0 \Rightarrow \frac{dn_t}{dh_t} > 0$, $\frac{de_t}{dh_t} \geq 0$ | (C) $\frac{dn_t}{dh_t} \geq 0$, $\frac{de_t}{dh_t} \geq 0$ |
| $\sigma_n < \sigma_e$, $\frac{\sigma_n}{\eta} \geq \sigma_e$ | (D) $\frac{dn_t}{dh_t} \leq 0$, $\frac{de_t}{dh_t} \geq \Rightarrow \frac{de_t}{dh_t} < 0$ | (E) $\frac{dn_t}{dh_t} \leq 0 \Rightarrow \frac{dn_t}{dh_t} > 0$, $\frac{de_t}{dh_t} \geq 0 \Rightarrow \frac{de_t}{dh_t} < 0$ | (F) $\frac{dn_t}{dh_t} \geq 0$, $\frac{de_t}{dh_t} \geq 0 \Rightarrow \frac{de_t}{dh_t} < 0$ |
| $\frac{\sigma_n}{\eta} \leq \sigma_e$ | (G) $\frac{dn_t}{dh_t} \leq 0$, $\frac{de_t}{dh_t} \leq 0$ | (H) $\frac{dn_t}{dh_t} \leq 0 \Rightarrow \frac{dn_t}{dh_t} > 0$, $\frac{de_t}{dh_t} \leq 0$ | (I) $\frac{dn_t}{dh_t} \geq 0$, $\frac{de_t}{dh_t} \leq 0$ |

see that the number of children is decided only by the elasticity of the cost of rearing a child. The quality of education is, on the other hand, decided by the relation between the elasticity of the cost of rearing a child and of the cost dependent on education. If the elasticity of the cost of rearing a child is, for example, over 1 and it is greater than that of the cost related to education, accumulation of human capital only decreases the number of children and only improves the quality of education over time. This is in situation (A). If $\sigma_n > 1$, $\frac{\sigma_n}{\eta} \geq \sigma_e$, and $\sigma_n < \sigma_e$ hold, the number of children decreases at all times but the quality of education improves for a while and decreases at last as human capital exceeds the threshold. There may be a cycle in accumulation of human capital. This is in situation (D). When the elasticity of the cost of rearing a child divided by the net burden ratio of rearing a child is less than that of the cost related to education even if it is over 1, the number of children and the quality of education decreases as human capital is accumulated. Thus, human capital is not accumulated and the number of children becomes larger. This is in situation (G).

There are many relations between the number of children and quality of education and human capital. Where the constancy of the cost of a child is assumed, we concluded there is a strictly positive relation between the number of children and the amount of human capital. This implies that the assumption describes the situation in developing countries. If the costs of a child are assumed to be on human capital, several relations can be described. If $\sigma_n \geq 1$ and $\sigma_n \geq \sigma_e$ hold, which means the economy is in situation (A), it describes the relation among the number of children, quality of education, and human capital in advanced countries. It is thought that there is a positive relation between the number of children and human capital and stagnation in human capital in developing countries. This is described as situation (I). If we can consider the stagnation as a cycle in human capital, developing countries are in situation (F). By dividing the cost of a child into two parts and assuming the costs depend on human capital, several situations can be described. This implies that the elasticity of the cost of rearing a child and that of the cost related to education with respect to human capital deserve study.

### 4.1 The Steady State with Increasing Costs of a Child

We define a steady state as a situation where the various quantities grow at constant rates. To consider the growth of human capital, I specialize the cost functions of a child as[3)]

$$\phi(h_t) = \phi h_t^{\varepsilon},\ 0 < \varepsilon < 1, \tag{23}$$

$$\pi(h_t) = \pi h_t^{\rho},\ 0 < \rho < 1. \tag{24}$$

Then the equilibrium law of motion for human capital per capita is

$$h_{t+1}=\begin{cases}\theta(1-l_t^{Ba})(e_t^{Ba})^{\gamma}h_t^{\delta} & \text{if } h_t\leq h^L,\\ \theta\{\frac{\phi\gamma\beta}{(\alpha-\beta\gamma)\pi}\}^{\gamma}h_t^{\gamma(\varepsilon-\rho)+\delta} & \text{if } h_t>h^L.\end{cases} \tag{25}$$

When $h_t>h^L$ holds, the growth rate of human capital per capita becomes

$$(1+g_t)\equiv\frac{h_{t+1}}{h_t}=\theta\left(\frac{\phi\gamma\beta}{(\alpha-\beta\gamma)\pi}\right)^{\gamma}h_t^{\gamma(\varepsilon-\rho)+\delta-1}. \tag{26}$$

It is dependent on the value of $\gamma$, $\varepsilon$, $\rho$, and $\delta$ whether human capital per capita grows or not. The following relations are concluded as long as $\theta\left(\frac{\phi\gamma\beta}{(\alpha-\beta\gamma)\pi}\right)^{\gamma}>1$ holds.

Proposition 4.2. *1. If* $\gamma(\varepsilon-\rho)+\delta<1$ *holds, human capital per capita converges to the value,* $h_L^*\equiv\{\frac{\beta\gamma\phi\theta^{\frac{1}{\gamma}}}{(\alpha-\beta\gamma)\pi}\}^{\frac{\gamma}{1-(\gamma(\varepsilon-\rho)+\delta)}}$. *It is human capital per capita in the stable steady state.*

*2. If* $\gamma(\varepsilon-\rho)+\delta=1$ *holds, human capital per capita grows at the constant rate,* $\theta\left(\frac{\phi\gamma\beta}{(\alpha-\beta\gamma)\pi}\right)^{\gamma}$.

*3. If* $\gamma(\varepsilon-\rho)+\delta>1$ *holds, human capital per capita grows at the increasing rate over accumulation of human capital. The value of* $h_H^*\equiv\{\frac{\beta\gamma\phi\theta^{\frac{1}{\gamma}}}{(\alpha-\beta\gamma)\pi}\}^{\frac{\gamma}{\gamma(\varepsilon-\rho)+\delta-1}}$ *is human capital per capita in the unstable steady state.*

For $\theta\left(\frac{\phi\gamma\beta}{(\alpha-\beta\gamma)\pi}\right)^{\gamma}>1$ to hold, the following condition is required; $\frac{\frac{\beta\gamma}{\pi}}{\frac{\alpha-\beta\gamma}{\phi}}>\left(\frac{1}{\theta}\right)^{\frac{1}{\gamma}}$. If $\theta=1$ is assumed, the condition becomes simplified,

$$\frac{\frac{\beta\gamma}{\pi}}{\frac{\alpha-\beta\gamma}{\phi}}>1. \tag{27}$$

This implies that the rate of direct return from education, $\frac{\beta\gamma}{\pi}$, needs to be over the rate of direct return from having a child, $\frac{\alpha-\beta\gamma}{\phi}$.

## 5 Concluding Remarks

I have examined a model where the parents choose the number of children they have, and the length and quality of their education. Where the cost of a child are independent of parental human capital, the optimal length and quality of education are constant and the fertility rate is an increasing function of human capital. The stock of variable quantities grows with the number of children.

Where the cost of a child depend on human capital per capita, the optimal amount of time spent working in youth decreases as human capital is accumulated. The number of children is only dependent on the elasticity of the cost of rearing a child. The quality of education is, on the other hand, dependent on the relation between the elasticity of the cost of rearing a child and that of the cost of the improving the quality of education.

This paper describes the importance of

the relation between the cost of a child and human capital. If the costs of a child are not related to human capital, they gradually become a lesser burden as human capital is accumulated; thus, the fertility rate becomes higher over accumulation of human capital. There is a simple relation between the number of children and human capital. If the costs of a child depend on human capital, they may be a proportional burden on parents although human capital is accumulated. Whether or not the number of children decreases depends on the elasticity of the cost of rearing a child. The quality of education is decided by the relation between the elasticity of the cost of improving the quality of education and that of the cost of rearing a child.

In this model, there are assumed only two ways to invest for parents: children and accumulation of human capital. That is why the quality of education is decided entirely according to the difference between the elasticity of the cost of rearing a child and of the cost related to education. There is actually an another type of investment: accumulation of physical capital. It will be necessary to consider this alternative method.

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## Notes

1) This implies that I neglect the young-age consumption for simplicity of calcula-tion.
2) See Barro and Becker. This assumption is calculated as the condition where giving a birth to a child has positive utility for parents.
3) This specialization does not satisfy the assumption 1 when $h_t=0$ holds. If individuals are born with at least one unit of human capital, the cost of rearing a child can satisfy the assumption 1. Or if the cost function is specialized as $\phi h_t^{\varepsilon}+p$, the cost of rearing a child can be over the value of $\lambda$ even if $h_t=0$ holds. $p$ is the minimum cost of rearing a child. When human capital per capita is over the threshold, $p$ becomes relatively small. It can be negligible. By considering the case of $h_t=0$ in this way, the discussion of this specialization which follows is thought to be reasonable.

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