The Difference in the Optimal Faculty-Student Ratio between Public Schools and Private Schools*

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Differences exist in the faculty-student ratio between the public and private schools. The faculty-student ratio in private schools is very low in Japan and this tendency is easily discerned in higher education. This paper presents various reasons for the noticeable differences between the public and private schools.

Keywords: the optimal ratio of faculty-student, public and private educative sector.

1 Introduction

Differences exist in the faculty-student ratios between the public and private schools. The faculty-student ratio in the private schools is very low and this tendency is easily discerned at Universities. Kindly refer to the figure 1. National (F/S) represents the ratio of faculty to students in national universities, Local (F/S) represents the local universities, and Private (F/S) represents the private universities. The faculty-student ratios in the national and local universities are almost equal and very high. The faculty-student ratios in the private universities are very low. Little attention has been devoted to this difference in the ratio of faculty between public and private universities. This paper discusses the above problem in detail.

Over the past few decades, a considerable number of studies have been conducted on education economics. Becker (1964) had attracted our attention with the theory of 'human capital.' Education is also regarded as an investment method, which acts as a catalyst to improve the productivity of labor. In this case, education has external effects and the public organizations, for instance, the government should support individual education. On the contrary, the signaling theory propounded by Spence (1973) emphasized that education does not improve the ability of individuals but only makes it clear, as to how much ability an individual has got by passing the entrance examination. In this case, education is transformed into an individual problem, thereby causing little or no reason for the government to encourage individuals to invest in education, which is pointed out by Oshio (2002). He also pointed out that education itself has the aspects of invest-

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ment and signalizing the ability along with consumption. The concern with the function of education has grown over the decades.

There are many aspects regarding the function of education. It is of extreme importance to follow up on this point further, but not necessary to discuss in detail as far as the purpose of this paper is concerned. I would like to focus on a concrete problem: the difference in the ratio of facultystudents between the private and public schools. Along with the external effects of education on economic growth, the role of education as consumption has gained importance. Hence, I wish emphasize the investment and consumption aspects of education in this paper on an expedient basis.

There were some studies, Balestrino (1997) and Zhang (1997), which focused on the investment aspect of education as possessing endogenous fertility. Their points of discussion dwell mainly on how the government should subsidize those who are educated. In Glomm's (1997) study, both the investment and consumption aspects of education were emphasized without the endogenous fertility aspect. He presented a model in which parents take decisions regarding the schooling for their children and compared two educational regimes: public and private. In the private educational regime, individuals decide the investment in education. In the public educational regime, it is decided by the public sector. Many studies like Glomm have paid attention on who should decide the investment in education: the government or individuals. Little attention was devoted to the difference between the public and private schools. In other words, it means that considerable attention has been paid to the demand side of education but little attention has been paid to the supply side of education, the faculty. If the number of students per faculty increases, lectures become more vacant but the wage rate of faculty becomes higher. There must be a trade-off problem of the faculty, which is different from that of the demand side of education, namely the individuals and government. I would like to focus on how the faculty evaluates the trade-off problem between the efficiency of investment in education and their own utility in terms of the investment and consumption aspect of education with endogenous fertility.

This paper is organized as follows. The general model is presented in the section 2. In the section 3, the effects of taxation on the ratio of facultystudent are investigated. I summarize the discussion and offer a few concluding comments in the last section.

2 The Model

There are an infinite number of periods and overlapping generations of twoperiod-lived individuals, young and adult, many firms and educational institutions and the public educative sector. A single good can

be produced and consumed. There are two kinds of individual. Those who are engaged in education (productive) sector are called faculty (labors) in this model. Individuals who are engaged in the same occupation are identical. Let N_t denote the number of adult individuals, L_t the number of labors and E_t the number of faculty in period t. The ratio of faculty to N_t is denoted by h_t. The number of labors and faculty are denoted by h_t as

$$L_t=(1-h_t)N_t, \tag{1}$$

$$E_t=h_tN_t. \tag{2}$$

They learn when young and works when old. The 'Parents' income is typically spent on their own consumption, rearing children and as investments in 'child education'. Investments in 'child education' are collected as schooling fees, which are then divided among the faculty as wages. They are concerned about their own consumption as adults, the number of children, and the stock of human capital that a child would gain.

The utility, u_t^X $(X=LandE)$, who is born at time $t-1$ and would be an adult at time t, depends positively on his own consumption, c_t^X, the number of children, n_t^X, and human capital of the offspring, H_{t+1}^X. The superscript denotes the kind of occupation, for example, u_t^L denotes the utility of a labor and u_t^E denotes that of faculty. The utility function is the logarithmic utility function[1]:

$$u(c_t^X, n_t^X, H_{t+1}^X)=\alpha\ln c_t^X+\beta\ln n_t^X+\gamma\ln H_{t+1}^X. \tag{3}$$

Each adult in period t spends θw_t^X on rearing a child and invests πe_t^X in education per child. θ denotes the relative price of rearing a child, w_t^X denotes the wage rate, π denotes the relative price of investment in education, and e_t does the amount of investment in education. The budget constraint for a parent becomes

$$c_t^X=(1-n_t^X\theta)w_t^X-\pi n_t^Xe_t^X. \tag{4}$$

Like Glomm (1997)[2], human capital is accumulated according to the learning technology,

$$H_{t+1}^X=\frac{E_t}{N_{t+1}}(e_t^X)^{\sigma_1}(H_t^X)^{\sigma_2}. \tag{5}$$

where $\sigma_1+\sigma_2=1$. Human capital per child, H_{t+1}^X, depends positively on the faculty-student ratio, $\frac{E_t}{N_{t+1}}$, the parent's investment in education, e_t^X, and human capital per parent, H_t^X. $\frac{E_t}{N_{t+1}}$ denotes the effect of the ratio of faculty to student in school and e_t^X denotes that of the private investment in education on the accumulation of human capital. This implies that human capital is accumulated by the sum of the public and private investment in education and the stock of parental human capital.

By substituting equation (4) into (3), the maximizing utility problem becomes

$$\Omega=\max_{n_t^X,e_t^X}\alpha\ln\{(1-n_t^X\theta)w_t^X-\pi n_t^Xe_t^X\}+\beta\ln n_t^X+\gamma\ln\left(\left(\frac{E_t}{N_{t+1}}\right)(e_t^X)^{\sigma_1}(H_t^X)^{\sigma_2}\right). \tag{6}$$

I assume that N_{t+1} in the last term is not affected by the individual's decision. It denotes the effect of the faculty-student

ratio in school on the accumulation of human capital. This is beyond the control of an individual. That is why N_{t+1} in the last term is not affected by an individual's decision.

The first-order conditions for a parent are as follows:

$$\frac{-(\theta w_t^X+\pi e_t^X)\alpha}{(1-n_t^X\theta)w_t^X-\pi n_t^X e_t^X}+\frac{\beta}{n_t^X}=0, \tag{7}$$

$$\frac{-\alpha\pi n_t^X}{(1-n_t^X\theta)w_t^X-\pi n_t^X e_t^X}+\frac{\gamma\sigma_1}{e_t^X}=0. \tag{8}$$

Equation (7) means that the utility forgone from giving up $\theta+\pi e_t^X$ units of consumption to have an additional child equals the utility obtained from enjoying the child. Equation (8) equates the utility forgone from increasing in investment in education with the utility obtained from increasing the level of education per capita.

Equation (4), (7) and (8) lead to solutions for fertility and investment in education

$$(n^X)^*=\frac{\beta-\gamma\sigma_1}{\theta(\alpha+\beta)}\equiv n^*, \tag{9}$$

$$(e_t)^*=Aw_t^X. \tag{10}$$

where $A\equiv\frac{\gamma\sigma_1\theta}{\pi(\beta-\gamma\sigma_1)}$. I define D as

$$\begin{bmatrix} \frac{-\beta\theta^2(\alpha+\beta)^3}{\alpha(\beta-\gamma\sigma_1)^2} & \frac{-\pi(\alpha+\beta)^2}{\alpha w_t^X} \\ \frac{-\pi(\alpha+\beta)^2}{\alpha w_t^X} & \frac{-(\alpha+\gamma\sigma_1)\pi^2(\beta-\gamma\sigma_1)^2}{\alpha\gamma\sigma_1(\theta w_t^X)^2} \end{bmatrix}$$

which is the Hessian matrix. It satisfies the second order conditions:

$$\frac{-\beta\theta^2(\alpha+\beta)^3}{\alpha(\beta-\gamma\sigma_1)^2}<0, \tag{11a}$$

$$\begin{vmatrix} \frac{-\beta\theta^2(\alpha+\beta)^3}{\alpha(\beta-\gamma\sigma_1)^2} & \frac{-\pi(\alpha+\beta)^2}{\alpha w_t^X} \\ \frac{-\pi(\alpha+\beta)^2}{\alpha w_t^X} & \frac{-(\alpha+\gamma\sigma_1)\pi^2(\beta-\gamma\sigma_1)^2}{\alpha\gamma\sigma_1(\theta w_t^X)^2} \end{vmatrix} = \frac{(\beta-\gamma\sigma_1)\pi^2(\alpha+\beta)^3}{\alpha\gamma\sigma_1(w_t^X)^2}>0. \tag{11b}$$

as long as $\beta>\gamma\sigma_1$ holds. The condition means that the preference to enjoying a child need to be more than that to raising the level of education per capita. I assume it after this.

In equation (9), fertility is not related to any kind of occupation. It is equal among adults because the preferences are the same and the cost of having a child is positively dependent on the parent's own wage rate at the same rate. Equation (10) represents that the investment in education positively depends on the parent's own wage rate.

The organization associated with education collects the investment in education to finance wages of the faculty. The budget constraint becomes

$$e_t^L n_t^L L_t+e_t^E n_t^E E_t=E_t w_t^E. \tag{12}$$

Substituting equation (10) into equation (12) represents the relation between w_t^E and w_t^L,

$$w_t^E=\frac{(1-h_t)n^*A}{(1-n^*A)h_t}w_t^L. \tag{13}$$

If $\frac{\gamma\sigma_1}{(\alpha+\beta)\pi}>h_t$ holds, the wage rates of faculty is higher than those of labors.

Lemma 2.1. *If the ratio of faculty is sufficiently low, the wage rates of faculty become higher than those of labors.*

The average wage rate, $\overline{w_t}\equiv\frac{w_t^L L_t+w_t^E E_t}{N_t}$,

and the average investment in education, $\overline{e_t} \equiv \frac{e_t^L n_t^L L_t + e_t^E n_t^E E_t}{n^* N_t}$, become

$$\overline{w_t} = \frac{(1-h_t)w_t^L}{1-n^*A}, \tag{14}$$

$$\overline{e_t} = \frac{(1-h_t)Aw_t^L}{1-n^*A}. \tag{15}$$

According to equation (9) and (10), the consumption and the utility of adult become

$$c_t^X = \frac{\alpha w_t^X}{(\alpha+\beta)}, \tag{16}$$

$$u_t^X = \alpha \ln\left(\frac{\alpha w_t^X}{\alpha+\beta}\right) + \beta \ln\left(\frac{\beta-\gamma\sigma_1}{(\alpha+\beta)\theta}\right) + \gamma \ln\left(\left(\frac{E_t}{N_{t+1}}\right)\left(\frac{\gamma\sigma_1\theta w_t^X}{(\beta-\gamma\sigma_1)\pi}\right)^{\sigma_1}(H_t^X)^{\sigma_2}\right). \tag{17}$$

The consumption positively depends on α and negatively on β. Like Glomm (1997), labors earn wages in proportion to the stock of human capital:

$$w_t^L = qH_t. \tag{18}$$

where $q>0$.

2.1 The Optimal Ratio of Faculty

Individuals decide consumption, the number of children, and the amount of investment in education, as stated above. Next, we focus on the ratio of faculty. Either the public educative sector or the private educative sector decides the ratio of faculty. Two organizations are different in form for the social welfare function. The public educative sector decides the ratio of faculty to maximize the following social welfare function:

$$\max_{h_t} W_t^u \equiv \sum_{t=0}^{\infty} \delta^t u_t^a(w_t^a, n_t^a, e_t^a), \tag{19}$$

where δ is the social discount rate. The variables superscripted by a denote the average variables. This function implies that the social welfare, which the public educative sector considers, depends on the utility of an individual with an average income. This function is not either the utilitarianism or the Rawls' social welfare function[3]. However, this function is a kind of the utilitarianism social welfare function. This type of function is more treatable because the social welfare function is only represented by the per capita utility.

The private educative sector decides the ratio of faculty to maximize the following social welfare function:

$$\max_{h_t} W_t^r \equiv \sum_{t=0}^{\infty} \delta^t u_t^E(w_t^E, n_t^E, e_t^E), \tag{20}$$

This function implies that the educative social welfare function depends only on the utility per faculty. The utilities of laborers are not included in this function. The ratio of faculty derived from this function is decided for the faculty only. The two social welfare functions are different in the factors they consider.

I assume the public educative sector and the private educative sector think that children born by laborers will become laborers and that children born from faculty will become faculty. First, the optimal ratio of faculty that the public educative sector considers is calculated.

Substituting equation (14), (15), (17), and (18) into equation (19) rewrites the social welfare function,

$$\max_{h_t} W_t^u = \delta^t((\alpha+\gamma\sigma_1)\ln(1-h_t)+\gamma\ln h_t$$
$$+(\alpha+\gamma)\ln H_t+C)+\delta^{t+1}((\alpha+\gamma\sigma_1)$$
$$\ln(1-h_{t+1})+\gamma\ln h_{t+1}+(\alpha+\gamma)$$
$$\ln\left(\left(\frac{(1-h_t)n^*A^2q}{1-n^*A}\right)^{\sigma_1}\left(\frac{h_t}{n^*}\right)H_t\right)+C)$$
$$+\delta^{t+2}((\alpha+\gamma\sigma_1)\ln(1-h_{t+2}) \qquad (21)$$
$$+\gamma\ln h_{t+2}+(\alpha+\gamma)\ln$$
$$\left(\left(\frac{(1-h_{t+1})(1-h_t)(n^*A^2q)^2}{(1-n^*A)^2}\right)^{\sigma_1}\right.$$
$$\left.\left(\frac{(h_{t+1})(h_t)}{(n^*)^2}\right)H_t\right)+C)$$
$$+\cdots$$

where $C\equiv(\alpha+\gamma\sigma_1)\ln\left(\frac{n^*Aq}{1-n^*A}\right)+(\beta-\gamma)\ln n^*+\alpha\ln(1-n^*(\theta+\pi A))+2\gamma\sigma_1\ln(A)$.

The first order condition for an interior solution of h_t becomes

$$\frac{\partial W_t^u}{\partial h_t}=\frac{-(\alpha+\gamma\sigma_1)}{1-h_t}+\frac{\gamma}{h_t}$$
$$+\eta(\alpha+\gamma)\left(\frac{1}{h_t}+\frac{-\sigma_1}{1-h_t}\right)=0 \qquad (22)$$

where $\eta\equiv\frac{\delta}{1-\delta}$. The optimal ratio of faculty for individuals with the average income becomes

$$h_a^*=\frac{\chi}{\alpha+(1+\sigma_1)\chi}, \qquad (23)$$

where $\chi\equiv\gamma+\eta(\alpha+\gamma)$.

Next, the optimal ratio of faculty for faculty is calculated. Substituting equation (10), (17), (13) and (18) into equation (24) rewrites the social welfare function:

$$\max_{h_t} W_t^r=\delta^t\left((\alpha+\gamma\sigma_1)\ln\left(\frac{(1-h_t)}{h_t}\right)+\gamma\ln h_t\right.$$
$$\left.+(\alpha+\gamma)\ln H_t+C\right)$$
$$+\delta^{t+1}((\alpha+\gamma\sigma_1)\ln\left(\frac{(1-h_{t+1})}{h_{t+1}}\right)+\gamma\ln h_{t+1}$$
$$+(\alpha+\gamma)\ln\left(\left(\frac{(1-h_t)n^*A^2q}{(1-n^*A)h_t}\right)^{\sigma_1}\left(\frac{h_t}{n^*}\right)H_t\right)$$
$$+C)+\delta^{t+2}((\alpha+\gamma\sigma_1)\ln\left(\frac{(1-h_{t+2})}{h_{t+1}}\right)$$
$$+\gamma\ln h_{t+2}$$
$$+(\alpha+\gamma)\ln\left(\left(\frac{(1-h_{t+1})(1-h_t)(n^*A^2q)^2}{(1-n^*A)^2h_{t+1}h_t}\right)^{\sigma_1}\right.$$
$$\left.\left(\frac{(h_{t+1})(h_t)}{(n^*)^2}\right)H_t\right)+C)$$
$$+\cdots \qquad (24)$$

The first order condition for an interior solution of h_t becomes

$$\frac{\partial W_t^r}{\partial h_t}=-(\alpha+\gamma\sigma_1)\left(\frac{1}{1-h_t}+\frac{1}{h_t}\right)+\gamma\frac{1}{h_t}$$
$$+\eta(\alpha+\gamma)\left((1-\sigma_1)\frac{1}{h_t}+\frac{-\sigma_1}{1-h_t}\right)=0. \qquad (25)$$

The optimal ratio of faculty for the private educative sector becomes

$$h_E^*=\frac{\sigma_2\chi-\alpha}{\chi}. \qquad (26)$$

The comparison between the optimal ratios of faculty is

$$h_a^*>h_E^*. \qquad (27)$$

The optimal ratio of faculty the public educative sector decides is higher than that the private educative sector decides. When the ratio of faculty is decided, the change in the ratio of faculty has an effect on the social welfare through two channels. First, the channel through the faculty-student ratio in school has a positive effect. Sec-

ond, the channel through the wage rate has a negative effect. The second channel is very important for faculty. An increase in the ratio of faculty directly decreases the share of the investment in education per faculty. Thus, the ratio of faculty, which is decided by the private educative sector, becomes lower. The lower ratio of faculty implies faculty-student ratio in schools also becomes lower.

Next, we discuss the average growth of human capital. I define $(1+G)$ as $\left(\frac{h}{n}\right)\left(\frac{(1-h)qnA^2}{(1-nA)}\right)^{\sigma_1}$. $(1+G)$ is a strictly concave function of h for any $0<h<1$[4] and is maximized at $h^G \equiv \frac{1}{1+\sigma_1}$.

The average growth of human capital, $(1+g^X) \equiv \frac{H^a_{t+1}}{H^a_t}(h^*_X)$, becomes

$$(1+g^a)=\left(\frac{h^*_a}{n^*}\right)\left(\frac{(1-h^*_a)qn^*A^2}{(1-n^*A)}\right)^{\sigma_1}, \quad (28)$$

$$(1+g^E)=\left(\frac{h^*_E}{n^*}\right)\left(\frac{(1-h^*_E)qn^*A^2}{(1-n^*A)}\right)^{\sigma_1}. \quad (29)$$

The comparison among h^*_a, h^*_E and h^G becomes

$$h^G > h^*_a > h^*_E. \quad (30)$$

According to equation (30) and because $(1+G)$ is a strictly concave function, the comparison between the average growth of human capital becomes

$$(1+g^a)>(1+g^E). \quad (31)$$

If the average growth of human capital is emphasized, the ratio of the faculty should be decided by the public educative sector. In this point, for the public education it is reasonable to accept. However, the order of the average wage rate is different from that of the average growth of human capital. With the same stock of human capital, the comparison of the average wage rate becomes

$$\overline{w^a}(\overline{H})<\overline{w^E}(\overline{H}). \quad (32)$$

where $\overline{H}$ implies the same stock of human capital. The lower ratio of the optimal ratio of faculty leads to a higher wage rate of the faculty because the share of the investment in education per faculty rises because the possibility of unemployment is not considered in this model; the rise in the wage rate of faculty directly raises the average wage rate. If the possibility of unemployment is considered, the result may be different. The lower ratio of faculty results in the higher number of laborers. The wage rate of laborers would be decreased. Thus, the lower ratio of faculty does not always raise the average wage rate. Though we should note that the possibility of unemployment is not considered in this model, we have the following proposition:

Proposition 2.1. *When the public educative sector decides the ratio of faculty to maximize the social welfare, the average growth rate of human capital becomes higher but the average wage rate becomes lower under the same stock of human capital. When the private educative sector decides the ratio of faculty, the average wage rate becomes higher but the growth of human capital becomes lower under the same stock of human capital.*

3 Conclusion

In this model, individuals decide consumption, the number of children, and the investment in education. The ratio of faculty is decided either by the public educative sector or by the private educative sector. The public educative sector maximizes the welfare of individuals with the average income. The private educative sector maximizes the welfare of faculty. The ratio of faculty decided by the public educative sector is higher and raises the growth of human capital. The ratio of faculty decided by the private educative sector raises the wages of faculty. This paper shows the reason that faculty-student ratio becomes low in private schools.

This paper only shows the reason as to why there is a difference in the ratio of faculty-students between the public and the private universities. There is considerable scope for expanding this model. First, we should study whether the difference in the ratio should be decreased or not. Second, how would it be done, if the difference in the ratio should be decreased? Recently, public universities were turned into independent agencies. It implies that 'affection' from the supply side of education would increase. We should pay more attention to this aspect of the education problem. Third, individuals decide the number of children and the investment in education in this paper. According to the investment in education, the wages of faculty will be decided. The private educative sector can decide not only the faculty-student ratio but also the school fee. The value of the school fee has a significant impact on the decision of the number of children. We should take into consideration the mechanism of how the school fee is decided in this kind of problem.

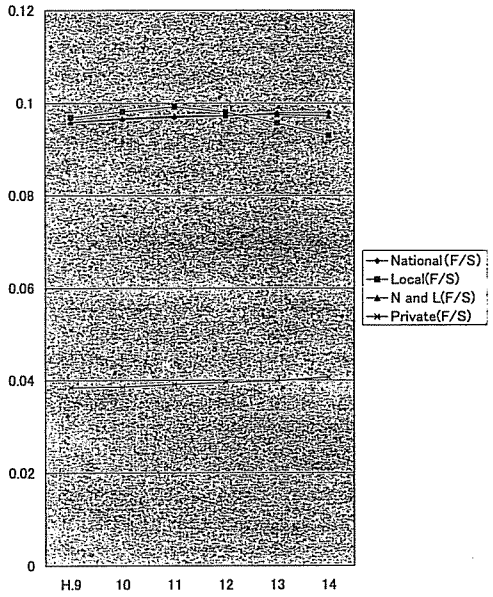

Figure 1 The faculty-student ratios in national, local and private universities.
This figure is calculated from "Statistical Abstract" Ministry of Education, Culture, Sports, Science and Technology (2003).

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paper.

Notes

1) This utility function is the same utility function in Glomm's (1997), except for the inclusion of the number of children in the utility.

2) In Glomm (1997) human capital is accumulated according to the time allocated to schooling, the quality of the school, and the stock of parental knowledge. As we are not concerned with the time allocation, it is ignored. As a substitute for considering the time allocation, we consider faculty-student ratio because this substitution calculates the optimal ratio of faculty numerically. The quality of school in Glomm corresponds to the investment in education in this paper.

3) The utilitarianism social welfare function implies that the social welfare is the sum of the utility of individuals with equal incomes. The Rawls' social welfare function is that the social welfare only depends on the utility of the individual in the worst situation. See Johansson (1991).

4) It is proofed because $\frac{d^2(1+G)}{dh^2}<0$ holds for any $0<h<1$.

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