Intrinsic anisotropic magnetoresistance in spin-polarized two-dimensional electron gas with Rashba spin-orbit interaction

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We analyze the effects of disorder on the intrinsic anisotropic magnetoresistance (AMR) of a spin-polarized two-dimensional electron gas with a Rashba-type spin-orbit interaction in the linear response theory. We show that the AMR vanishes unless the lifetime is spin dependent, analogous with the anomalous and spin Hall effects, which indicates a strong similarity between the anomalous and spin Hall effects and AMR. Furthermore, we perform numerical simulations of the ballistic and diffusive AMR.

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Recently, the spin-orbit interaction (SOI) in metals and semiconductors has attracted significant attention in the field of spintronics, since it allows for electrical control of spin without the use of ferromagnets or a magnetic field. It is well known that the SOI gives rise to unusual transport phenomena, such as the anomalous Hall (AH) effect in ferromagnets,[1] spin Hall (SH) effect in normal conductors,[2–5] and anisotropic magnetoresistance (AMR).[6] There has been great progress in the understanding of the role of SOI on the AH and SH effects over the last several years. Thus far, extrinsic (impurity induced skew and side jump) mechanisms[7,8] have been widely accepted to explain the experimental results; however, an intrinsic mechanism originating from the Berry phase[9] has recently been reexamined.[10,11] Analogous to this, intrinsic origins[4,5] have been raised for the SH effect after the original prediction of the SH effect caused by extrinsic mechanisms.[2,3] The effects of scattering on the intrinsic AH and SH effects have recently been studied intensively for two-dimensional electron gas (2DEG) with Rashba-type SOI.[12–16] The present authors have shown that both the AH and SH effects vanish unless the lifetime is spin dependent when the carrier density is high, indicating that there is a strong similarity between the two.[14]

The AMR is magnetoresistance caused by the change in the relative angle between the current and magnetization in a ferromagnet, and it also originates from the SOI. A phenomenological model of AMR is widely used in experimental analyses; however, the microscopic interpretation of AMR may still be ambiguous.[17–19] In view of the progress in understanding the AH and SH effects mentioned above, questions may be raised as to whether the AMR caused by "intrinsic" origins can exist and if there is any similarity between AMR and AH and SH effects. Actually, the AMR in the ballistic transport regime has been predicted for transition metals using a first-principles method,[20] and AMR in 2DEG with Rashba and Dresselhaus SOIs has been studied in detail in the Boltzmann formalism.[21] An analysis of AMR has also been done for realistic p-type semiconductors.[22] However, the role of electron scattering on AMR caused by intrinsic (uniform) SOI is far from being fully understood. The treatment of AMR within the same framework used for AH and SH effects may yield fruitful results for the questions raised.

To this end, we have extended the previous method applied for AH and SH conductivities[12,14,23] to obtain an analytical result of the intrinsic AMR in the spin-polarized Rashba-split 2DEG, and we show that intrinsic AMR actually occurs but vanishes unless the lifetime is spin dependent, similar to the AH and SH effects. The results indicate the strong similarity among the intrinsic AH and SH effects and AMR.[24] We further calculate the intrinsic AMR in the ballistic and diffusive regime by using numerical methods for a tight-binding version of the 2DEG with the Rashba SOI. It will be shown that the AMR in the ballistic regime yields a nonlinear dependence on the exchange magnetic field, and the AMR in the diffusive regime can be large at low carrier densities. The former result may be attributed to a Fermi-surface (circle) effect, and the latter to the energy dependence of the density of states characteristic of the tight-binding model. The present study may provide a fundamental understanding of the transport properties in 2DEG with the Rashba SOI.

We consider a system of a 2DEG located on an xy plane, with magnetization $\boldsymbol{M}$ parallel to the x, y, or z direction, as shown in Fig. 1(a). The Hamiltonian is given by $\mathbf{H}=\mathbf{H}_0^\alpha+\mathbf{V}$, where the first and second terms are the unperturbed Hamiltonian[25] and isotropic random impurity potentials with a δ-function type and are given as

$$\mathbf{H}_0^\alpha = \begin{pmatrix} \frac{\hbar^2}{2m}k^2 & i\lambda\hbar k_- \\ -i\lambda\hbar k_+ & \frac{\hbar^2}{2m}k^2 \end{pmatrix} - \Delta_{ex}\sigma_\alpha \quad (1)$$

and

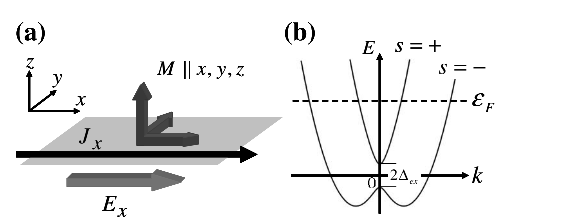

FIG. 1. (a) A schematic figure of 2DEG magnetized along the x, y, and z directions. The longitudinal conductivity σ_{xx} is calculated as a response of the current density J_x to an electric field E_x applied along the x direction and (b) the energy dispersion curve for magnetization parallel to the z direction.

$$\mathbf{V} = \begin{pmatrix} V_\uparrow & 0 \\ 0 & V_\downarrow \end{pmatrix} \sum_i \delta(\boldsymbol{r} - \boldsymbol{R}_i), \tag{2}$$

respectively. Here, λ indicates the SOI, $k_\pm = k_x \pm ik_y$, $\Delta_{ex}\sigma_\alpha$ is the Zeeman term due to an exchange splitting Δ_{ex} with a Pauli spin matrix σ_α, with $\alpha = x, y, z$, and $\boldsymbol{R}_i$ indicates an atomic site. The energy dispersion of the unperturbed Hamiltonian is given as

$$E^x_{ks} = \frac{\hbar^2}{2m}k^2 + s\sqrt{F_k - 2\Delta_{ex}\lambda\hbar k_y}, \tag{3}$$

$$E^y_{ks} = \frac{\hbar^2}{2m}k^2 + s\sqrt{F_k + 2\Delta_{ex}\lambda\hbar k_x}, \tag{4}$$

and

$$E^z_{ks} = \frac{\hbar^2}{2m}k^2 + s\sqrt{F_k}, \tag{5}$$

where $s = \pm$, and $F_k = \lambda^2\hbar^2k^2 + \Delta^2_{ex}$. Since the analysis to obtain the conductivity is rather complicated due to the spin-dependent potential, we perform the analysis in the Pauli spin space ($|k\sigma\rangle$). However, it is often more convenient to perform manipulations in chiral spin space ($|ks\rangle$), and in this case, we make use of the unitary transformation given by the eigenvectors. The energy eigenvalues for $M \parallel z$ are indicated in Fig. 1(b).

We first present the analytical result of the longitudinal conductivity for the diffusive transport regime using the Kubo formula,

$$\sigma^\alpha_{xx} = \frac{\hbar}{2\pi L^2}\mathrm{Tr}\langle J^\alpha_x G^{\alpha R} J^\alpha_x G^{\alpha A}\rangle_{\mathrm{imp}}, \tag{6}$$

where x and α indicate the direction of the current and magnetization, respectively, L^2 the volume of 2DEG, J^α_x is a current operator, $G^{\alpha R(A)}$ is the retarded (advanced) Green's function, and $\langle\cdots\rangle_{\mathrm{imp}}$ indicates a random average over the distribution of impurities. The random average for the Green's function and the conductivity will be dealt with using the Born and ladder approximations, respectively.

The current operators used in the Kubo formula are given by $J^\alpha_x = e\,\partial H^\alpha_0/\partial p_x$. The unperturbed Green's function is diagonal in the chiral spin space and is expressed in terms of the Green's function,

$$g^{\alpha R(A)}_{ks} = [\varepsilon + (-)i0 - E^\alpha_{ks}]^{-1}. \tag{7}$$

In the Born approximation, the random average of the impurity potential is usually neglected since it only gives a constant potential. In the present formalism, however, we retain the spin-dependent part of the constant term,

$$\langle\langle k|\mathbf{V}|k\rangle\rangle_{\mathrm{imp}} = (nV\mathbf{1} - n\delta V\sigma_z)\delta_{kk'}, \tag{8}$$

since the spin symmetry is broken in the present case. Here, $\delta V = (V_\downarrow - V_\uparrow)/2$, and n represents the concentration of the impurities. The Born approximation gives the following self-energy for which the spin axis of the Pauli spin space is taken along the magnetization direction α,

$$\langle\langle k|\mathbf{V}G^{\alpha R(A)}_0\mathbf{V}|k\rangle\rangle_{\mathrm{imp}} = -(+)i\pi nD\begin{pmatrix} V^2_\uparrow & 0 \\ 0 & V^2_\downarrow \end{pmatrix}\delta_{kk'}, \tag{9}$$

where $-$ and $+$ correspond to the retarded and advanced Green's function, respectively. Then, the effective (random averaged) Green's function may be written as

$$\tilde{G}^{\alpha R(A)} = [G^{\alpha R(A)^{-1}}_0 - \Sigma^{R(A)}]^{-1}, \tag{10}$$

where

$$\Sigma^{R(A)} = -n\delta V\delta_{kk'} - (+)i\begin{pmatrix} \dfrac{\hbar}{2\tau_\uparrow} & 0 \\ 0 & \dfrac{\hbar}{2\tau_\downarrow} \end{pmatrix}\delta_{kk'}, \tag{11}$$

with $\hbar/2\tau_s = n\pi DV^2_s$. It should be noted that Eq. (9) is exact within the Born approximation for $M \parallel z$ but correct up to λ^2 for $M \parallel x, y$.

The averaged conductivity is given by the sum of the nonvertex and the vertex parts as

$$\sigma^\alpha_{xx} = \sigma^{\alpha\mathrm{nv}}_{xx} + \sigma^{\alpha\mathrm{v}}_{xx}, \tag{12}$$

$$\sim \frac{\hbar}{2\pi L^2}\mathrm{Tr}(J^\alpha_x\tilde{G}^{\alpha R}J^\alpha_x\tilde{G}^{\alpha A} + J^\alpha_x\tilde{G}^{\alpha R}\langle V\Gamma^\alpha V\rangle_{\mathrm{imp}}\tilde{G}^{\alpha A}). \tag{13}$$

In the ladder approximation, the vertex function Γ^α can be written as

$$\Gamma^\alpha = \Gamma^\alpha_0 + \tilde{G}^{\alpha R}\langle V\Gamma^\alpha V\rangle_{\mathrm{imp}}\tilde{G}^{\alpha A}, \tag{14}$$

with

$$\Gamma^\alpha_0 = \sum_k \tilde{G}^{\alpha R}J^\alpha_x\tilde{G}^{\alpha A}, \tag{15}$$

which should be solved self-consistently. In the following, we manipulate the expressions (14) and (15) to obtain equations suitable for numerical integration over the momentum, and we also deduce an analytic expression for the conductivity by expanding them in terms of the spin-dependent part of the lifetime $\delta\tau/\tau$ with $\tau_{\uparrow(\downarrow)} = \tau + (-)\delta\tau$, and the SOI λ.

The analytical expression for the conductivity thus obtained is

$$\sigma^\alpha_{xx} = 2\frac{e^2[\epsilon_F + \tilde{\Delta}(\delta\tau/\tau)]D\tau}{m} + e^2D\tau\lambda^2\left[2 - \left(c^\alpha_1 + c^\alpha_2\frac{\epsilon^2_F}{\tilde{\Delta}^2 + \Sigma^2}\right)\left(\frac{\delta\tau}{\tau}\right)^2\right], \tag{16}$$

with $(c^\alpha_1, c^\alpha_2) = (3,2)$, (2,1), and (1,1) for $\alpha = z$, x, and y, respectively, $\tilde{\Delta} \equiv n\delta V + \Delta_{ex}$, and D is the density of states of the 2DEG at ϵ_F. The first term corresponds to the Drude conductivity with a fixed Fermi energy. In the analysis, we have assumed that the Fermi energy $\epsilon_F > \Delta_{ex}$.

Since the values of (c^α_1, c^α_2) in Eq. (16) are dependent on the magnetization direction, we find that an effect of the AMR exists and that $\sigma^y_{xx} > \sigma^x_{xx} > \sigma^z_{xx}$. However, the AMR vanishes when $\delta\tau/\tau = 0$ even though $M \neq 0$. This result is

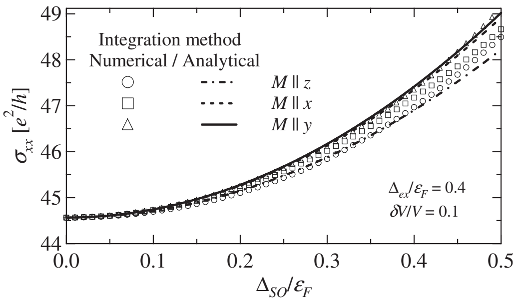

FIG. 2. Calculated results of the conductivity as a function of the SOI $\Delta_{SO} \equiv \lambda\sqrt{2m\epsilon_F}$ over ϵ_F for different directions of the magnetization with an impurity concentration of 10%. The curves and symbols are analytical results and those in the numerical integration, respectively.

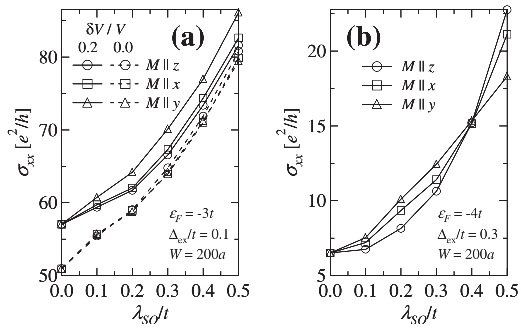

FIG. 3. Longitudinal conductivity as a function of $\lambda_{SO}=\lambda\hbar/2a$ calculated using numerical simulations for (a) $\epsilon_F=-3t$ with $\delta V/V=0$ and 0.2 and (b) $\epsilon_F=-4t$ with $\delta V/V=0$. The impurity concentration is 10%.

caused by the cancellation of the nonvertex part and vertex part of the conductivity.

Figure 2 compares the results calculated by using the expression (16) (solid, broken, and chained curves) with those obtained by performing numerical integration over the momentum for the general equations (triangles, squares, and circles). We find that the agreement between them is rather satisfactory. The numerical integration gives the same result $\sigma_{xx}^y > \sigma_{xx}^x > \sigma_{xx}^z$ consistently with that obtained using Eq. (16). The effect of AMR, however, is rather small for the present choice of parameter values.

The cancellation of the nonvertex and vertex parts mentioned above is correct only when the Fermi energy crosses the two energy branches ($s=+$ and $-$ bands). Unless this condition is satisfied, we can show that the self-energy is dependent on both spin and α even if $V_\uparrow = V_\downarrow$ because the unperturbed Green's function depends on the direction of the magnetization. In this case, we can expect AMR to appear even if $\delta\tau/\tau=0$.

The results obtained above are very similar to those obtained for AH and SH conductivities. In the AH and SH effects, the conductivity in the diffusive transport regime vanishes identically when the Fermi energy crosses the two energy branches and when skew scattering contributions are neglected, while it has a complicated expression otherwise.

We next present results obtained in the numerical simulation. We adopt the Lee-Fisher formalism of the conductance with the recursive Green's function method and calculate the conductance for finite-size systems, with a width of $W=200a$, where a is the lattice spacing and length $L=200a \sim 1000a$ for a square lattice with SOI and random potentials. The conductance values calculated are averaged over 400 different configurations of random impurities with a fixed concentration of 10%. A scaling relation between the averaged conductance $\langle\Gamma\rangle_{\text{imp}}$ and conductivity σ_{xx},

$$1/\langle\Gamma\rangle_{\text{imp}} = R_c + L/\sigma_{xx}W, \tag{17}$$

is used, where R_c is the contact resistance. Since the Lee-Fisher formalism is based on the Kubo-Greenwood formula, and the random potentials are treated numerically, the present method goes beyond the Born and ladder approximations used for the analytical one.

Figure 3(a) shows the calculated results for the conductivity as a function of the SOI $\lambda_{SO} \equiv \lambda\hbar/2a$ over t, where t is the effective hopping integral in the tight-binding version of the Hamiltonian. The calculated results show the same trend obtained in the analytical method. When $\delta V/t=0.0$, the AMR vanishes while it survives for $\delta V/t=0.2$. The magnitude of AMR, however, can be larger than those obtained in the analytical method. This may be due to the change in the electronic states caused by spin-dependent random potentials.

Figure 3(b) shows the calculated results of the conductivity as a function of λ_{SO}/t for $\delta V/t=0.0$, and $\epsilon_F=-4t$. In this case, the AMR appears even though $\delta V/t=0.0$, since the Fermi level crosses only one branch. Note that the band bottom in the paramagnetic state without SOI is $-4t$. The relative magnitude of the conductivity σ_{xx}^x, σ_{xx}^y, and σ_{xx}^z is the same with that shown in Fig. 3(a) only for small SOI; however, the difference between them is rather large in this case. This means that a large AMR can be realized when the Fermi energy is located near the band bottom, that is, when the carrier density is low.

The conductance in the ballistic transport regime is calculated for periodical 2DEG with SOI. Figure 4(a) shows the conductance Γ as a function of Δ_{ex}/t. We find that the relative magnitude of the conductance for $M\parallel x$, y, and z is the same with that of the conductivity σ_{xx}^x, σ_{xx}^y, and σ_{xx}^z calcu-

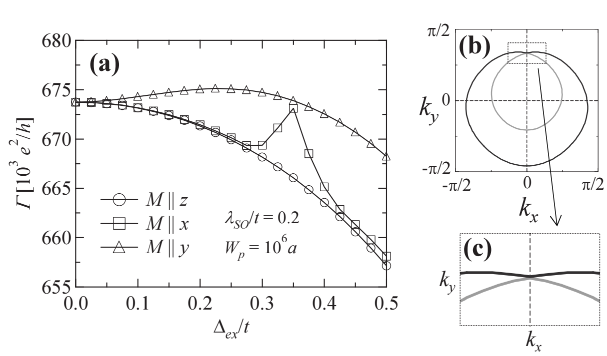

FIG. 4. (a) Ballistic conductance for various directions of magnetization calculated as a function of Δ_{ex}/t. The parameter values are $\lambda_{SO}/t=0.2$ and $\epsilon_F=-3t$. (b) The Fermi circle for $M\parallel x$ at the conductance peak position $\Delta_{ex}/t=0.2\sqrt{3}$ and (c) the vicinity of contacting.

lated before, and the effect of AMR appears even in the ballistic transport regime. As can be seen in the figures, Γ for $M \| x$ shows a peak, which is interpreted as a Fermi-surface effect. With increasing Δ_{ex}, for example, the Fermi surfaces of the two branches come in contact, as shown in Figs. 4(b) and 4(c). This is the origin of the peak structure in the curve of the calculated Γ.

Finally, we comment on some relevance for experiments to confirm the present results. Since we are dealing with longitudinal conductivity, the experimental setup may be much simpler compared to the Hall measurements. Possible materials used could be lightly doped n-type GaAs with magnetic impurities. Care should be taken for the well-known p-type magnetic semiconductors (Ga-Mn)As since the magnetic anisotropy may produce a change in the density of states near the top of the valence band and gives rise to the AMR.[26–28]

In conclusion, we have studied the intrinsic AMR in the diffusive and ballistic transport regime for an exchange-split 2DEG with a Rashba-type SOI. We have formulated the explicit expression for the longitudinal conductivity in the diffusive transport regime and found that the effect of AMR vanishes when the lifetime is spin independent when the Fermi energy is located above the band gap caused by the exchange splitting and SOI. The numerical simulation of the longitudinal conductivity essentially gives the same results. The results suggest a strong similarity between the AH and SH effect, and AMR. It has also been shown that a characteristic Fermi-surface (circle) effect may appear in the AMR in the ballistic transport regime. The present results may contribute toward a deeper understanding of the transport properties characteristic of the Rashba-type spin-orbit interaction.

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