Inflationary cosmology and the late-time accelerated expansion of the universe in nonminimal Yang-Mills- $F(R)$ gravity and nonminimal vector-$F(R)$ gravity

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We study inflationary cosmology and the late-time accelerated expansion of the universe in nonminimal Yang-Mills (YM) theory, in which the YM field couples to a function of the scalar curvature. It is shown that power-law inflation can be realized due to the nonminimal YM field-gravitational coupling which may be caused by quantum corrections. Moreover, it is demonstrated that both inflation and the late-time accelerated expansion of the universe can be realized in a modified YM-$F(R)$ gravity which is consistent with solar-system tests. Furthermore, it is shown that this result can be realized also in a nonminimal vector-$F(R)$ gravity. In addition, we consider the duality of the nonminimal electromagnetic theory and that of the nonminimal YM theory, and also discuss the cosmological reconstruction of the YM theory.

I. INTRODUCTION

Recent observations confirmed that the inflationary stage existed in the early universe, and that also at present the expansion of the universe is accelerating [1,2]. Various scenarios for the late-time acceleration in the expansion of the universe have been proposed. In fact, however, the cosmic acceleration mechanism is not well understood yet (for recent reviews, see [3–7]).

There exist two approaches to account for the late-time acceleration of the universe. One is dark energy, i.e., the general relativistic approach. The other is dark gravity, i.e., the modified gravity approach. Among the latter approaches studied so far, the modification to the Einstein-Hilbert action, e.g., the addition of an arbitrary function of the scalar curvature to it, is one of the most promising latter approaches (for a review, see [7]). Such a modified theory must pass cosmological bounds and solar-system tests because it is considered as an alternative gravitational theory.

A very realistic modified gravitational theory that evades solar-system tests has recently been proposed by Hu and Sawicki [8] (for related studies, see [9]). In this theory, an effective epoch described by the cold dark matter model with cosmological constant (ΛCDM), which accounts for high-precision observational data, is realized in general relativity with the cosmological constant (for a review of observational data confronted with modified gravity, see [10]). This theory can successfully explain the late-time

acceleration of the universe. In Ref. [8], however, the possibility of the realization of inflation has not been discussed. In Refs. [11–13], therefore, modified gravities in which both inflation and the late-time acceleration of the universe can be realized, following the previous inflation-acceleration unification proposal [14], have been presented and investigated. The classification of viable $F(R)$ gravities has also been suggested in Ref. [12]. Here, $F(R)$ is an arbitrary function of the scalar curvature R.

Furthermore, there exists another gravitational source of inflation and the late-time acceleration of the universe: a coupling between the scalar curvature and matter Lagrangian [15,16] (see also [17]). Such a coupling may be applied for the realization of the dynamical cancellation of cosmological constant [18]. In Refs. [19–21], the criteria for the viability of such theories has been considered. As a simple case, a coupling between a function of the scalar curvature and the kinetic term of a massless scalar field in a viable modified gravity has been considered [22].

Recently, inflation and the late-time acceleration of the universe in nonminimal electromagnetism, in which the electromagnetic field couples to a function of the scalar curvature, has been studied in Ref. [23] by using the analyzing procedure in the electromagnetic field considered in Ref. [24]. It is known that the coupling between the scalar curvature and the Lagrangian of the electromagnetic field arises in curved spacetime due to one-loop vacuum-polarization effects in quantum electrodynamics (QED) [25]. As a result, it has been shown that power-law inflation can be realized due to the nonminimal gravitational coupling of the electromagnetic field, and that large-scale magnetic fields can be generated due to the breaking of the conformal invariance of the electromagnetic field

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through its nonminimal gravitational coupling [1] (see also [27]). The mechanism of inflation in this model is as follows. In the very early universe before inflation, electromagnetic quantum fluctuations are generated due to the breaking of the conformal invariance of the electromagnetic field and they act as a source for inflation. Furthermore, also during inflation electromagnetic quantum fluctuations are newly generated and the scale is stretched due to inflation, so that the scale can be larger than the Hubble horizon at that time, and they lead to the large-scale magnetic fields observed in galaxies and clusters of galaxies. This idea is based on the assumption that a given mode is excited quantum mechanically while it is subhorizon sized and then as it crosses outside the horizon "freezes in" as a classical fluctuation [28]. These large-scale magnetic fields can be the origin of the large-scale magnetic fields with the field strength 10^{-7}–10^{-6} G on 10 kpc–1 Mpc scale observed in clusters of galaxies [29] (for reviews of cosmic magnetic fields, see [30]). Furthermore, it has been demonstrated that both inflation and the late-time acceleration of the universe can be realized in a modified Maxwell-$F(R)$ gravity proposed in Ref. [13] which is consistent with solar-system tests.

In the present paper, we consider inflationary cosmology and the late-time accelerated expansion of the universe in nonminimal non-Abelian gauge theory, called the Yang-Mills (YM) theory, in which the non-Abelian gauge field (the YM field) couples to a function of the scalar curvature, in order to investigate the cosmological consequences of the nonminimal gravitational coupling of the YM field. Furthermore, we consider a nonminimal vector-$F(R)$ gravity. In past studies, inflation driven by a vector field has been discussed [31,32]. Moreover, as a candidate for dark energy, the effective YM condensate [33,34], the Born-Infeld quantum condensate [35], and a vector field [36–40] have been proposed. In particular, the possibility that the accelerated expansion of the universe is driven by a field with an anisotropic equation of state has been considered in Ref. [40]. As a result, we show that power-law inflation can be realized due to the nonminimal gravitational coupling of the YM field.[2] Moreover, we demonstrate that both inflation and the late-time accelerated expansion of the universe can be realized in a modified Yang-Mills- $F(R)$ gravity which is consistent with solar-system tests. Furthermore, we show that this result can be realized also in a nonminimal vector-$F(R)$ gravity. In addition, we consider the duality of the nonminimal electromagnetic theory and that of the nonminimal YM theory, and also discuss the reconstruction of the YM theory.

There are several motivations to study nonminimal YM theory. First of all, we show that the appearance of such nonminimal terms in the early universe is compatible with current formulations of YM theory due to specific choice of nonminimal function. Second, some string compactification may lead to effective scalars-YM-Einstein theory (plus higher order corrections). In some cases, one can delete scalars in such a way that extra curvature terms (nonminimal ones) appear in front of the YM Lagrangian. Third, the celebrated asymptotic freedom phenomenon may be understood as appearance of nonminimal terms at the early universe.

This paper is organized as follows. In Sec. II we consider a nonminimal gravitational coupling of the $SU(N)$ YM field in general relativity. First, we describe our model and derive equations of motion from it. Next, we analyze the gravitational field equation, and then show that power-law inflation can be realized. In Sec. III we consider a nonminimal gravitational coupling of the $SU(N)$ YM field in a modified gravitational theory proposed in Ref. [13]. We show that in this theory both inflation and the late-time acceleration of the universe can be realized. In Sec. IV we consider a nonminimal vector-$F(R)$ gravity. Furthermore, in Sec. V we consider the duality of the nonminimal electromagnetic theory and that of the nonminimal YM theory. In addition, in Sec. VI we discuss the reconstruction of the YM theory. Finally, a summary is given in Sec. VII. We use units in which $k_{\mathrm{B}} = c = \hbar = 1$ and denote the gravitational constant $8\pi G$ by κ^2, so that $\kappa^2 \equiv 8\pi/M_{\mathrm{Pl}}^2$, where $M_{\mathrm{Pl}} = G^{-1/2} = 1.2 \times 10^{19}$ GeV is the Planck mass. Moreover, in terms of electromagnetism we adopt Heaviside-Lorentz units.

II. INFLATION IN GENERAL RELATIVITY

In this section, following the discussion given in Ref. [23], we first consider a nonminimal gravitational coupling of the YM field in general relativity.

A. Model

We consider the following model action:

$$S_{\mathrm{GR}} = \int d^4x \sqrt{-g}[\mathcal{L}_{\mathrm{EH}} + \mathcal{L}_{\mathrm{YM}}], \tag{2.1}$$

$$\mathcal{L}_{\mathrm{EH}} = \frac{1}{2\kappa^2} R, \tag{2.2}$$

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} I(R) F_{\mu\nu}^a F^{a\mu\nu}$$
$$\times \left[1 + b\tilde{g}^2 \ln \left| \frac{-(1/2)F_{\mu\nu}^a F^{a\mu\nu}}{\mu^4} \right| \right], \tag{2.3}$$

with

$$I(R) = 1 + f(R), \tag{2.4}$$

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[1] In Ref. [26], gravitational-electromagnetic inflation from a 5-dimensional vacuum state has been considered.

[2] In Ref. [41], the spontaneous generation of chromomagnetic fields at high temperature has been investigated.

$$b = \frac{1}{4} \frac{1}{8\pi^2} \frac{11}{3} N, \qquad (2.5)$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c, \qquad (2.6)$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, R is the scalar curvature arising from the spacetime metric tensor $g_{\mu\nu}$, and $\mathcal{L}_{\text{EH}}$ is the Einstein-Hilbert action. Moreover, $\mathcal{L}_{\text{YM}}$ with $I(R) = 1$ is the effective Lagrangian of the $SU(N)$ YM theory up to one-loop order [42,43], $f(R)$ is an arbitrary function of R, b is the asymptotic freedom constant, $F_{\mu\nu}^a$ is the field strength tensor, A_μ^a is the $SU(N)$ YM field with the internal symmetry index a (Roman indices, a, b, c, run over $1, 2, \ldots, N^2 - 1$, and in $F_{\mu\nu}^a F^{a\mu\nu}$ the summation in terms of the index a is also made), and f^{abc} is a set of numbers called structure constants and completely antisymmetric [44]. Furthermore, μ is the mass scale of the renormalization point, and a field-strength-dependent running coupling constant is given by [43]

$$\tilde{g}^2(X) = \frac{\tilde{g}^2}{1 + b\tilde{g}^2 \ln|X/\mu^4|}, \qquad (2.7)$$

where

$$X \equiv -\frac{1}{2} F_{\mu\nu}^a F^{a\mu\nu}. \qquad (2.8)$$

Hence, $\tilde{g}$ is the value of the running coupling constant when $X = \mu^4$.

The field equations can be derived by taking variations of the action in Eq. (2.1) with respect to the metric $g_{\mu\nu}$ and the $SU(N)$ YM field A_μ^a as follows:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 T_{\mu\nu}^{(\text{YM})}, \qquad (2.9)$$

with

$$T_{\mu\nu}^{(\text{YM})} = I(R)(g^{\alpha\beta} F_{\mu\beta}^a F_{\nu\alpha}^a \varepsilon - \frac{1}{4} g_{\mu\nu} \mathcal{F})$$
$$+ \frac{1}{2} \{ f'(R) \mathcal{F} R_{\mu\nu} + g_{\mu\nu} \Box[f'(R) \mathcal{F}]$$
$$- \nabla_\mu \nabla_\nu [f'(R) \mathcal{F}] \}, \qquad (2.10)$$

$$\varepsilon = 1 + b\tilde{g}^2 \ln \left| e\left[\frac{-(1/2) F_{\mu\nu}^a F^{a\mu\nu}}{\mu^4} \right] \right|$$
$$= 1 + b\tilde{g}^2 \ln \left| e\left(\frac{X}{\mu^4} \right) \right|, \qquad (2.11)$$

$$\mathcal{F} = F_{\mu\nu}^a F^{a\mu\nu} \left[1 + b\tilde{g}^2 \ln \left| \frac{-(1/2) F_{\mu\nu}^a F^{a\mu\nu}}{\mu^4} \right| \right]$$
$$= -2X\left(1 + b\tilde{g}^2 \ln \left| \frac{X}{\mu^4} \right| \right), \qquad (2.12)$$

and

$$\frac{1}{\sqrt{-g}} \partial_\mu[\sqrt{-g} I(R) \varepsilon F^{a\mu\nu}] - I(R) \varepsilon f^{abc} A_\mu^b F^{c\mu\nu} = 0, \qquad (2.13)$$

where the prime denotes differentiation with respect to R, ∇_μ is the covariant derivative operator associated with $g_{\mu\nu}$, and $\Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$ is the covariant d'Alembertian for a scalar field. In addition, $R_{\mu\nu}$ is the Ricci curvature tensor, while $T_{\mu\nu}^{(\text{YM})}$ is the contribution to the energy-momentum tensor from the $SU(N)$ YM field. Moreover, ε is a field-strength-dependent effective dielectric constant [43], and $e \approx 2.72$ is the Napierian number. In deriving the second equalities in Eqs. (2.11) and (2.12), we have used Eq. (2.8).

We assume the spatially flat Friedmann-Robertson-Walker spacetime with the metric

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2 = a^2(\eta)(-d\eta^2 + d\mathbf{x}^2), \qquad (2.14)$$

where a is the scale factor, and η is the conformal time. In this spacetime, $g_{\mu\nu} = \text{diag}(-1, a^2(t), a^2(t), a^2(t))$, and the components of $R_{\mu\nu}$ and R are given by

$$R_{00} = -3(\dot{H} + H^2), \qquad R_{0i} = 0,$$
$$R_{ij} = (\dot{H} + 3H^2)g_{ij}, \qquad R = 6(\dot{H} + 2H^2), \qquad (2.15)$$

where $H = \dot{a}/a$ is the Hubble parameter. Here, a dot denotes a time derivative, $\dot{} = \partial/\partial t$.

B. Power-law inflation

The $(\mu, \nu) = (0, 0)$ component and the trace part of the $(\mu, \nu) = (i, j)$ component of Eq. (2.9), where i and j run from 1 to 3, read

$$H^2 + J_1 = \frac{\kappa^2}{6} \{ I(R)(b\tilde{g}^2 X + \varepsilon Y) + 3[-f'(R)(\dot{H} + H^2)$$
$$+ 6f''(R)H(\ddot{H} + 4H\dot{H})]\mathcal{F} + 3f'(R)H\dot{\mathcal{F}} \}, \qquad (2.16)$$

$$J_1 = \frac{1}{6} F(R) - F'(R)(\dot{H} + H^2), \qquad (2.17)$$

and

$$2\dot{H} + 3H^2 + J_2 = \frac{\kappa^2}{2}\left\{I(R)X\left(-\frac{1}{3}\varepsilon + b\tilde{g}^2\right) + \left[-f'(R)(\dot{H} + 3H^2) + 6f''(R)(\dddot{H} + 7H\ddot{H} + 4\dot{H}^2 + 12H^2\dot{H})\right.\right.$$

$$\left.\left. + 36f'''(R)(\ddot{H} + 4H\dot{H})^2\right]\mathcal{F} + 3[f'(R)H + 4f''(R)(\ddot{H} + 4H\dot{H})]\dot{\mathcal{F}} + f'(R)\ddot{\mathcal{F}}\right\}, \tag{2.18}$$

$$J_2 = \tfrac{1}{2}F(R) - F'(R)(\dot{H} + 3H^2) + 6F''(R)[\dddot{H} + 4(\dot{H}^2 + H\ddot{H})] + 36F'''(R)(\ddot{H} + 4H\dot{H})^2, \tag{2.19}$$

respectively. Here, X and Y are given by

$$X = |E_i^{a(\text{proper})}(t)|^2 - |B_i^{a(\text{proper})}(t)|^2, \tag{2.20}$$

$$Y = |E_i^{a(\text{proper})}(t)|^2 + |B_i^{a(\text{proper})}(t)|^2, \tag{2.21}$$

where $E_i^{a(\text{proper})}(t)$ and $B_i^{a(\text{proper})}(t)$ are the quantities corresponding to proper electric and magnetic fields in the $SU(N)$ YM theory, respectively. In this paper, because we consider the case in which there exist the YM electric and magnetic fields as background quantities at the 0th order, we here consider that the YM electric and magnetic fields do not have the dependence on the space components x. Moreover, J_1 and J_2 are correction terms in a modified gravitational theory described by the action in Eq. (3.1) in the next section. Hence, because in this section we consider general relativity, i.e., the case $F(R) = 0$ in the action in Eq. (3.2), here both J_1 and J_2 are zero. Furthermore, in deriving Eqs. (2.16) and (2.18), we have used equations in (2.15) and the following equation:

$$\mathcal{F} = -2X(\varepsilon - b\tilde{g}^2), \tag{2.22}$$

which follows from Eqs. (2.8), (2.11), and (2.12).

In the search of exact solutions for nonminimal YM (electromagnetic)-gravity theory (see [45,46]), there exists the problem of off-diagonal components of YM (electromagnetic) stress tensor being nonzero while the right-hand side of Einstein equations is zero. In our case, we consider the following. As a simple case, we can consider the following case in which the off-diagonal components of $T_{\mu\nu}^{(\text{YM})}$ in Eq. (2.10) vanishes: (i) Only (YM) magnetic fields are generated and hence (YM) electric fields are negligible. (ii) $\boldsymbol{B}^a = (B_1^a, B_2^a, B_3^a)$, where $B_1^a = B_2^a = 0$, $B_3^a \neq 0$, namely, we consider the case in which only one component of $\boldsymbol{B}^a$ is nonzero and hence the other two components are zero. In such a case, it follows from div$\boldsymbol{B}^a = 0$ that the off-diagonal components of the last term on the right-hand side of $T_{\mu\nu}^{(\text{YM})}$, i.e., $\nabla_\mu\nabla_\nu[f'(R)\mathcal{F}]$, are zero. Thus, all of the off-diagonal components of $T_{\mu\nu}^{(\text{YM})}$ are zero. In this paper (including Secs. III and IV) we consider the above case in order to investigate the cosmological consequences of the nonminimal gravitational coupling of the YM field.

In Eq. (2.13), because the amplitude of A_μ^a is small, we can neglect the higher order than or equal to the quadratic terms in A_μ^a and investigate the linearized equation of Eq. (2.13) in terms of A_μ^a. The linearized equation of

motion in the Coulomb gauge, $\partial^j A_j^a(t, \boldsymbol{x}) = 0$, and the case of $A_0^a(t, \boldsymbol{x}) = 0$, reads

$$\ddot{A}_i^a(t, \boldsymbol{x}) + \left(H + \frac{\dot{I}}{I}\right)\dot{A}_i^a(t, \boldsymbol{x}) - \frac{1}{a^2}\Delta^{(3)}A_i^a(t, \boldsymbol{x}) = 0, \tag{2.23}$$

where $\Delta^{(3)} = \partial^i\partial_i$ is the flat 3-dimensional Laplacian. It follows from Eq. (2.23) that the Fourier mode $A_i^a(k, t)$ satisfies the equation

$$\ddot{A}_i^a(k, t) + \left(H + \frac{\dot{I}}{I}\right)\dot{A}_i^a(k, t) + \frac{k^2}{a^2}A_i^a(k, t) = 0. \tag{2.24}$$

Replacing the independent variable t by η, we find that Eq. (2.24) becomes

$$\frac{\partial^2 A_i^a(k, \eta)}{\partial\eta^2} + \frac{1}{I(\eta)}\frac{dI(\eta)}{d\eta}\frac{\partial A_i^a(k, \eta)}{\partial\eta} + k^2 A_i^a(k, \eta) = 0. \tag{2.25}$$

By using the WKB approximation on subhorizon scales and the long-wavelength approximation on superhorizon scales, and matching these solutions at the horizon crossing, we find [24]

$$|A_i^a(k, \eta)|^2 = |C(k)|^2$$

$$= \frac{1}{2kI(\eta_k)}\left|1 - \left[\frac{1}{2}\frac{1}{kI(\eta_k)}\frac{dI(\eta_k)}{d\eta}\right.\right.$$

$$\left.\left. + i\right]k\int_{\eta_k}^{\eta_f}\frac{I(\eta_k)}{I(\tilde{\eta})}d\tilde{\eta}\right|^2, \tag{2.26}$$

where η_k and η_f are the conformal time at the horizon crossing and one at the end of inflation, respectively. Consequently, from Eq. (2.26) we obtain the amplitude of the proper YM magnetic fields on a comoving scale $L = 2\pi/k$ in the position space

$$|B_i^{a(\text{proper})}(t)|^2 = \frac{k|C(k)|^2}{\pi^2}\frac{k^4}{a^4}\left[1 + \frac{1}{2}f^{abc}u^b u^c\frac{k|C(k)|^2}{2\pi^2}\right], \tag{2.27}$$

where $u^b(= 1)$ and $u^c(= 1)$ are the quantities denoting the dependence on the indices b and c, respectively. Thus, from Eq. (2.27) we see that the YM magnetic fields evolves as $|B_i^{a(\text{proper})}(t)|^2 = |\bar{B}^a|^2/a^4$, where $|\bar{B}^a|$ is a constant.

In this case, we find that Eqs. (2.16) and (2.18) are reduced to

$$H^2 = \kappa^2 \left\{ \frac{1}{6} I(R)(\varepsilon - b\tilde{g}^2) + [-f'(R)(\dot{H} + H^2) + 6f''(R)H(\ddot{H} + 4H\dot{H})](\varepsilon - b\tilde{g}^2) - 4f'(R)H^2\varepsilon \right\} \frac{|\bar{B}^a|^2}{a^4}, \qquad (2.28)$$

and

$$2\dot{H} + 3H^2 = \kappa^2 \left\{ \frac{1}{6} I(R)(\varepsilon - 3b\tilde{g}^2) + [-f'(R)(\dot{H} + 3H^2) + 6f''(R)(\dddot{H} + 7H\ddot{H} + 4\dot{H}^2 + 12H^2\dot{H}) \right.$$

$$+ 36f'''(R)(\ddot{H} + 4H\dot{H})^2](\varepsilon - b\tilde{g}^2) + 4[f'(R)(-\dot{H} + H^2) - 12f''(R)H(\ddot{H} + 4H\dot{H})]\varepsilon + 16f'(R)H^2 b\tilde{g}^2 \right\}$$

$$\times \frac{|\bar{B}^a|^2}{a^4}, \qquad (2.29)$$

respectively. Eliminating $I(R)$ from Eqs. (2.28) and (2.29), we obtain

$$\dot{H} + \frac{\varepsilon}{\varepsilon - b\tilde{g}^2} H^2 = \kappa^2 \left(f'(R) \left\{ -(2\varepsilon + b\tilde{g}^2)\dot{H} + \left[\left(\frac{3\varepsilon - 7b\tilde{g}^2}{\varepsilon - b\tilde{g}^2} \right)\varepsilon + 8b\tilde{g}^2 \right] H^2 \right\} \right.$$

$$+ 3f''(R)[(\varepsilon - b\tilde{g}^2)\dddot{H} - 2(\varepsilon + 2b\tilde{g}^2)H\ddot{H} + 4(\varepsilon - b\tilde{g}^2)\dot{H}^2 - 24\varepsilon H^2\dot{H}]$$

$$+ 18f'''(R)(\varepsilon - b\tilde{g}^2)(\ddot{H} + 4H\dot{H})^2 \right) \frac{|\bar{B}^a|^2}{a^4}. \qquad (2.30)$$

We here note the following point. From Eq. (2.11), we see that the value of ε depends on the field strength, in other words, it varies in time. In fact, however, the change in time of ε is smaller than that of other quantities because the dependence of ε on the field strength is logarithmic, so that we can approximately regard ε as constant in Eq. (2.30). (Thus, from this point we regard ε as constant.)

Here we consider the case in which $f(R)$ is given by the following form:

$$f(R) = f_{\mathrm{HS}}(R) \equiv \frac{c_1(R/m^2)^n}{c_2(R/m^2)^n + 1}, \qquad (2.31)$$

which satisfies the conditions

$$\lim_{R \to \infty} f_{\mathrm{HS}}(R) = \frac{c_1}{c_2} = \mathrm{const}, \qquad (2.32)$$

$$\lim_{R \to 0} f_{\mathrm{HS}}(R) = 0. \qquad (2.33)$$

Here, c_1 and c_2 are dimensionless constants, n is a positive constant, and m denotes a mass scale. This form, $f_{\mathrm{HS}}(R)$, has been proposed by Hu and Sawicki [8]. The second condition (2.33) means that there could exist a flat space-time solution. Hence, because in the late-time universe the value of the scalar curvature becomes zero, the YM coupling I becomes unity, so that the standard YM theory can be naturally recovered.

In order to show that power-law inflation can be realized, we consider the case in which the scale factor is given by $a(t) = \bar{a}(t/\bar{t})^p$, where $\bar{t}$ is some fiducial time during inflation, $\bar{a}$ is the value of $a(t)$ at $t = \bar{t}$, and p is a positive constant. In this case, $H = p/t$, $\dot{H} = -p/t^2$, $\ddot{H} = 2p/t^3$, and $\dddot{H} = -6p/t^4$. Moreover, it follows from the fourth equation in (2.15) that $R = 6p(2p - 1)/t^2$. At the inflationary stage, because $R/m^2 \gg 1$, we are able to use the following approximate relations:

$$f_{\mathrm{HS}}(R) = \frac{c_1}{c_2} \left[1 - \frac{1}{c_2} \left(\frac{R}{m^2} \right)^{-n} \right]. \qquad (2.34)$$

Substituting the above relations in terms of a, H, and R, and the approximate expressions of $f'_{\mathrm{HS}}(R)$, $f''_{\mathrm{HS}}(R)$, and $f'''_{\mathrm{HS}}(R)$ derived from Eq. (2.34) into Eq. (2.30), we find

$$p = \frac{n + 1}{2}, \qquad (2.35)$$

$$\frac{\bar{a}}{\bar{t}^p} = \left\{ \frac{1}{3^{n+1}n^n(n+1)^{n+1}} \frac{(-c_1)}{c_2^2} \frac{(n+1)\varepsilon^2 + 3(n-1)b\tilde{g}^2\varepsilon + 6(b\tilde{g}^2)^2}{(n-1)\varepsilon + 2b\tilde{g}^2} |\bar{B}^a|^2 \kappa^2 m^{2n} \right\}^{1/4}. \qquad (2.36)$$

Hence, if $n \gg 1$, p becomes much larger than unity, so that power-law inflation can be realized. Consequently, it follows from this result that the YM field with a nonminimal gravitational coupling in Eq. (2.3) can be a source of inflation. This result is the same as in nonminimal Maxwell theory [23].

In this paper we consider only the case in which the values of the terms proportional to $f'(R)$, $f''(R)$, and $f'''(R)$ in the right-hand sides of Eqs. (2.28) and (2.29) are dominant to the value of the term proportional to $I(R)$. Among the terms proportional to $f'(R)$, $f''(R)$, and $f'''(R)$, the term proportional to $f'(R)$ is dominant, and its value is order

$f'(R)H^2 \approx n(c_1/c_2^2)(H^2/m^2)(R/m^2)^{-(n+1)}$, which can be derived by using Eq. (2.34). Here, it follows from $H = p/t$ and $R = 6p(2p - 1)/t^2$ that R is order $10H^2$. The condition that the term proportional to $f'(R)$ is dominant in the source term would be $I(R)/[f'(R)H^2] \sim 10c_2(R/m^2)^n/n \ll 1$. This would require extremely small c_2 because at the inflationary stage $R/m^2 \gg 1$ and $n \gg 1$. In such a case, the value of the right-hand side of Eq. (2.30), which is order $\kappa^2 f'(R)H^2|\bar{B}^a|^2/a^4$, can be order H^2. Consequently, the right-hand side of Eq. (2.30) can balance with the left-hand side of Eq. (2.30), and hence Eq. (2.30) can be satisfied without contradiction to the result, i.e., power-law inflation in which p is much larger than unity can be realized. The reason why we consider the case in which the term proportional to $I(R)$ on the right-hand side of Eqs. (2.28) and (2.29) is so small in comparison to the term proportional to $f'(R)$ that it can be neglected is as follows [23]: If the opposite case, namely, the term proportional to $I(R)$, is dominant to the term proportional to $f'(R)$, Eqs. (2.28) and (2.29) are approximately written as $H^2 \approx (1/6)\kappa^2 I(R)|\bar{B}^a|^2/a^4$ and $2\dot{H} + 3H^2 \approx (1/6)\kappa^2 I(R)|\bar{B}^a|^2/a^4$, respectively. Thus, in this case it follows from Eqs. (2.28) and (2.29) that H^2 and $2\dot{H} + 3H^2$ are the same order and their difference, $2\dot{H} + 2H^2$, must be much smaller than H^2. In fact, Eq. (2.30) implies that $\dot{H} + [\varepsilon/(\varepsilon - b\tilde{g}^2)]H^2$ balances with a much smaller quantity than $\kappa^2 I(R)|\bar{B}^a|^2/a^4$. Now, $\{\dot{H} + [\varepsilon/(\varepsilon - b\tilde{g}^2)]H^2\}/H^2 = \varepsilon/(\varepsilon - b\tilde{g}^2) - 1/p \ll 1$ and hence p must be smaller than unity because $\varepsilon > 0$ and $b > 0$. Consequently, in this case power-law inflation cannot be realized.

Finally, we note the following point. The constraint on a nonminimal gravitational coupling of matter from the observational data of the central temperature of the Sun has been proposed [21]. Furthermore, the existence of the nonminimal gravitational coupling of the electromagnetic field changes the value of the fine structure constant, i.e., the strength of the electromagnetic coupling. Hence, the deviation of the nonminimal electromagnetism from the ordinary Maxwell theory can be constrained from the observations of radio and optical quasar absorption lines [47], those of the anisotropy of the cosmic microwave background (CMB) radiation [48,49], those of the absorption of CMB radiation at 21 cm hyperfine transition of the neutral atomic hydrogen [50], and big bang nucleosynthesis [51,52] as well as solar-system experiments [53] (for a recent review, see [54]). On the other hand, because the energy scale of the YM theory is higher than the electroweak scale, the existence of the nonminimal gravitational coupling of YM field might influence on models of the grand unified theories (GUT).

III. INFLATION AND LATE-TIME COSMIC ACCELERATION IN MODIFIED GRAVITY

Next, in this section we consider a nonminimal gravitational coupling of the YM field in a modified gravitational theory proposed in Ref. [13].

We consider the following model action:

$$S_{\mathrm{MG}} = \int d^4x\sqrt{-g}[\mathcal{L}_{\mathrm{MG}} + \mathcal{L}_{\mathrm{YM}}], \qquad (3.1)$$

$$\mathcal{L}_{\mathrm{MG}} = \frac{1}{2\kappa^2}[R + F(R)], \qquad (3.2)$$

where $F(R)$ is an arbitrary function of R. Here, $\mathcal{L}_{\mathrm{YM}}$ is given by Eq. (2.3). We note that $F(R)$ is the modified part of gravity, and hence $F(R)$ is completely different from the nonminimal gravitational coupling of the YM field $f(R)$ in Eq. (2.4).

Taking variations of the action Eq. (3.1) with respect to the metric $g_{\mu\nu}$, we find that the field equation of modified gravity is given by [13]

$$[1 + F'(R)]R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}[R + F(R)] + g_{\mu\nu}\square F'(R)$$
$$- \nabla_\mu\nabla_\nu F'(R) = \kappa^2 T_{\mu\nu}^{(\mathrm{YM})}. \qquad (3.3)$$

The $(\mu, \nu) = (0, 0)$ component and the trace part of the $(\mu, \nu) = (i, j)$ component of Eq. (3.3), where i and j run from 1 to 3, are given by Eqs. (2.16) and (2.18), respectively.

Here we consider the same case as in the preceding section. In this case, eliminating $I(R)$ from Eqs. (2.16) and (2.18), we obtain

$$\dot{H} + \frac{\varepsilon}{\varepsilon - b\tilde{g}^2}H^2 + \left\{\frac{\varepsilon}{6(\varepsilon - b\tilde{g}^2)}F(R) - F'(R)\left(\frac{b\tilde{g}^2}{\varepsilon - b\tilde{g}^2}\dot{H} + \frac{\varepsilon}{\varepsilon - b\tilde{g}^2}H^2\right) + 3F''(R)[\ddot{H} + 4(\dot{H}^2 + H\ddot{H})]\right.$$
$$\left. + 18F'''(R)(\ddot{H} + 4H\dot{H})^2\right\}$$
$$= \kappa^2\left(f'(R)\left\{-(2\varepsilon + b\tilde{g}^2)\dot{H} + \left[\left(\frac{3\varepsilon - 7b\tilde{g}^2}{\varepsilon - b\tilde{g}^2}\right)\varepsilon + 8b\tilde{g}^2\right]H^2\right\} + 3f''(R)[(\varepsilon - b\tilde{g}^2)\ddot{H} - 2(\varepsilon + 2b\tilde{g}^2)H\ddot{H}\right.$$
$$\left. + 4(\varepsilon - b\tilde{g}^2)\dot{H}^2 - 24\varepsilon H^2\dot{H}] + 18f'''(R)(\varepsilon - b\tilde{g}^2)(\ddot{H} + 4H\dot{H})^2\right)\frac{|\bar{B}^a|^2}{a^4}. \qquad (3.4)$$

Here we consider the case in which $F(R)$ is given by

$$F(R) = -M^2 \frac{[(R/M^2) - (R_0/M^2)]^{2l+1} + (R_0/M^2)^{2l+1}}{c_3 + c_4\{[(R/M^2) - (R_0/M^2)]^{2l+1} + (R_0/M^2)^{2l+1}\}}, \quad (3.5)$$

which satisfies the following conditions: $\lim_{R\to\infty} F(R) = -M^2/c_4 = $ const, $\lim_{R\to 0} F(R) = 0$. Here, c_3 and c_4 are dimensionless constants, l is a positive integer, and M denotes a mass scale. We consider that in the limit $R \to \infty$, i.e., at the very early stage of the universe, $F(R)$ becomes an effective cosmological constant, $\lim_{R\to\infty} F(R) = -M^2/c_4 = -2\Lambda_i$, where $\Lambda_i(\gg H_0^2)$ is an effective cosmological constant in the very early universe, and that at the present time $F(R)$ becomes a small constant, $F(R_0) = -M^2(R_0/M^2)^{2l+1}/[c_3 + c_4(R_0/M^2)^{2l+1}] = -2R_0$, where $R_0(\approx H_0^2)$ is current curvature. Here, H_0 is the Hubble constant at the present time: $H_0 = 100h \,\mathrm{km\,s^{-1}\,Mpc^{-1}} = 2.1h \times 10^{-42} \,\mathrm{GeV} \approx 1.5 \times 10^{-33} \,\mathrm{eV}$ [55], where we have used $h = 0.70$ [56].

Furthermore, we consider the case in which $f(R)$ is given by the following form:

$$f(R) = f_{\mathrm{NO}}(R)$$
$$\equiv \frac{[(R/M^2) - (R_0/M^2)]^{2q+1} + (R_0/M^2)^{2q+1}}{c_5 + c_6\{[(R/M^2) - (R_0/M^2)]^{2q+1} + (R_0/M^2)^{2q+1}\}}, \quad (3.6)$$

which satisfies the following conditions: $\lim_{R\to\infty} f_{\mathrm{NO}}(R) = 1/c_6 = $ const, $\lim_{R\to 0} f_{\mathrm{NO}}(R) = 0$. Here, c_5 and c_6 are dimensionless constants, and q is a positive integer. The form of $F(R)$ in Eq. (3.5) and $f_{\mathrm{NO}}(R)$ in Eq. (3.6) is taken from Ref. [13]. This form corresponds to the extension of the form of $f_{\mathrm{HS}}(R)$ in Eq. (2.31). It has been shown in Ref. [13] that modified gravitational theories described by the action (3.2) with $F(R)$ in Eq. (3.5) successfully pass the solar-system tests as well as cosmological bounds and they are free of instabilities.

Making the same considerations as in Ref. [23], we find that at the very early stage of the universe, it follows from Eq. (3.4) that $a(t) \propto \exp(\sqrt{\Lambda_i/3}t)$, so that exponential inflation can be realized, and that at the present time, it follows from Eq. (3.4) that $a(t) \propto \exp(\sqrt{R_0/3}t)$, so that the late-time acceleration of the universe can be realized. These results are also the same as in nonminimal Maxwell-$F(R)$ gravity [23].

Finally, we note the following point about the logarithmic contribution to modified gravity, namely, the case in which the Lagrangian of modified gravity in Eq. (3.2) is given by $\mathcal{L}_{\mathrm{MG}} = 1/(2\kappa^2)[R + F(R) + \ln(R/M^2)]$. Following the considerations in the previous subsections, because the logarithmic term is a subleading contribution, also in this case both inflation and the late-time acceleration of the universe can be realized. The qualitative difference from the case of the previous subsections is only that in the limit $R \to \infty$ the gravitational modification term, $F(R) + \ln(R/M^2)$, does not become constant. In fact, how-

ever, if it is considered that some cutoff scale of R in the very early universe should exist, the logarithmic contribution does not diverge in this limit, and hence the cosmology of this case is the same as that of the previous sections.

IV. NONMINIMAL VECTOR MODEL

In this section, we consider the cosmology in the non-Abelian nonminimal vector-$F(R)$ gravity.

We consider the following model action:

$$\bar{S}_{\mathrm{MG}} = \int d^4x \sqrt{-g}[\mathcal{L}_{\mathrm{MG}} + \mathcal{L}_{\mathrm{V}}], \quad (4.1)$$

$$\mathcal{L}_{\mathrm{V}} = I(R)\{-\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - V[A^{a2}]\}, \quad (4.2)$$

where $\mathcal{L}_{\mathrm{MG}}$ is given by Eq. (3.2), $F^a_{\mu\nu}$ is given by Eq. (2.6), and $A^{a2} = g^{\mu\nu}A^a_\mu A^a_\nu$. (As the generalization of the above nonminimal vector model, one can consider a model in which the derivative in $F^a_{\mu\nu}$ is the gauge covariant derivative given by $D_\mu = \nabla_\mu - i\tilde{g}A_\mu$, where $A_\mu = A^a_\mu \tau^a$. Here, τ^a are matrices and their commutation relations are conventionally written as the standard form $[\tau^a, \tau^b] = if^{abc}\tau^c$ [44]. In the present paper, however, as a simple nonminimal vector model we consider the theory described by the action in Eq. (4.2).)

We should note that the last term $V[A^{a2}]$ in the action (4.2) is not gauge invariant but can be rewritten in a gauge invariant way. For example, if the gauge group is a unitary group, we may introduce a σ-model-like field U, which satisfies $U^\dagger U = 1$. Then the last term could be rewritten in the gauge invariant form:

$$V[A^{a2}] \to V[\bar{c}\,\mathrm{tr}(U^\dagger A^a_\mu U)(U^\dagger A^{a\mu}U)]. \quad (4.3)$$

Here $\bar{c}$ is a constant for the normalization. If we choose the unitary gauge $U = 1$, the term in (4.3) reduces to the original one: $V[A^{a2}]$. This may tell that the action (4.2) described the theory where the gauge group is spontaneously broken.

The field equations can be derived by taking variations of the action in Eq. (4.1) with respect to the metric $g_{\mu\nu}$ and the vector field A^a_μ as follows:

$$[1 + F'(R)]R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}[R + F(R)] + g_{\mu\nu}\Box F'(R)$$
$$- \nabla_\mu\nabla_\nu F'(R) = \kappa^2 T^{(\mathrm{V})}_{\mu\nu}, \quad (4.4)$$

with

$$T_{\mu\nu}^{(V)} = I(R)\left\{g^{\alpha\beta}F_{\mu\beta}^a F_{\nu\alpha}^a + 2A_\mu^a A_\nu^a \frac{dV[A^{a2}]}{dA^{a2}} - \frac{1}{4}g_{\mu\nu}\bar{\mathcal{F}}\right\}$$
$$+ \frac{1}{2}\{f'(R)\bar{\mathcal{F}}R_{\mu\nu} + g_{\mu\nu}\Box[f'(R)\bar{\mathcal{F}}]$$
$$- \nabla_\mu\nabla_\nu[f'(R)\bar{\mathcal{F}}]\}, \tag{4.5}$$

$$\bar{\mathcal{F}} = F_{\mu\nu}^a F^{a\mu\nu} + 4V[A^{a2}], \tag{4.6}$$

and

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}I(R)F^{a\mu\nu}]$$
$$- I(R)\left\{f^{abc}A_\mu^b F^{c\mu\nu} + 2\frac{dV[A^{a2}]}{dA^{a2}}A^{a\nu}\right\} = 0, \tag{4.7}$$

where $T_{\mu\nu}^{(V)}$ is the contribution to the energy-momentum tensor from A_μ^a.

Here, as an example, we consider the case in which $V[A^{a2}]$ is given by a class of the following power-law potential:

$$V[A^{a2}] = \bar{V}\left(\frac{A^{a2}}{\bar{m}^2}\right)^{\bar{n}}, \tag{4.8}$$

where $\bar{V}$ is a constant, $\bar{m}$ denotes a mass scale, and $\bar{n}(>1)$ is a positive integer.

Similarly to Sec. II B, because the amplitude of A_μ^a is small, we neglect the higher order than or equal to the quadratic terms in A_μ^a and consider the linearized equation of Eq. (4.7) in terms of A_μ^a. For the power-law potential given by Eq. (4.8), the linearized equation of motion under the ansatz $\partial^j A_j^a(t, \boldsymbol{x}) = 0$ and $A_0^a(t, \boldsymbol{x}) = 0$ is the same as Eq. (2.23).[3]

The $(\mu, \nu) = (0, 0)$ component and the trace part of the $(\mu, \nu) = (i, j)$ component of Eq. (4.4), where i and j run from 1 to 3, read

$$H^2 + \frac{1}{6}F(R) - F'(R)(\dot{H} + H^2)$$
$$= \frac{\kappa^2}{6}(I(R)\{Y + 2V[A^{a2}]\} + 3[-f'(R)(\dot{H} + H^2)$$
$$+ 6f''(R)H(\ddot{H} + 4H\dot{H})]\bar{\mathcal{F}} + 3f'(R)H\dot{\bar{\mathcal{F}}}), \tag{4.9}$$

and

$$2\dot{H} + 3H^2 + \frac{1}{2}F(R) - F'(R)(\dot{H} + 3H^2) + 6F''(R)[\ddot{H} + 4(\dot{H}^2 + H\ddot{H})] + 36F'''(R)(\ddot{H} + 4H\dot{H})^2$$
$$= \frac{\kappa^2}{2}\left(\frac{1}{3}I(R)\left\{-X + 6V[A^{a2}] - 4\frac{1}{a^2}A_i^a A_i^a \frac{dV[A^{a2}]}{dA^{a2}}\right\} + [-f'(R)(\dot{H} + 3H^2) + 6f''(R)(\dddot{H} + 7H\ddot{H} + 4\dot{H}^2 + 12H^2\dot{H})$$
$$+ 36f'''(R)(\ddot{H} + 4H\dot{H})^2]\bar{\mathcal{F}} + 3[f'(R)H + 4f''(R)(\ddot{H} + 4H\dot{H})]\dot{\bar{\mathcal{F}}} + f'(R)\ddot{\bar{\mathcal{F}}}\right), \tag{4.10}$$

respectively. In deriving Eqs. (4.9) and (4.10), we have used equations in (2.15).

Here we consider the same case as in the previous sections. Moreover, we here consider the case in which $A_0^a = 0$. In this case, we have $(1/a^2)A_i^a A_i^a dV[A^{a2}]/(dA^{a2}) = \bar{n}V[A^{a2}]$. Consequently, using this relation, we find that Eqs. (4.9) and (4.10) are reduced to

$$H^2 + \frac{1}{6}F(R) - F'(R)(\dot{H} + H^2) = \kappa^2\left(\left[\frac{1}{6}I(R) - f'(R)(\dot{H} + 5H^2) + 6f''(R)H(\ddot{H} + 4H\dot{H})\right]\frac{|\bar{B}^a|^2}{a^4}\right.$$
$$\left. + \left\{\frac{1}{3}I(R) - 2f'(R)[\dot{H} + (1 + 2\bar{n})H^2] + 12f''(R)H(\ddot{H} + 4H\dot{H})\right\}V[A^{a2}]\right), \tag{4.11}$$

and

$$2\dot{H} + 3H^2 + \frac{1}{2}F(R) - F'(R)(\dot{H} + 3H^2) + 6F''(R)[\ddot{H} + 4(\dot{H}^2 + H\ddot{H})] + 36F'''(R)(\ddot{H} + 4H\dot{H})^2$$
$$= \kappa^2\left(\left[\frac{1}{6}I(R) + f'(R)(-5\dot{H} + H^2) + 6f''(R)(\dddot{H} - H\ddot{H} + 4\dot{H}^2 - 20H^2\dot{H}) + 36f'''(R)(\ddot{H} + 4H\dot{H})^2\right]\frac{|\bar{B}^a|^2}{a^4}\right.$$
$$+ \left\{\frac{1}{3}I(R) - 2f'(R)[(1 + 2\bar{n})\dot{H} + (3 + 6\bar{n} - 4\bar{n}^2)H^2] + 12f''(R)[\dddot{H} + (7 - 4\bar{n})H\ddot{H} + 4\dot{H}^2 + 4(3 - 4\bar{n})H^2\dot{H}]\right.$$
$$\left.\left. + 72f'''(R)(\ddot{H} + 4H\dot{H})^2\right\}V[A^{a2}]\right), \tag{4.12}$$

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[3]This is similar to the Coulomb gauge but since the action (4.2) is not gauge invariant, or gauge symmetry is completely fixed by the unitary gauge as in after (4.3), this condition is only a working hypothesis.

respectively. Eliminating $I(R)$ from Eqs. (4.11) and (4.12), we obtain

$$\dot{H} + H^2 + \left\{\frac{1}{6}F(R) - F'(R)H^2 + 3F''(R)[\ddot{H} + 4(\dot{H}^2 + H\ddot{H})] + 18F'''(R)(\ddot{H} + 4H\dot{H})^2\right\}$$

$$= \kappa^2([f'(R)(-2\dot{H} + 3H^2) + 3f''(R)(\dddot{H} - 2H\ddot{H} + 4\dot{H}^2 - 24H^2\dot{H}) + 18f'''(R)(\ddot{H} + 4H\dot{H})^2]\frac{|\bar{B}^a|^2}{a^4}$$

$$+ 2\{-f'(R)[\bar{n}\dot{H} + (1 + 2\bar{n} - 2\bar{n}^2)H^2] + 3f''(R)[\dddot{H} + 2(3 - 2\bar{n})H\ddot{H} + 4\dot{H}^2 + 8(1 - 2\bar{n})H^2\dot{H}]$$

$$+ 18f'''(R)(\ddot{H} + 4H\dot{H})^2\}V[A^{a2}]). \tag{4.13}$$

In the case that $|B_i^{a(\text{proper})}(t)|^2 = |\bar{B}^a|^2/a^4$, $V[A^{a2}] \propto a^{-2\bar{n}}$. Hence, if $\bar{n} = 2$, the time evolution of $V[A^{a2}]$ is the same as that of $|B_i^{a(\text{proper})}(t)|^2$. On the other hand, if $\bar{n} \geq 2$, $V[A^{a2}]$ decreases much more rapidly than $|B_i^{a(\text{proper})}(t)|^2$ during inflation. Thus, in the latter case we can neglect the terms proportional to $V[A^{a2}]$ on the right-hand side of Eq. (4.13). Consequently, it follows from Eq. (4.13) that when we consider the case in which similarly to the preceding section, $F(R)$ and $f(R)$ are given by Eqs. (3.5) and (3.6), respectively, we can make the same consideration as the preceding section, and hence power-law inflation and the late-time acceleration of the universe can be realized.

Furthermore, as another case, we consider the case in which $F(R)$ is given by Eq. (3.5) and $f(R)$ is given by the following form:

$$f(R) = \bar{f}(R) \equiv \frac{c_7(R/\bar{M}^2)^{\bar{q}} - 1}{c_8(R/\bar{M}^2)^{\bar{q}} + 1}, \tag{4.14}$$

which satisfies the following conditions: $\lim_{R\to\infty}\bar{f}(R) = c_7/c_8 = \text{const}$, $\lim_{R\to 0}\bar{f}(R) = -1$. Here, c_7 and c_8 are dimensionless constants, $\bar{q}$ is a positive constant, and $\bar{M}$ denotes a mass scale. In this case, the value of $I(R) = 1 + f(R)$ becomes close to zero when that of R is very small, namely, at the present time. Making the same consideration as the preceding section, we can also find in this case that power-law inflation and the late-time acceleration of the universe can be realized.

V. DUALITY

In this section, we consider the duality of the nonminimal electromagnetic theory and that of the nonminimal YM theory.

A. Duality of the nonminimal electromagnetic theory

We consider the duality of the action of $\tilde{f}(R)$-coupled electromagnetic theory:

$$S_{\tilde{f}A} = \frac{1}{4}\int d^4x\sqrt{-g}\tilde{f}(R)F_{A\mu\nu}F_A^{\mu\nu},$$
$$F_{A\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{5.1}$$

where $\tilde{f}(R)$ is an arbitrary function of R, and A_ν is the $U(1)$ gauge field. Before going to the duality of the action in

Eq. (5.1), we consider the duality without gravity:

$$S_A = \frac{1}{4}\int d^4x F_{A\mu\nu}F_A^{\mu\nu}, \qquad F_{A\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{5.2}$$

By introducing a new field $\bar{B}_\mu$, the action can be rewritten as

$$S_{\bar{F}\bar{B}} = \frac{1}{4}\int d^4x\left(\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\partial_\mu\bar{B}_\nu)\bar{F}_{\rho\sigma}\right). \tag{5.3}$$

Here, $\bar{F}_{\mu\nu}$ is an independent field (not given in terms of A_μ or $\bar{B}_\mu$ as in $F_{A\mu\nu}$). The variation of $\bar{B}_\mu$ gives

$$\epsilon^{\mu\nu\rho\sigma}\partial_\nu\bar{F}_{\rho\sigma} = 0, \tag{5.4}$$

which tells that $\bar{F}_{\mu\nu}$ can be given in terms of a vector field A_μ as

$$\bar{F}_{\mu\nu} = F_{A\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{5.5}$$

Then the action $S_{\bar{F}\bar{B}}$ in Eq. (5.3) reduces to S_A in Eq. (5.2). On the other hand, by the variation of $\bar{F}_{\mu\nu}$, we obtain

$$\bar{F}^{\mu\nu} = -\frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\partial_\rho\bar{B}_\sigma. \tag{5.6}$$

By substituting Eq. (5.6) into the action in Eq. (5.2), we obtain

$$S_{\bar{B}} = \frac{1}{4}\int d^4x F_{\bar{B}\mu\nu}F_{\bar{B}}^{\mu\nu}, \qquad F_{\bar{B}\mu\nu} \equiv \partial_\mu\bar{B}_\nu - \partial_\nu\bar{B}_\mu. \tag{5.7}$$

Equations (5.5) and (5.6) give

$$F_A^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\bar{B}\rho\sigma}, \tag{5.8}$$

which tells that $F_{B\mu\nu}$ are dual to $F_{A\mu\nu}$, that is, the magnetic field exchanges with the electric field.

We now consider the action in Eq. (5.1), which can be rewritten as

$$S_{\tilde{f}\bar{F}\bar{B}} = \frac{1}{4}\int d^4x\left\{\sqrt{-g}\tilde{f}(R)\bar{F}_{\mu\nu}\bar{F}^{\mu\nu}\right. $$
$$\left. + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\partial_\mu\bar{B}_\nu)\bar{F}_{\rho\sigma}\right\}. \tag{5.9}$$

Now $\bar{F}_{\mu\nu}$ is an independent field again. From the variation of $\bar{B}_\mu$, we obtain Eq. (5.4), which can be solved as

Eq. (5.5), and we find the action $S_{\tilde{f}\bar{F}\bar{B}}$ in Eq. (5.9) is equivalent to Eq. (5.1). On the other hand, by the variation of $\bar{F}_{\mu\nu}$, instead of Eq. (5.6), we obtain

$$\bar{F}^{\mu\nu} = -\frac{1}{4}\frac{\epsilon^{\mu\nu\rho\sigma}\partial_\rho\bar{B}_\sigma}{\tilde{f}(R)\sqrt{-g}}. \quad (5.10)$$

Then substituting Eq. (5.10) into Eq. (5.9) and using the identity

$$\epsilon^{\alpha\beta\rho\sigma}\epsilon^{\mu\nu\gamma\delta}g_{\mu\alpha}g_{\nu\beta} = 2g(g^{\rho\gamma}g^{\sigma\delta} - g^{\rho\delta}g^{\sigma\gamma}), \quad (5.11)$$

we obtain an action dual to Eq. (5.1):

$$S_{fB} = \frac{1}{4}\int d^4x\sqrt{-g}\frac{1}{\tilde{f}(R)}F_{\bar{B}\mu\nu}F_{\bar{B}}^{\mu\nu},$$
$$F_{\bar{B}\mu\nu} \equiv \partial_\mu\bar{B}_\nu - \partial_\nu\bar{B}_\mu. \quad (5.12)$$

B. Duality of the nonminimal Yang-Mills theory

As in the case of the electromagnetic theory, we may consider the duality of the action of $\tilde{f}(R)$-coupled Yang-Mills theory:

$$S_{\tilde{f}A} = \frac{1}{4}\int d^4x\sqrt{-g}\tilde{f}(R)F_{A\mu\nu}F_A^{\mu\nu},$$
$$F_{A\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + f^{abc}A_\mu^b A_\nu^c. \quad (5.13)$$

The action can be rewritten in the following form:

$$S_{\tilde{f}\bar{F}\bar{B}} = \frac{1}{4}\int d^4x\left\{\sqrt{-g}\tilde{f}(R)\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^A\bar{F}_{\rho\sigma}\right\}. \quad (5.14)$$

Now $\bar{F}_{\mu\nu}$ is an independent field again. From the variation of A_μ^a in $F_{\mu\nu}^a$, we obtain

$$\epsilon^{\mu\nu\rho\sigma}D_\nu\bar{F}_{\rho\sigma} = 0. \quad (5.15)$$

Here D_μ is a covariant derivative. The solution of (5.15) is given by

$$\bar{F}_{\mu\nu} = F_{A\mu\nu}. \quad (5.16)$$

By substituting (5.16) into (5.14), we obtain (5.13). On the other hand, by the variation of $\bar{F}_{\mu\nu}$, we obtain

$$\bar{F}^{\mu\nu} = -\frac{1}{8}\frac{\epsilon^{\mu\nu\rho\sigma}F_{A\rho\sigma}}{\tilde{f}(R)\sqrt{-g}}. \quad (5.17)$$

Then substituting Eq. (5.17) into Eq. (5.14) and using the identity (5.11), we obtain a dual action:

$$S_{fB} = \frac{1}{4}\int d^4x\sqrt{-g}\frac{1}{\tilde{f}(R)}F_{A\mu\nu}F_A^{\mu\nu}. \quad (5.18)$$

Note that dual form of the action may be useful in the cosmological considerations.

VI. RECONSTRUCTION OF THE YM THEORY

In this section, we indicate how to reconstruct the YM theory from the known universe evolution (for a review, see [57]).

We now consider the following action:

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} + \tilde{\mathcal{F}}(F_{\mu\nu}^a F^{a\mu\nu})\right). \quad (6.1)$$

By introducing an auxiliary scalar field ϕ, we may rewrite the action (6.1) in the following form:

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} + \frac{1}{4}P(\phi)F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{4}Q(\phi)\right). \quad (6.2)$$

By the variation of ϕ, we obtain

$$0 = P'(\phi)F_{\mu\nu}^a F^{a\mu\nu} + Q'(\phi), \quad (6.3)$$

which could be solved with respect to ϕ as $\phi = \phi(F_{\mu\nu}^a F^{a\mu\nu})$. Here, the prime denotes differentiation with respect to ϕ. Then by substituting the expression into (6.2), we obtain the action (6.1) with

$$\tilde{\mathcal{F}}(F_{\mu\nu}^a F^{a\mu\nu}) = \frac{1}{4}\{P(\phi(F_{\mu\nu}^a F^{a\mu\nu}))F_{\mu\nu}^a F^{a\mu\nu} + Q(\phi(F_{\mu\nu}^a F^{a\mu\nu}))\}. \quad (6.4)$$

By the variation of the action (6.2) with respect to the metric tensor $g_{\mu\nu}$, we obtain the Einstein equation:

$$\frac{1}{2\kappa^2}\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) = -\frac{1}{2}P(\phi)F_{\mu\rho}^a F^a_{\nu}{}^\rho + \frac{1}{8}g_{\mu\nu}(P(\phi)F_{\rho\sigma}^a F^{a\rho\sigma} + Q(\phi)). \quad (6.5)$$

On the other hand, by the variation with respect to A_μ^a, we obtain

$$0 = \partial_\nu(\sqrt{-g}P(\phi)F^{a\nu\mu}) - \sqrt{-g}P(\phi)f^{abc}A_\nu^b F^{c\nu\mu}. \quad (6.6)$$

For simplicity, we only consider the case that the gauge algebra is $SU(2)$, where $f^{abc} = \epsilon^{abc}$, and we assume the gauge fields are given in the following form:

$$A_\mu^a = \begin{cases} \bar{\alpha}e^{\lambda(t)}\delta^a{}_i & (\mu = i = 1, 2, 3) \\ 0 & (\mu = 0). \end{cases} \quad (6.7)$$

Here $\bar{\alpha}$ is a constant with mass dimension and λ is a proper function of t. In general, if the vector field is condensed, the rotational invariance of the universe could be broken. In case of (6.7), the direction of the vector field is gauge variant. Then all the gauge invariant quantities given by (6.7) do not break the rotational invariance.

By the assumption, (6.3) has the following form:

$$0 = 6(-\bar{\alpha}^2\dot{\lambda}^2e^{2\lambda}a^{-2} + \bar{\alpha}^4e^{4\lambda}a^{-4})P'(\phi) + Q'(\phi), \quad (6.8)$$

and the (t, t)-component of Eq. (6.5) is given by

$$0 = \frac{3}{\kappa^2} H^2 - \frac{3}{2}(\bar{\alpha}^2 \dot{\lambda}^2 e^{2\lambda} a^{-2} + \bar{\alpha}^4 e^{4\lambda} a^{-4}) - \frac{1}{4} Q(\phi). \tag{6.9}$$

The $\mu = 0$ component of (6.6) becomes identity and the $\mu = i$ component gives

$$0 = \partial_t(aP(\phi)\dot{\lambda}e^{\lambda}) - 2\bar{\alpha}^2 a^{-1} P(\phi)e^{3\lambda}. \tag{6.10}$$

Since we can always define the scalar field ϕ properly, we may identify the scalar field with the time coordinate $\phi = t$. Then by differentiating Eq. (6.9) with respect to t and eliminating $\dot{Q} = Q'(\phi)$, we obtain

$$0 = \frac{2}{\kappa^2} H\dot{H} + \bar{\alpha}^2 \dot{\lambda}^2 e^{2\lambda} a^{-2} \dot{P}$$
$$- P\{\bar{\alpha}^2(\dot{\lambda}\,\ddot{\lambda} + \dot{\lambda}^3 - \lambda^2 H)e^{2\lambda} a^{-2}$$
$$+ 2\bar{\alpha}^4(\dot{\lambda} - H)e^{4\lambda} a^{-4}\}. \tag{6.11}$$

Furthermore by eliminating $\dot{P}$ by using (6.10), we find

$$P = \frac{2H\dot{H}}{\kappa^2\{2\bar{\alpha}^2 a^{-2} e^{2\lambda}(\dot{\lambda}^2 + \dot{\lambda}\,\ddot{\lambda}) - \bar{\alpha}^4 e^{4\lambda} a^{-4} H\}}. \tag{6.12}$$

Then by using (6.12), we can eliminate P (and $\dot{P}$) in (6.10) and obtain

$$0 = 2(\dot{\lambda}\,\ddot{\lambda} + \ddot{\lambda}^2 + 3\dot{\lambda}^2\ddot{\lambda}) - \bar{\alpha}^2 e^{2\lambda} a^{-2}\dot{H} + 4\{\dot{\lambda}^3 + \dot{\lambda}\,\ddot{\lambda} - \bar{\alpha}^2$$
$$\times e^{2\lambda} a^{-2}H\}(\dot{\lambda} - H) + \{2(\dot{\lambda}^3 + \dot{\lambda}\,\ddot{\lambda})$$
$$- \bar{\alpha}^2 e^{2\lambda} a^{-2}H\}\left\{\frac{\dot{H}}{H} + \frac{\ddot{H}}{\dot{H}} + H + \frac{\ddot{\lambda}}{\dot{\lambda}} + \dot{\lambda} - \frac{2\bar{\alpha}^2 a^{-2} e^{2\lambda}}{\dot{\lambda}}\right\}. \tag{6.13}$$

If we give a proper $a = a(t)$ and therefore $H = H(t)$, Eq. (6.13) can be regarded as a third order differential equation with respect to λ. If we find the solution of λ with three constants of the integration, we find the explicit form of $P(\phi) = P(t)$ by using (6.12) and further obtain $Q(\phi)$ by using (6.9). Then we find the explicit form of three parameter families of the action (6.2). This tells that almost arbitrary time development of the university could be realized by the action (6.2) or (6.1).

As an example, we may consider the case of the power-law expansion:

$$a = \left(\frac{t}{t_1}\right)^{h_1} \qquad \left(H = \frac{h_1}{t}\right). \tag{6.14}$$

Here t_1 and h_1 are constants. By assuming

$$\lambda = (h_1 - 1)\ln\left(\frac{t}{t_1}\right) + \lambda_1 \tag{6.15}$$

(λ_1 is a constant), Eq. (6.3) reduces to the algebraic equation:

$$0 = \frac{2h_1}{h_1 - 1}\bar{X}^2 + (-4h_1^2 + 13h_1 + 2)\bar{X}$$
$$+ (h_1 - 1)^2(h_1 - 2)(4H_1 - 20). \tag{6.16}$$

Here $\bar{X} = \bar{\alpha}^2 t_1^2 e^{2\lambda}$. If (6.16) has a real positive solution with respect to $\bar{X}$, we obtain λ_1 and therefore the exact form of λ. Then we can reconstruct a model to give the power expansion (6.14). Similarly, any other universe evolution history may be reproduced by specific form of the action under consideration.

VII. CONCLUSION

In the present paper, we have considered inflationary cosmology and the late-time accelerated expansion of the universe in the YM theory, in which the YM field couples to a function of the scalar curvature, in order to investigate the cosmological consequences of the nonminimal gravitational coupling of the YM field. As a result, we have shown that power-law inflation can be realized due to the nonminimal gravitational coupling of the YM field. Moreover, we have demonstrated that both inflation and the late-time accelerated expansion of the universe can be realized in a modified YM-$F(R)$ gravity proposed in Ref. [13] which is consistent with solar-system tests. Furthermore, we have shown that this result can be realized also in a nonminimal vector-$F(R)$ gravity. In addition, we have considered the duality of the nonminimal electromagnetic theory and that of the nonminimal YM theory. Furthermore, we also discussed the reconstruction of the YM theory from the known universe history expansion. As an example, it has been shown that a model to give the power expansion of the scale factor can be reconstructed.

Finally, we remark the following point. It is interesting that our models may be extended by another gauge-noninvariant nonminimal coupling with the curvature like the ones done in Ref. [58] (for models of vector curvaton, see [59]). Such a nonminimal vector curvaton may give an extra contribution to curvature perturbations if compared with the present models. Another important point is related to the exit from the inflation. In the models under consideration, it may be realized via the gravitational scenario, as the instability of de Sitter universe, due to extra gravitational terms. This scenario will be investigated in detail elsewhere. It may be also relevant for the study of the future universe: if our universe will stay as a Λ-CDM one forever or it will evolve to another singular/nonsingular state.

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[1] D. N. Spergel *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **148**, 175 (2003); H. V. Peiris *et al.* (WMAP Collaboration), *ibid.* **148**, 213 (2003); D. N. Spergel *et al.* (WMAP Collaboration), *ibid.* **170**, 377 (2007).

[2] S. Perlmutter *et al.* (Supernova Cosmology Project Collaboration), Astrophys. J. **517**, 565 (1999); A. G. Riess *et al.* (Supernova Search Team Collaboration), Astron. J. **116**, 1009 (1998); P. Astier *et al.* (SNLS Collaboration), Astron. Astrophys. **447**, 31 (2006); A. G. Riess *et al.*, Astrophys. J. **659**, 98 (2007).

[3] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003); V. Sahni, AIP Conf. Proc. **782**, 166 (2005).

[4] T. Padmanabhan, Phys. Rep. **380**, 235 (2003).

[5] E. J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006).

[6] R. Durrer and R. Maartens, Gen. Relativ. Gravit. **40**, 301 (2008).

[7] S. Nojiri and S. D. Odintsov, Int. J. Geom. Methods Mod. Phys. **4**, 115 (2007); arXiv:0801.4843.

[8] W. Hu and I. Sawicki, Phys. Rev. D **76**, 064004 (2007).

[9] S. A. Appleby and R. A. Battye, Phys. Lett. B **654**, 7 (2007); S. Nojiri and S. D. Odintsov, *ibid.* **652**, 343 (2007); L. Pogosian and A. Silvestri, Phys. Rev. D **77**, 023503 (2008); S. Capozziello and S. Tsujikawa, Phys. Rev. D **77**, 107501 (2008); S. A. Appleby and R. A. Battye, arXiv:0803.1081; S. Tsujikawa, K. Uddin, and R. Tavakol, Phys. Rev. D **77**, 043007 (2008); S. Tsujikawa, K. Uddin, S. Mizuno, R. Tavakol, and J. Yokoyama, Phys. Rev. D **77**, 103009 (2008).

[10] S. Capozziello and M. Francaviglia, Gen. Relativ. Gravit. **40**, 357 (2008).

[11] S. Nojiri and S. D. Odintsov, Phys. Rev. D **77**, 026007 (2008).

[12] G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov, L. Sebastiani, and S. Zerbini, Phys. Rev. D **77**, 046009 (2008).

[13] S. Nojiri and S. D. Odintsov, Phys. Lett. B **657**, 238 (2007).

[14] S. Nojiri and S. D. Odintsov, Phys. Rev. D **68**, 123512 (2003).

[15] S. Nojiri and S. D. Odintsov, Phys. Lett. B **599**, 137 (2004); Proc. Sci., WC2004 (2004) 024 [arXiv:hep-th/0412030].

[16] G. Allemandi, A. Borowiec, M. Francaviglia, and S. D. Odintsov, Phys. Rev. D **72**, 063505 (2005).

[17] N. Deruelle, M. Sasaki, and Y. Sendouda, arXiv:0803.2742 [Phys. Rev. D (to be published)].

[18] S. Mukohyama and L. Randall, Phys. Rev. Lett. **92**, 211302 (2004); T. Inagaki, S. Nojiri, and S. D. Odintsov, J. Cosmol. Astropart. Phys. 06 (2005) 010; A. D. Dolgov and M. Kawasaki, arXiv:astro-ph/0307442.

[19] O. Bertolami, C. G. Boehmer, T. Harko, and F. S. N. Lobo, Phys. Rev. D **75**, 104016 (2007); T. Koivisto, Classical Quantum Gravity **23**, 4289 (2006).

[20] V. Faraoni, Phys. Rev. D **76**, 127501 (2007).

[21] O. Bertolami and J. Páramos, Phys. Rev. D **77**, 084018 (2008).

[22] S. Nojiri, S. D. Odintsov, and P. V. Tretyakov, arXiv:0710.5232.

[23] K. Bamba and S. D. Odintsov, J. Cosmol. Astropart. Phys. 04 (2008) 024.

[24] K. Bamba and M. Sasaki, J. Cosmol. Astropart. Phys. 02 (2007) 030; K. Bamba, *ibid.* 10 (2007) 015.

[25] I. T. Drummond and S. J. Hathrell, Phys. Rev. D **22**, 343 (1980).

[26] A. Raya, J. E. M. Aguilar, and M. Bellini, Phys. Lett. B **638**, 314 (2006); F. A. Membiela and M. Bellini, arXiv:0712.3032.

[27] L. Campanelli, P. Cea, G. L. Foglin and L. Tedesco, arXiv:0802.2630.

[28] M. S. Turner and L. M. Widrow, Phys. Rev. D **37**, 2743 (1988).

[29] K.-T. Kim, P. P. Kronberg, P. E. Dewdney, and T. L. Landecker, Astrophys. J. **355**, 29 (1990); K.-T. Kim, P. C. Tribble, and P. P. Kronberg, *ibid.* **379**, 80 (1991); T. E. Clarke, P. P. Kronberg, and H. Boehringer, *ibid.* **547**, L111 (2001).

[30] Y. Sofue, M. Fujimoto, and R. Wielebinski, Annu. Rev. Astron. Astrophys. **24**, 459 (1986); P. P. Kronberg, Rep. Prog. Phys. **57**, 325 (1994); D. Grasso and H. R. Rubinstein, Phys. Rep. **348**, 163 (2001); A. D. Dolgov, arXiv:hep-ph/0110293; C. L. Carilli and G. B. Taylor, Annu. Rev. Astron. Astrophys. **40**, 319 (2002); L. M. Widrow, Rev. Mod. Phys. **74**, 775 (2002); M. Giovannini, Int. J. Mod. Phys. D **13**, 391 (2004); V. B. Semikoz and D. D. Sokoloff, *ibid.* **14**, 1839 (2005); M. Giovannini, Lect. Notes Phys. **737**, 863 (2008).

[31] L. H. Ford, Phys. Rev. D **40**, 967 (1989).

[32] A. Golovnev, V. Mukhanov, and V. Vanchurin, arXiv:0802.2068.

[33] W. Zhao and Y. Zhang, Classical Quantum Gravity **23**, 3405 (2006); Phys. Lett. B **640**, 69 (2006); Y. Zhang, T. Y. Xia, and W. Zhao, Classical Quantum Gravity **24**, 3309 (2007).

[34] T. Y. Xia and Y. Zhang, Phys. Lett. B **656**, 19 (2007).

[35] E. Elizalde, J. E. Lidsey, S. Nojiri, and S. D. Odintsov, Phys. Lett. B **574**, 1 (2003).

[36] C. Armendariz-Picon, J. Cosmol. Astropart. Phys. 07 (2004) 007.

[37] H. Wei and R. G. Cai, Phys. Rev. D **73**, 083002 (2006).

[38] H. Wei and R. G. Cai, J. Cosmol. Astropart. Phys. 09 (2007) 015.

[39] J. B. Jiménez and A. L. Maroto, arXiv:0801.1486.

[40] T. Koivisto and D. F. Mota, arXiv:0707.0279; arXiv:0801.3676.

[41] V. I. Demchik and V. V. Skalozub, Eur. Phys. J. C **25**, 291 (2002); V. V. Skalozub and A. V. Strelchenko, *ibid.* **33**, 105 (2004).

[42] S. G. Matinyan and G. K. Savvidy, Nucl. Phys. **B134**, 539 (1978); H. Pagels and E. Tomboulis, *ibid.* **B143**, 485 (1978).

[43] S. L. Adler and T. Piran, Rev. Mod. Phys. **56**, 1 (1984).

[44] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Westview Press, Boulder, Colorado, 1995).

[45] A. B. Balakin and J. P. S. Lemos, Classical Quantum Gravity **22**, 1867 (2005).

[46] A. B. Balakin, H. Dehnen, and A. E. Zayats, Phys. Rev. D **76**, 124011 (2007).

[47] P. Tzanavaris, M. T. Murphy, J. K. Webb, V. V. Flambaum, and S. J. Curran, Mon. Not. R. Astron. Soc. **374**, 634 (2007).

[48] R. A. Battye, R. Crittenden, and J. Weller, Phys. Rev. D **63**, 043505 (2001).

[49] P. Stefanescu, New Astron. Rev. **12**, 635 (2007).

[50] R. Khatri and B. D. Wandelt, Phys. Rev. Lett. **98**, 111301 (2007).

[51] L. Bergstrom, S. Iguri, and H. Rubinstein, Phys. Rev. D **60**, 045005 (1999).

[52] P. P. Avelino *et al.*, Phys. Rev. D **64**, 103505 (2001).

[53] Y. Fujii and M. Sasaki, Phys. Rev. D **75**, 064028 (2007).

[54] E. Garcia-Berro, J. Isern, and Y. A. Kubyshin, Astron. Astrophys. Rev. **14**, 113 (2007).

[55] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990).

[56] W. L. Freedman *et al.*, Astrophys. J. **553**, 47 (2001).

[57] S. Nojiri and S. D. Odintsov, J. Phys. Conf. Ser. **66**, 012005 (2007).

[58] K. Dimopoulos and M. Karciauskas, arXiv:0803.3041.

[59] K. Dimopoulos, Phys. Rev. D **74**, 083502 (2006); **76**, 063506 (2007).