Chirality Sensitive Effect on Surface States in Chiral *p*-Wave Superconductors

Takehito Yokoyama,[1,2] Christian Iniotakis,[3] Yukio Tanaka,[1,2] and Manfred Sigrist[3]

[1]*Department of Applied Physics, Nagoya University, Nagoya, 464-8603, Japan*
[2]*CREST, Japan Science and Technology Corporation (JST), Nagoya, 464-8603, Japan*
[3]*Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland*

We study the local density of states at the surface of a chiral *p*-wave superconductor in the presence of a weak magnetic field. As a result, the formation of low-energy Andreev bound states is either suppressed or enhanced by an applied magnetic field, depending on its orientation with respect to the chirality of the *p*-wave superconductor. Similarly, an Abrikosov vortex, which is situated not too far from the surface, leads to a zero-energy peak of the density of states, if its chirality is the same as that of the superconductor, and to a gap structure for the opposite case. We explain the underlying principle of this effect and propose a chirality sensitive test on unconventional superconductors.

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Much attention has been paid to unconventional superconductors because they can exhibit a sign or general phase change of their gap function as a function of momentum. This property induces many intriguing phenomena, which can be observed directly by so-called phase sensitive experiments providing powerful tools to test the symmetry of the gap function [1]. One important consequence of the sign change of the gap function is the possible existence of Andreev bound states at the surface of the superconductor [2–4]. The formation of Andreev bound states increases the local zero-energy quasiparticle density of states (DOS) at the surface, leading to a pronounced zero-bias conductance peak in the tunneling conductance observable both in singlet *d*-wave superconductors like the cuprates and in triplet *p*-wave superconductors such as Sr_2RuO_4 [5–13]. For the case of *d*-wave superconductors it is well known that an applied magnetic field or an applied electric current result in a split of this zero-bias conductance peak, since the zero-energy spectral weight of the bound states is effectively Doppler shifted towards higher energies [6,14–16]. The same effect also appears for an Abrikosov vortex, which is pinned not too far from the boundary. Here, the zero-energy DOS is suppressed in a shadowlike region "behind" the vortex [17,18].

Regarding the chiral *p*-wave superconducting phase as it is likely realized in Sr_2RuO_4, a further aspect appears. The chiral *p*-wave state characterized by the vector $\boldsymbol{d}(\boldsymbol{k}) = (0, 0, k_x \pm ik_y)$ breaks time reversal symmetry [19–23]. In this Letter we will show that the influence of an external magnetic field on the surface density of states is selective for the chirality. The quasiparticle density of states at the surface increases or decreases depending on the relative orientation of the applied magnetic field and the chirality. Similarly, we find that the influence of a vortex on the surface states depends on the orientation of vorticity with respect to chirality. These characteristic effects could open an alternative way to chirality sensitive probes, in contrast

to phase or spin sensitive setups, which have been intensively used in the scientific community already.

For our calculations, we use the quasiclassical Eilenberger theory of superconductivity [24,25] in the so-called Riccati parametrization [26], which allows us to achieve numerically stable solutions for the quasiclassical propagators (and thus also for the DOS) in spatially non-homogeneous systems. Concretely, in our case we consider a superconducting half-space $x > 0$ exhibiting a gap function of chiral *p*-wave symmetry. The surface at $x = 0$ is included in our calculations in a straightforward way by specific boundary conditions [27–29]. We assume a cylindrical Fermi surface for the superconductor with the symmetry axis pointing along the *z* direction, so that the Fermi velocity can be parametrized by the polar angle θ via $\boldsymbol{v}_F = v_F(\hat{\boldsymbol{x}} \cos\theta + \hat{\boldsymbol{y}} \sin\theta)$. For a given point $\boldsymbol{r}_0$ in space and angle θ parametrizing the Fermi surface, a quasiclassical trajectory is then defined according to $\boldsymbol{r}(x') = \boldsymbol{r}_0 + x'\hat{\boldsymbol{v}}_F$. Along such a trajectory the Eilenberger equations can be transformed into 2×2 matrix differential equations in spin space, which are of the Riccati type and can be solved much easier [29]. In our case, we deal with a one-component gap function, so that the corresponding Riccati equations take the simpler form [10,26]

$$\hbar v_F \partial_{x'} a(x') + [2\tilde{\epsilon}_n + \Delta^\dagger a(x')]a(x') - \Delta = 0$$
$$\hbar v_F \partial_{x'} b(x') - [2\tilde{\epsilon}_n + \Delta b(x')]b(x') + \Delta^\dagger = 0 \tag{1}$$

for the two scalar coherence functions a and b. Here, $i\tilde{\epsilon}_n = i\epsilon_n + \mathbf{v}_F \cdot \frac{e}{c}\boldsymbol{A}$ denotes Matsubara frequencies, which are shifted due to the presence of a magnetic vector potential $\boldsymbol{A}$. The pairing potential Δ can be factorized in the following form:

$$\Delta(\boldsymbol{r}, \theta) = \Delta_0 \exp(i\theta)\Psi(\boldsymbol{r}). \tag{2}$$

Here, Ψ denotes a factor that covers the spatial dependence of the pairing potential in general. Since we are only

interested in the main qualitative aspects of the local DOS, namely, whether the zero-energy spectral weight at the surface is suppressed or increased, we may take the modulus of Ψ to be constant [2,17,18]. For the calculation of physical properties, the Riccati Eqs. (1) have to be integrated numerically using proper starting values in the bulk. The local DOS, which is already normalized to the DOS in the normal state, is then achieved by an integration over the Fermi surface. In terms of the coherence functions a and b, we have

$$N(\boldsymbol{r}_0, E) = \int_0^{2\pi} \frac{d\theta}{2\pi} \operatorname{Re}\left[\frac{1 - ab}{1 + ab}\right]_{i\epsilon_n \to E + i\delta}, \quad (3)$$

where E denotes the quasiparticle energy with respect to the Fermi level and δ is an effective scattering parameter that corresponds to an inverse mean free path. For all numerical calculations, we fix this value as $\delta = 0.1\Delta_0$.

In order to study the basic effect of chirality, we consider a magnetic field applied along the z axis at the surface, represented by a nearly homogeneous vector potential $\boldsymbol{A}$. We choose the real gauge; i.e., the spatially dependent part Ψ of the pairing potential is taken to be real. Since we additionally assumed a spatially constant modulus of the pairing potential, it is possible to get analytical solutions for the coherence functions a and b in this case, which also allows us to examine the corresponding behavior of the local DOS analytically. Directly at the surface, we get

$$N(E) = 2\operatorname{Re}\left\langle \frac{1}{1 + a_{\text{in}}b_{\text{out}}} \right\rangle_{i\epsilon_n \to E + i\delta} - 1 \quad (4)$$

with $a_{\text{in}} = s\Delta_0 e^{i(\pi - \theta)}$, $b_{\text{out}} = s\Delta_0 e^{-i\theta}$ and the abbreviation $s = 1/(\tilde{\varepsilon}_n + \sqrt{\tilde{\varepsilon}_n^2 + \Delta_0^2})$. Furthermore, $\langle \ldots \rangle$ denotes angular averaging, which we may restrict to outgoing angles $-\pi/2 \leq \theta \leq \pi/2$ only. This directly yields

$$N(E) = 2\operatorname{Re}\left\langle \frac{1}{1 - (1 - 2\tilde{\varepsilon}_n s)e^{-2i\theta}} \right\rangle_{i\epsilon_n \to E + i\delta} - 1. \quad (5)$$

Expanding the zero-energy DOS in orders of the vector potential $\boldsymbol{A}$, we obtain in the clean limit of $\delta \to 0^+$

$$N(E = 0) = 1 + \frac{e\upsilon_F}{c\Delta_0}A_y + \ldots. \quad (6)$$

Physically, this result displays the influence of a Doppler shift due to a superfluid velocity on the local quasiparticle spectrum. In terms of chiral surface states [30,31], the Doppler shift leads to a change of the slope of the quasiparticle dispersion [$\epsilon(k_y) = \Delta_0 k_y/k_F$], which has a direct impact on the corresponding DOS. As is seen from the result on the right-hand side of Eq. (6), the term of the vector potential, which belongs to the direction perpendicular to the chirality of the p-wave superconductor, survives in linear order. We have explicitly checked that this linear behavior qualitatively remains also for a self-consistent pair potential. This is in contrast to other super-

conductors like s-, d-, or p-wave superconductors without chirality, where we obtain similarly

$$N(E = 0) = C + \langle F(\theta)\sin\theta \rangle A_y + \ldots \quad (7)$$

with a constant C and a function F, which satisfies $F(\theta) = F(-\theta)$. Thus, after angular averaging, the coefficient of the linear term vanishes in these cases, reflecting the presence of inversion symmetry with respect to the x-y plane. Since an applied magnetic field is related to the vector potential as $B_z = \partial_x A_y$, the zero-energy DOS in the chiral p-wave superconductor depends on the sign of the magnetic field. Applying a weak magnetic field along the chirality direction suppresses the zero-energy bound states, while applying it in the opposite direction leads to a zero-energy peak of the surface DOS. It is important to realize that the derived Eq. (6) qualitatively implies this chirality sensitive effect also for the case of a more general vector potential. Especially, this chirality effect is remarkable in the presence of a vortex near the surface.

As a next example of the chirality effect, we study the case of a single Abrikosov vortex line parallel to the z axis. For such a vortex in the bulk of a chiral p-wave superconductor, the interplay between vorticity and chirality has been intensively studied already [32]. In the present work, however, we focus on important surface effects, which appear at the boundary due to the presence of the vortex. We assume the vortex to be at a distance x_V from the boundary. With y denoting the coordinate along the boundary, the vortex shall sit at $y_V = 0$. For convenience, we set the vector potential $\boldsymbol{A} = \boldsymbol{0}$ and incorporate the vortex by a properly chosen phase factor $\Psi(\boldsymbol{r}) = e^{i\Phi(\boldsymbol{r})}$ instead, which (in standard complex notation $z = x + iy$) is given by [17,18]

$$e^{i\Phi(\boldsymbol{r})} = \frac{z - z_V}{|z - z_V|}\left(\frac{z - \bar{z}_V}{|z - \bar{z}_V|}\right)^*. \quad (8)$$

Here, the first factor is the phase of a single vortex at position z_V, while the second factor corresponds to a virtual antivortex placed at the mirrored position $\bar{z}_V = -x_V$, which ensures the correct implementation of boundary conditions. In the following, we consider two different cases: The chiralities of the p-wave state and the vortex are the same (a) or opposite (b). The latter case is implemented by a complex conjugation of the phase factor given in Eq. (8), replacing the vortex by an antivortex and vice versa. Our results for the corresponding local zero-energy DOS near the surface of the p-wave superconductor are shown in Fig. 1. The Abrikosov vortex position is fixed at a distance of $x_V = 2\xi$ from the boundary with $\xi = \hbar\upsilon_F/\Delta_0$ denoting the coherence length. Apart from zero-energy bound states in the vortex core, there are also bound states at the surface of the p-wave superconductor. Far away from the vortex, these surface bound states have the spectral weight of $N(E = 0) = 1$ [cf. Eq. (6)]. However, as can be seen quite clearly in Fig. 1, the local DOS drastically
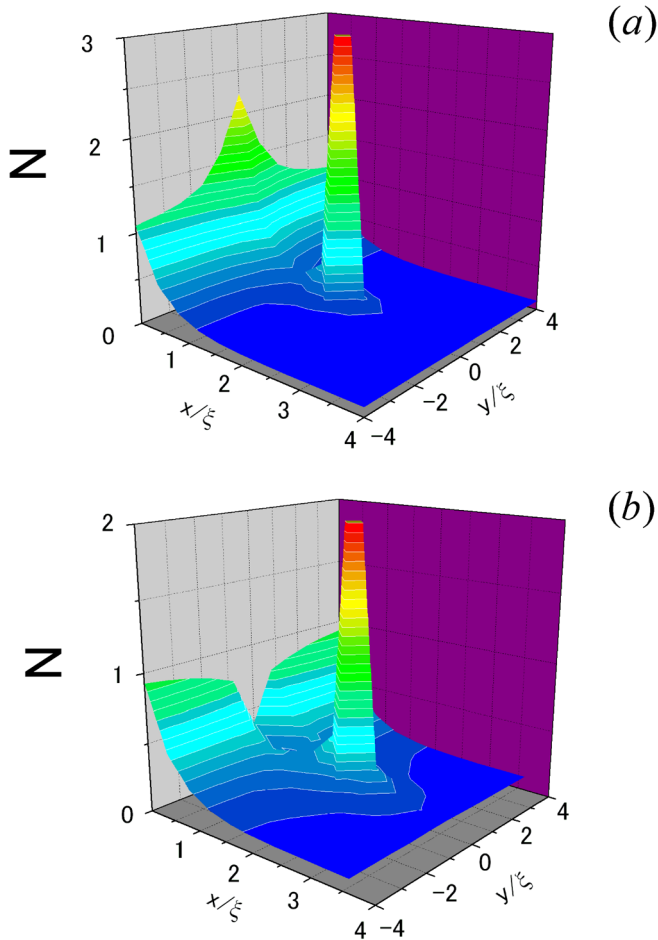
FIG. 1 (color online). $N(E = 0)$ in a chiral p-wave supercon-
ductor in the presence of a vortex, which is situated at a distance
of $x_V = 2\xi$ from the surface. The vortex and the p-wave state
have (a) the same chirality and (b) the opposite chirality. Clearly,
there is a remarkable difference in the quasiparticle spectral
weight at the surface ($x = 0$) behind the vortex, showing a strong
increase in (a) and a suppression in (b).

changes in a shadowlike region behind the vortex. If the
vortex and the p-wave state have the same chirality, the
bound states are strongly enhanced (a), and for opposite
chirality they are suppressed (b). The latter suppression
resembles a similar effect appearing in d-wave supercon-
ductors [17,18].

In Fig. 2, we show the local DOS at the point $x = y = 0$
for different vortex to boundary distances x_V as a function
of energy. Around zero energy, we find a sharp peak or dip
structure, respectively, again depending on the chirality.
These structures get less pronounced when the vortex
distance is increased; nevertheless, they persist. More-
over, the quasiparticle spectrum starts to exhibit some
kind of mirror symmetry around the value $N(E) = 1$ for
the two different chiralities. Note that this is in qualitative
agreement with Eq. (6) since after a transformation to the
real gauge we eventually have $A_y > 0$ due to the vortex,
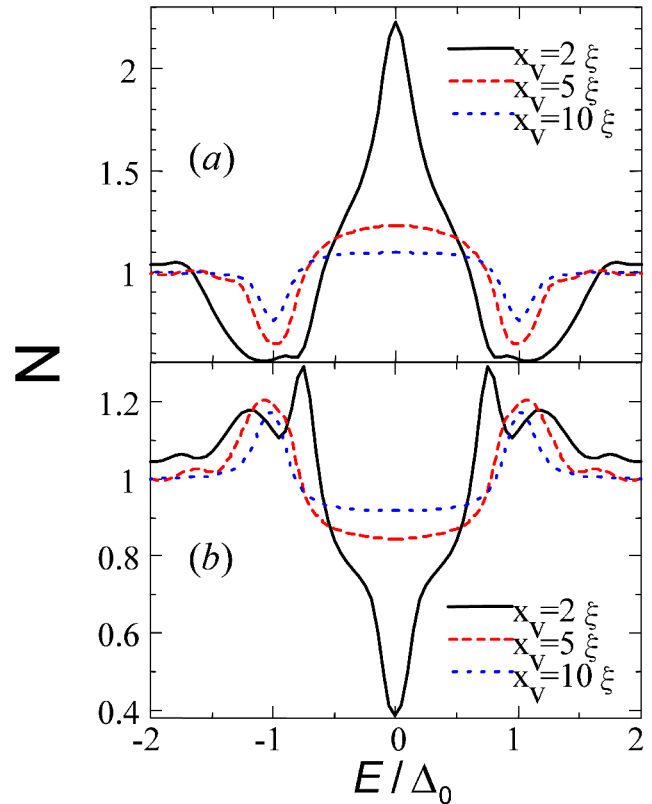leading to the strong increase of Andreev bound states at

FIG. 2 (color online). Local DOS at the point $x = y = 0$ for
different vortex to boundary distances x_V as a function of energy.
Again, p-wave state and Abrikosov vortex have the same chi-
rality in (a) and the opposite in (b), resulting in peak and dip
structures, respectively, around zero energy.

the surface for the same chirality. The result for opposite
chirality is obtained due to $A_y < 0$ for the antivortex,
accordingly. It is worth noting that the modification of
the surface quasiparticle states due to the presence of
vortices has an effect on the force acting on vortices near
the surface. An increase of the DOS leads to a repulsion of
the vortex from the boundary towards the bulk, whereas a
decrease results in an attraction towards the boundary.
Thus, in both cases the Bean-Livingston barrier would be
modified, which influences the escape and entrance of
vortices to the superconductor [33,34].

Our results allow us to propose a chirality sensitive test
on superconductors, based on well-established experimen-
tal techniques, which are capable of indicating the weight
of Andreev bound states, for example, tunneling conduc-
tance experiments or low-temperature scanning tunneling
spectroscopy [35]. For a chiral superconductor, it is ex-
pected to observe a suppression of the zero-energy DOS at
the surface, when a weak magnetic field is applied parallel
to the chirality. Inverting the field, however, leads to an
enhancement of the DOS. In this way chirality could be
detected. This unusual reversal effect does not appear in
nonchiral superconductors. Moreover, the experiment al-
lows us to detect the chirality and possibly even the do-

mains of different chirality, if domain walls reach the surface.

In summary, we have studied the DOS at the surface of a chiral p-wave superconductor in the presence of a magnetic vector potential due to, for example, an applied magnetic field or an Abrikosov vortex. We clarified that the weight of low-energy surface bound states gets either suppressed or increased, depending on the orientation of both chirality and magnetic field. Because of this chirality sensitive effect on the Andreev bound states, a setup to test the chirality of an unconventional superconductor could be accessible experimentally. In particular, tunneling experiments for the material Sr_2RuO_4 are promising probes for the observation of the effect presented here. Furthermore, for vortices this effect also has a chirality selective influence on the Bean-Livingston barrier, which could give rise to a different escape rate of vortices from the two kinds of chiral domains.

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