Strong magnetic field enhancement of spin triplet pairing arising from coexisting $2k_F$ spin and $2k_F$ charge fluctuations

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We study the effect of the magnetic field (Zeeman splitting) on spin triplet superconductivity. We show generally that the enhancement of spin triplet pairing mediated by coexisting $2k_F$ spin and $2k_F$ charge fluctuations can be much larger than in the case of triplet pairing mediated by ferromagnetic spin fluctuations. We propose that this may be related to the recent experiment on $(TMTSF)_2ClO_4$, in which the possibility of a singlet to triplet pairing transition has been suggested.

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I. INTRODUCTION

Spin triplet superconductivity is one of the most fascinating unconventional superconducting states. The investigation of the mechanism of this pairing state has been a theoretical challenge. Spin triplet pairing mediated by ferromagnetic spin fluctuations has been studied for a long time in the context of superfluid ^{3}He, but another possibility has arisen in the past several years: triplet pairing mediated by coexisting $2k_F$ spin and $2k_F$ charge (or orbital) fluctuations proposed for Sr_2RuO_4,[1] and for the organic superconductors $(TMTSF)_2X$ (TMTSF=tetramethyl-tetraselenafulvalene, X $=PF_6$, ClO_4, etc.) (Refs. 2–7) and θ-$(BEDT\text{-}TTF)_2I_3$ (BEDT-TTF=bisethylenedithio-tetrathiafulvalene).[8]

For $(TMTSF)_2X$ in particular, the possibility of spin triplet pairing has been pointed out for a long time,[9–19] and two of the present authors as well as several other groups have come up with the possibility of a close competition between singlet d-wave-like pairing and triplet f-wave-like pairing due to coexisting $2k_F$ spin and $2k_F$ charge fluctuations. Very recently, an NMR study on $(TMTSF)_2ClO_4$ has pointed out the possibility of a transition from spin singlet pairing at low magnetic fields to triplet pairing or a Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) state at high fields.[20] In fact, this possibility of a singlet to triplet pairing transition under high magnetic field was pointed out theoretically.[21–23]

In the present paper, we study the effect of the magnetic field (Zeeman splitting) on triplet pairing using the random phase approximation (RPA). We show generally that the enhancement of spin triplet pairing mediated by coexisting $2k_F$ spin and $2k_F$ charge fluctuations can be much larger than in the case of triplet pairing mediated by ferromagnetic spin fluctuations. This enhancement of triplet superconductivity under a magnetic field is different from the effect studied previously[21–23] in that it is peculiar to the $2k_F$ spin-charge fluctuation mechanism. Applying this idea to a microscopic model for $(TMTSF)_2X$ in which strong $2k_F$ spin and $2k_F$ charge fluctuations occur, we actually show that the magnetic field enhancement of spin triplet f-wave pairing is strong compared to the enhancement of triplet pairing mediated by ferromagnetic spin fluctuation that occurs in the triangular lattice Hubbard model.[24,25] Due to this strong effect, we show that even when spin singlet pairing dominates in the absence of the magnetic field, a transition to triplet pairing may take place when a magnetic field is applied. This is consistent with the possibility of the magnetic-field-induced singlet-triplet transition in $(TMTSF)_2ClO_4$.[20] Moreover, this strong magnetic field effect may be used as a general probe for identifying the pairing mechanism of triplet superconductors.

II. FORMULATION

The extended Hubbard model that takes into account the Zeeman effect is given by

$$H = \sum_{i,j,\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i} U n_{i\uparrow} n_{i\downarrow} + \sum_{i,j,\sigma,\sigma'} V_{ij} n_{i\sigma} n_{j\sigma'} + g\mu_B B \sum_{i,\sigma} \mathrm{sgn}(\sigma) c_{i\sigma}^{\dagger} c_{i\sigma}. \quad (1)$$

t_{ij} represents the hopping, U is the on-site interaction, and V_{ij} are the off-site interactions. $g\mu_B B$ is the Zeeman energy with the spin quantization axis $\hat{z} \parallel \mathbf{B}$. We ignore the orbital effect of the magnetic field, assuming an experimental configuration in which the magnetic field is applied parallel to the x-y plane. Within the RPA,[26] the pairing interactions mediated by spin and charge fluctuations are given by

$$V^{s}(\mathbf{q}) = U + V(\mathbf{q}) + \frac{U^2}{2}\chi_{\mathrm{sp}}^{zz}(\mathbf{q}) + U^2 \chi_{\mathrm{sp}}^{+-}(\mathbf{q}) - \frac{[U + 2V(\mathbf{q})]^2}{2}\chi_{\mathrm{ch}}(\mathbf{q}), \quad (2)$$

$$V^{t\sigma\sigma}(\mathbf{q}) = V(\mathbf{q}) - 2[U + V(\mathbf{q})]V(\mathbf{q})\chi^{\sigma\bar{\sigma}}(\mathbf{q}) - V(\mathbf{q})^2 \chi^{\sigma\sigma}(\mathbf{q}) - [U + V(\mathbf{q})]^2 \chi^{\bar{\sigma}\bar{\sigma}}(\mathbf{q}), \quad (3)$$

$$V^{t\sigma\bar{\sigma}}(\mathbf{q}) = V(\mathbf{q}) + \frac{U^2}{2}\chi_{\rm sp}^{zz}(\mathbf{q}) - U^2\chi_{\rm sp}^{+-}(\mathbf{q}) - \frac{[U+2V(\mathbf{q})]^2}{2}\chi_{\rm ch}(\mathbf{q}), \quad (4)$$

for the spin singlet and the triplets with $\mathbf{d}\perp\hat{z}$ ($S_z=\pm 1$) and with $\mathbf{d}\parallel\hat{z}$ ($S_z=0$), respectively. Here $V(\mathbf{q})$ is the Fourier transform of the off-site interactions. The longitudinal spin and charge susceptibilities are obtained from $\chi_{\rm sp}^{zz}=(\chi^{\uparrow\uparrow}+\chi^{\downarrow\downarrow}-\chi^{\uparrow\downarrow}-\chi^{\downarrow\uparrow})/2$ and $\chi_{\rm ch}=(\chi^{\uparrow\uparrow}+\chi^{\downarrow\downarrow}+\chi^{\uparrow\downarrow}+\chi^{\downarrow\uparrow})/2$. Here,

$$\chi^{\sigma\sigma}(\mathbf{q}) = \frac{[1+\chi_0^{\bar{\sigma}\bar{\sigma}}V(\mathbf{q})]\chi_0^{\sigma\sigma}}{[1+\chi_0^{\sigma\sigma}V(\mathbf{q})][1+\chi_0^{\bar{\sigma}\bar{\sigma}}V(\mathbf{q})]-[U+V(\mathbf{q})]^2\chi_0^{\sigma\sigma}\chi_0^{\bar{\sigma}\bar{\sigma}}}, \quad (5)$$

$$\chi^{\sigma\bar{\sigma}}(\mathbf{q}) = \frac{-\chi_0^{\sigma\sigma}[U+V(\mathbf{q})]\chi_0^{\bar{\sigma}\bar{\sigma}}}{[1+\chi_0^{\sigma\sigma}V(\mathbf{q})][1+\chi_0^{\bar{\sigma}\bar{\sigma}}V(\mathbf{q})]-[U+V(\mathbf{q})]^2\chi_0^{\sigma\sigma}\chi_0^{\bar{\sigma}\bar{\sigma}}}. \quad (6)$$

The longitudinal bare susceptibility is given by

$$\chi_0^{\sigma\sigma}(\mathbf{q}) = \frac{-1}{N}\sum_{\mathbf{k}}\frac{f(\xi_\sigma(\mathbf{k}+\mathbf{q}))-f(\xi_\sigma(\mathbf{k}))}{\xi_\sigma(\mathbf{k}+\mathbf{q})-\xi_\sigma(\mathbf{k})}, \quad (7)$$

where $\xi_\sigma(\mathbf{k})$ is the band dispersion taking account of the Zeeman effect measured from the chemical potential μ, and $f(\xi)$ is the Fermi distribution function. The transverse spin susceptibility is given by $\chi_{\rm sp}^{+-}=\chi_0^{+-}/(1-U\chi_0^{+-})$, where the transverse bare susceptibility is

$$\chi_0^{+-}(\mathbf{q}) = \frac{-1}{N}\sum_{\mathbf{k}}\frac{f(\xi_\sigma(\mathbf{k}+\mathbf{q}))-f(\xi_{\bar{\sigma}}(\mathbf{k}))}{\xi_\sigma(\mathbf{k}+\mathbf{q})-\xi_{\bar{\sigma}}(\mathbf{k})}. \quad (8)$$

To obtain the superconducting state, we solve the linearized BCS gap equation within weak-coupling theory,

$$\lambda^\gamma\phi^\gamma(\mathbf{k}) = -\sum_{\mathbf{k}'}V^\gamma(\mathbf{k}-\mathbf{k}')\frac{1-f(\xi_\sigma(\mathbf{k}'))-f(\xi_{\sigma'}(\mathbf{k}'))}{\xi_\sigma(\mathbf{k}')+\xi_{\sigma'}(\mathbf{k}')}\phi^\gamma(\mathbf{k}'). \quad (9)$$

We consider singlet and triplet pairings with $\mathbf{d}\parallel\hat{z}$ ($\gamma=s,t\sigma\bar{\sigma}$) for opposite spin pairing ($\sigma\neq\sigma'$), and triplet pairing with $\mathbf{d}\perp\hat{z}$ ($\gamma=t\sigma\sigma$) for parallel spin pairing ($\sigma=\uparrow,\downarrow$). $\phi^\gamma(\mathbf{k})$ is the gap function and the critical temperature T_c is determined as the temperature where the eigenvalue λ reaches unity. To give a reference for the values of the magnetic field, we calculate the Pauli limit by $\mu_B B_P=1.75k_B T_c/\sqrt{2}$. Although the RPA may be considered as quantitatively insufficient for discussing the absolute value of T_c, we expect this approach to be valid for studying the *competition* between different pairing symmetries. In fact, as we shall see, we find very good agreement between the RPA results and the already known results obtained by the dynamical cluster approximation (DCA).[25]

III. GENERAL ARGUMENT

Before giving the calculation results, we show generally using the above formula that the effect of the Zeeman splitting on the triplet pairing caused by the coexistence of $2k_F$ spin and $2k_F$ charge fluctuations can be very special. First, let us consider a case where off-site repulsions are not present, so that only the spin fluctuations are relevant, and therefore the possibility of triplet pairing superconductivity arises due to ferromagnetic spin fluctuations. In this case, the triplet pairing interactions reduce to

$$V^{t\sigma\sigma}(\mathbf{q}) = -U^2\chi^{\bar{\sigma}\bar{\sigma}}(\mathbf{q}), \quad (10)$$

$$V^{t\sigma\bar{\sigma}}(\mathbf{q}) = +\frac{U^2}{2}\chi_{\rm sp}^{zz}(\mathbf{q}) - U^2\chi_{\rm sp}^{+-}(\mathbf{q}), \quad (11)$$

where $\chi^{\sigma\sigma}$ also reduces to

$$\chi^{\sigma\sigma}(\mathbf{q}) = \frac{\chi_0^{\sigma\sigma}(\mathbf{q})}{1-U^2\chi_0^{\sigma\sigma}(\mathbf{q})\chi_0^{\bar{\sigma}\bar{\sigma}}(\mathbf{q})}. \quad (12)$$

Here, we assume without losing generality that $\chi_0^{\sigma\sigma}$ is enhanced while $\chi_0^{\bar{\sigma}\bar{\sigma}}$ is suppressed by the magnetic field. (Whether $\sigma=\uparrow$ or $\sigma=\downarrow$ depends on the band structure and the band filling of the system as we shall see later.) In the first order of the magnetic field, $\chi_{\rm sp}^{zz}$ and $\chi_{\rm sp}^{+-}$ are not affected because exchanging $\uparrow$ and $\downarrow$ does not affect these quantities. $\chi^{\sigma\sigma}$ is enhanced because the numerator $\chi_0^{\sigma\sigma}$ in Eq. (12) is enhanced, but again the term in the denominator, $\chi_0^{\sigma\sigma}\chi_0^{\bar{\sigma}\bar{\sigma}}$, is not affected by the magnetic field to first order. Thus, although $V^{t\bar{\sigma}\bar{\sigma}}$ should dominate over $V^{t\sigma\sigma}$, its enhancement due to the Zeeman splitting occurs only through the direct enhancement of $\chi_0^{\sigma\sigma}$, which may not be so large for realistic magnetic fields.

When the possibility of triplet pairing arises due to the coexistence of $2k_F$ spin and $2k_F$ charge fluctuations, where the latter are induced by off-site repulsions, the situation can change drastically. To make the discussion simple, let us consider a case with $-[U+2V(\mathbf{Q}_{2k_F})]\simeq U$, or equivalently $U+V(\mathbf{Q}_{2k_F})\simeq 0$, for which, in the absence of the magnetic field, $\chi_{\rm sp}^{zz}(\mathbf{Q}_{2k_F})=\chi_{\rm sp}^{+-}(\mathbf{Q}_{2k_F})\simeq\chi_{\rm ch}(\mathbf{Q}_{2k_F})$ and thus $V^{t\sigma\sigma}(\mathbf{Q}_{2k_F})=V^{t\sigma\bar{\sigma}}(\mathbf{Q}_{2k_F})\simeq -V^s(\mathbf{Q}_{2k_F})$. (We shall see later that our idea works for more general cases.) Here, the singlet and triplet pairing interactions have nearly the same absolute values because the $2k_F$ spin and the $2k_F$ charge contributions work constructively (destructively) in the spin triplet (singlet) pairing interaction.[1,3–7] In this case, $\chi^{\sigma\sigma}$ at $\mathbf{q}=\mathbf{Q}_{2k_F}$ can be given by the reduced form

$$\chi^{\sigma\sigma}(\mathbf{Q}_{2k_F}) \simeq \frac{\chi_0^{\sigma\sigma}(\mathbf{Q}_{2k_F})}{1+V(\mathbf{Q}_{2k_F})\chi_0^{\sigma\sigma}(\mathbf{Q}_{2k_F})}. \quad (13)$$

Here again, we assume without losing generality that $\chi_0^{\sigma\sigma}(\mathbf{Q}_{2k_F})$ [$\chi_0^{\bar{\sigma}\bar{\sigma}}(\mathbf{Q}_{2k_F})$] is enhanced (suppressed) by the Zeeman splitting. $\chi_{\rm sp}^{zz}$ and $\chi_{\rm sp}^{+-}$ again and also $\chi_{\rm ch}$ are unaffected by the magnetic field in first order, so V^s and $V^{t\sigma\bar{\sigma}}$ are unaffected, while $\chi^{\sigma\sigma}$ is again affected. The difference from the case with ferromagnetic spin fluctuations lies in that the denominator of Eq. (13) decreases as $\chi_0^{\sigma\sigma}$ increases, since $V(\mathbf{Q}_{2k_F})<0$. The enhancement of $\chi^{\sigma\sigma}$ due to this effect can be very large in the vicinity of $2k_F$ charge-density-wave (CDW) ordering, because $1+[U+2V(\mathbf{Q}_{2k_F})]\chi_0(\mathbf{Q}_{2k_F})=0$ sig-

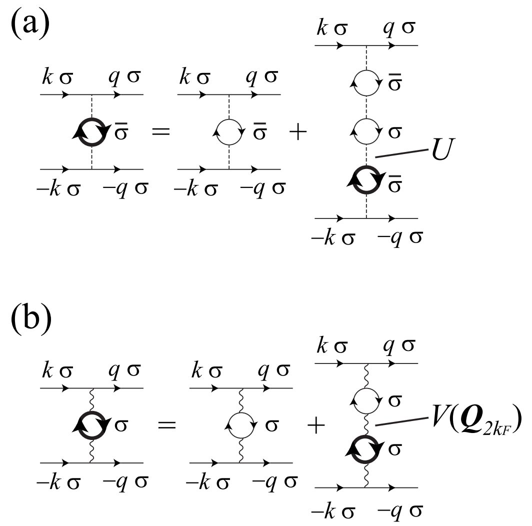

FIG. 1. Diagrammatic view for the presence of (a) only the on-site repulsion and (b) the off-site repulsions in $U+V(\mathbf{Q}_{2k_F})=0$.

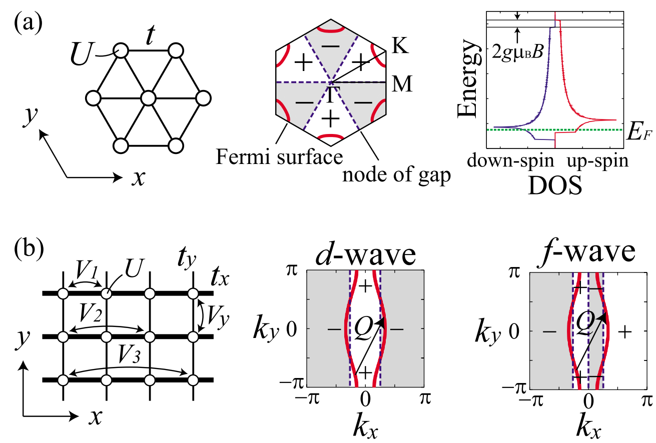

FIG. 2. (Color online) (a) The model on the triangular lattice (left), the f-wave gap (center), and the DOS in the presence of Zeeman splitting (right). The Fermi level is for a dilute band filling. (b) The Q1D model for $(TMTSF)_2X$ (left), and d-wave (center) and f-wave (right) gaps along with the Fermi surface. The arrows represent the nesting vector.

nals this ordering, which is the same as $1+V(\mathbf{Q}_{2k_F})\chi_0(\mathbf{Q}_{2k_F})=0$ when $U+V(\mathbf{Q}_{2k_F})=0$. Therefore, when the possibility of triplet pairing arises in the vicinity of coexisting $2k_F$ CDW and $2k_F$ spin-density-wave (SDW) phases, triplet pairing with parallel spins can be strongly favored by the Zeeman splitting, with the following reduced form of the triplet pairing interaction:

$$V^{t\sigma\sigma}(\mathbf{Q}_{2k_F}) = -V(\mathbf{Q}_{2k_F})^2\chi^{\sigma\bar{\sigma}}(\mathbf{Q}_{2k_F}), \tag{14}$$

when the condition $U+V(\mathbf{Q}_{2k_F})=0$ is satisfied. The difference between the two cases can be expressed in a diagrammatic representation as in Fig. 1. For the ferromagnetic spin fluctuation mechanism, the bubble diagrams in the triplet pairing interaction always enter as paired units consisting of $\chi_0^{\sigma\sigma}$ and $\chi_0^{\bar{\sigma}\bar{\sigma}}$ because of the Pauli principle. On the other hand, for the $2k_F$ spin+$2k_F$ charge fluctuation mechanism, the bubbles enter in the unpaired form of $\chi_0^{\sigma\sigma}$ because they are connected by off-site interactions.

In actual cases, superconductivity is usually degraded by the orbital effect under magnetic fields, but even in that case, the enhancement of triplet pairing due to the above effect should make the suppression moderate. For quasi-one-dimensional (Q1D) systems in particular, where the additional node in the superconducting (SC) gap required in the triplet pairing does not intersect the Fermi surface [see Fig. 2(b)], the coexistence of $2k_F$ spin and $2k_F$ charge fluctuations already results in a subtle competition between singlet and triplet pairings,[4,5] so that the strong enhancement of the triplet pairing interaction by the magnetic field may easily result in a singlet to triplet pairing transition.

IV. CALCULATION RESULTS

A. Triangular lattice

We now apply the above idea to actual systems. First, we consider a case where the possibility of triplet pairing occurs due to ferromagnetic spin fluctuations induced by on-site repulsion. As a typical example, we consider a case on a triangular lattice with dilute band filling as shown in Fig. 2(a). In this case, the possibility of spin triplet f-wave pairing due to ferromagnetic spin fluctuations has been pointed out previously.[24,25]

The band dispersion is given by $\xi_\sigma(\mathbf{k})=2t\cos k_x+2t\cos k_y+2t\cos(k_x+k_y)-\mu+g\mu_B B\,\mathrm{sgn}(\sigma)$. Here, the x and y directions are shown in Fig. 2(a). We take the transfer energy as the unit of energy, i.e., $t=1.0$ The on-site interaction is $U=3.0$, and the band filling is taken as $n=0.2$. We take 128×128 k-point meshes in the RPA calculation. When the magnetic field is absent, we obtain $k_BT_c\simeq0.014$. The Pauli limit corresponding to this T_c is $\mu_BB_P\simeq0.017$, which should be considered as a reference for the values of the magnetic field. When the Zeeman splitting is introduced, $\chi_0^{\downarrow\downarrow}$ becomes slightly larger than $\chi_0^{\uparrow\uparrow}$ because of the increase of the density of states (DOS) at the Fermi level of the down spin due to the Zeeman splitting [see Fig. 2(a)]. Thus, this corresponds to the case with $\sigma=\downarrow$ in our general argument, so that we should focus on the enhancement of $\chi^{\downarrow\downarrow}$. In order to measure how the magnetic field enhancement of $\chi_0^{\downarrow\downarrow}$ is reflected in the enhancement of $\chi^{\downarrow\downarrow}$, we introduce the parameter

$$\alpha_\sigma(\mathbf{q},B)=\frac{\chi^{\sigma\sigma}(\mathbf{q},B)/\chi^{\sigma\sigma}(\mathbf{q},0)}{\chi_0^{\sigma\sigma}(\mathbf{q},B)/\chi_0^{\sigma\sigma}(\mathbf{q},0)}. \tag{15}$$

This quantity measures the difference of the magnetic field effect between the bare and the nonbare susceptibilities. In Fig. 3(a), $\alpha_\downarrow$ at $\mu_BB=0.03$ is plotted as a function of $\mathbf{q}$. $\alpha_\downarrow\simeq1$ means that the effect of the denominator in Eq. (12) is small, as expected from our argument above. The effect of the magnetic field on the strength of the triplet pairing is shown in Fig. 4(a), where we plot the eigenvalues of the linearized gap equation. As expected, due to the enhancement in $\chi^{\downarrow\downarrow}$, the triplet $f^{\uparrow\uparrow}$ wave dominates in the presence of the magnetic field. We note that this result for λ closely

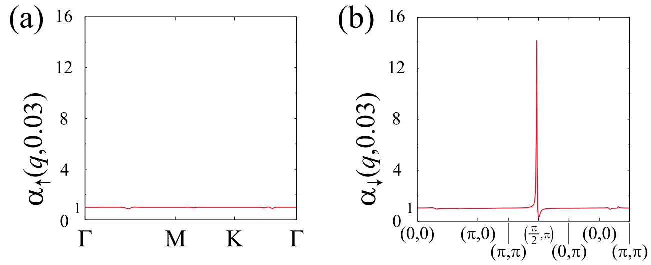

FIG. 3. (Color online) $\alpha_\sigma(\mathbf{q},B)$ at $T=0.02$ for (a) the triangular lattice with $n=0.2$ and (b) the Q1D model with $V_2+V_y=0.85=U/2$.

resembles the field dependence of the pairing susceptibility calculated by the DCA for the same system,[25] suggesting the reliability of the present approach.

B. Quasi-one-dimensional system

We now turn to the case where the possibility of triplet pairing arises due to the coexistence of $2k_F$ spin and $2k_F$ charge fluctuations. As a typical example, we consider the case of $(TMTSF)_2X$.[3–7] We adopt a 3/4-filled Q1D extended Hubbard model as shown in Fig. 2(b).[5] The band dispersion is given by $\xi_\sigma(\mathbf{k})=2t_x\cos k_x+2t_y\cos k_y-\mu+g\mu_B B\,\mathrm{sgn}(\sigma)$.[27] We take $t_x=1.0$ as the unit of energy, and $t_y=0.2$. As for the interaction parameters, we consider not only the on-site interaction U, but also off-site interactions: first, second, and third nearest neighbors (NNs) in the x direction, V_1, V_2, and V_3, and NN interaction V_y in the y direction, where the Fourier transform of these interactions is given by $V(\mathbf{q})=2V_1\cos(k_x)+2V_2\cos(2k_x)+2V_3\cos(3k_x)+2V_y\cos(k_y)$. The band filling is taken as $n=1.5$ in accord with $(TMTSF)_2X$.

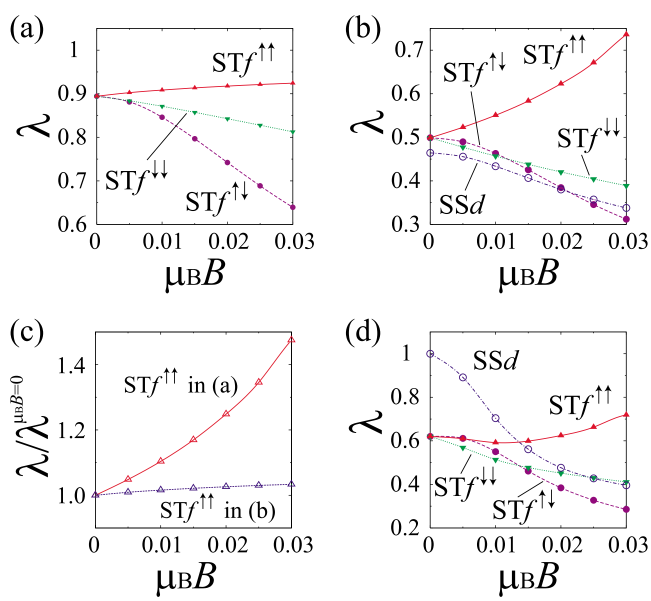

FIG. 4. (Color online) B dependence of λ at $T=0.02$ for (a) the triangular lattice with $n=0.2$ and (b) the Q1D model with $V_2+V_y=U/2$. (c) B dependence of the most dominant λ for the cases shown in (a) and (b) normalized by λ at $B=0$. (d) B dependence of λ in the Q1D model for $V_2+V_y=0.8<U/2$ at $T=0.012$.

We take 256×128 k-point meshes. As discussed in previous studies,[4–7] when $\chi_{\mathrm{ch}}(\mathbf{Q}_{2k_F})=\chi_{\mathrm{sp}}(\mathbf{Q}_{2k_F})$, singlet d-wave and triplet f-wave pairings are nearly degenerate in the absence of the magnetic field because the triplet and singlet pairing interactions are nearly equal at $\mathbf{Q}_{2k_F}$, and the additional gap node in the f-wave pairing does not intersect the Fermi surface, so that the nodal structure on the Fermi surface is the same for d and f, as shown in Fig. 2(b). In the calculation below, we take the interaction values as $U=1.7$, $V_1=0.9$, $V_2=0.45$, $V_3=0.1$, and $V_y=0.4$. This choice of parameter set has been taken in Refs. 5 and 28 as an example that satisfies $V_2+V_y=U/2$, which results in $U+V(\mathbf{Q}_{2k_F})=0$ $[\mathbf{Q}_{2k_F}\simeq(\pi/2,\pi)]$ and thus $\chi_{\mathrm{ch}}(\mathbf{Q}_{2k_F})=\chi_{\mathrm{sp}}(\mathbf{Q}_{2k_F})$. Although our argument in the presence of the magnetic field given above is general and does not depend on particular choices of parameter values as long as $2k_F$ spin and charge fluctuations coexist, we take this particular choice for the actual RPA calculation. The values of U, V_1, V_3, and V_y will be fixed throughout the paper, while V_2 will be varied later.

Now, introducing the magnetic field enhances $\chi_0^{\uparrow\uparrow}(\mathbf{Q}_{2k_F})$ and suppresses $\chi_0^{\downarrow\downarrow}(\mathbf{Q}_{2k_F})$ because the up-spin band becomes close to half filling and the electron-hole symmetry is somewhat restored. Thus, this corresponds to $\sigma=\uparrow$ in our general argument for the case with coexisting $2k_F$ spin and $2k_F$ charge fluctuations, so we should look at the enhancement of $\chi^{\uparrow\uparrow}$ due to the magnetic field. In Fig. 3(b), $\alpha_\uparrow$ is plotted at $\mu_B B=0.03$, which greatly exceeds unity at $\mathbf{Q}_{2k_F}$ as expected from our previous argument that the denominator in Eq. (13) comes close to 0. In Fig. 4(b), we show the magnetic field dependence of the λ. It can be seen that the triplet $f^{\uparrow\uparrow}$ wave is strongly enhanced due to the strong enhancement of $\chi^{\uparrow\uparrow}(\mathbf{Q}_{2k_F})$. In Fig. 4(c), we compare the B dependence of λ normalized by its value at $B=0$ for the two cases. The enhancement of the triplet pairing mediated by coexisting $2k_F$ spin and $2k_F$ charge fluctuations is indeed much larger.

In the above, we considered the case where $\chi_{\mathrm{ch}}(\mathbf{Q}_{2k_F})=\chi_{\mathrm{sp}}(\mathbf{Q}_{2k_F})$ and thus d-wave and f-wave pairings are nearly degenerate at $B=0$, but even when d dominates at $B=0$ a transition to f can take place within realistic values of B due to this strong enhancement of the triplet pairing interaction. To see this in detail, we consider the case with $V_2=0.4$, for which d-wave pairing dominates over f-wave pairing at $B=0$. Here $T_c=0.012$, which corresponds to $\mu_B B_P\simeq0.015$. In Fig. 4(d), it can be seen that the triplet dominates over the singlet for large enough magnetic field. By further varying V_2, we obtain in Fig. 5(a) a pairing "phase diagram" in the (V_2+V_y)-B space obtained by comparing λ at $k_BT=0.012$. With the magnetic field within a realistic range, the spin triplet (ST) f-wave pairing with $S_z=+1$ (SC-STf$^{\uparrow\uparrow}$) has a tendency to dominate over spin singlet (SS) d-wave pairing (indicated by SC-SSd) in a wide range of values V_2+V_y. The phase diagram for the superconducting states and the normal state in the T-B space is shown in Fig. 5(b). Although T_c of the $f^{\uparrow\uparrow}$ wave increases with B, but we expect that the orbital effect actually suppresses T_c. Even in that case, the effect of the Zeeman splitting should strongly favor the occurrence of triplet pairing over singlet pairing.

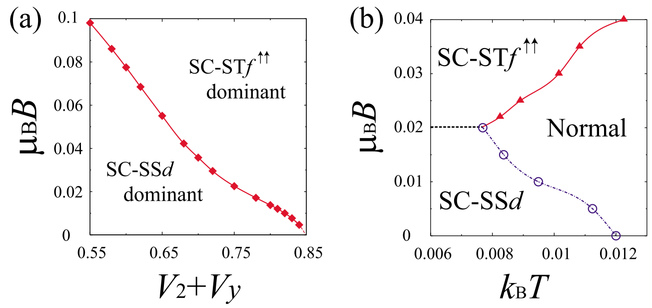

FIG. 5. (Color online) (a) Pairing "phase diagram" obtained by comparing λ at k_BT=0.012. (b) Calculated T-B phase diagram in V_2=0.4 and V_y=0.4. T_c is determined by the condition λ=1, and the boundary (dashed line) between d and f should be considered as just a guideline.

V. CONCLUSION

In conclusion, we have shown generally that the magnetic field enhancement of the spin triplet pairing due to the coexistence of $2k_F$ spin and $2k_F$ charge fluctuations can be extremely large compared to that mediated by ferromagnetic spin fluctuations. Thus, even when spin singlet pairing dominates in the absence of the magnetic field, a transition to spin triplet pairing can take place on application of a (not unrealistically large) magnetic field. This is consistent with the possibility of the magnetic-field-induced singlet-triplet transition in $(TMTSF)_2ClO_4$.[20] The orbital effect and the possibility of the FFLO state also mentioned in Ref. 20 remain as interesting future problems. Also, reliable evaluation of the interactions is necessary in order to confirm the validity of the parameter choice that results in closely competing $2k_F$ spin and $2k_F$ charge fluctuations.

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[27] In this model, the y component of $\mathbf{Q}_{2k_F}$ is π, which may be different from the wave vector of the SDW for certain anions. Nevertheless, our theory is general and does not depend on the wave vector $\mathbf{Q}_{2k_F}$. The key is that the spin and the charge fluctuations have the same wave vector.
[28] The definition of the parameters is different from that in Ref. 5 for clarity. (V, V', V'', V_p) in Ref. 5 corresponds to (V_1, V_2, V_3, V_y) here.