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主論文

ON BASE POINT FREE THEOREM

(固定点自由化定理について)

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§ 1. Introduction

Let X be a non-singular projective variety with $\dim X = n$ over \mathbf{C} . And let $\mathcal{A} = \sum_{i=1}^s \mathcal{A}_i$ be a reduced divisor on X with only simple normal crossings.

Let $\mathbf{Strata}(\mathcal{A}) := \{\Gamma \mid 1 \leq k \leq n, 1 \leq i_1 < i_2 < \cdots < i_k \leq s, \Gamma \text{ is an irreducible component of } \mathcal{A}_{i_1} \cap \mathcal{A}_{i_2} \cap \cdots \cap \mathcal{A}_{i_k} \neq \emptyset\}$. A divisor R on (X, \mathcal{A}) is, by definition, *nef and log big* if R is nef and big and $R|_\Gamma$ is nef and big for any member Γ of $\mathbf{Strata}(\mathcal{A})$ (due to Reid [8, 10.4]).

Note that if R is ample then R is nef and log big on (X, \mathcal{A}) .

The purpose of this paper is to prove the following:

MAIN THEOREM (“log effective freeness”). *Let L be a nef divisor on X such that $aL - (K_X + \mathcal{A})$ is nef and log big on (X, \mathcal{A}) for some $a \geq 0$.*

Then there exists a natural number l_1 , depending only on n and a , such that the complete linear system $|l_1 L|$ is base point free.

The paper is organized as follows. In Section 2 we recall known base point free theorems. In Section 3 we give a proof to Reid’s “log eventual freedom” theorem in the “smooth” case. In Section 4 we prove the main theorem. In Section 5 we show some results concerning the log canonical divisor.

We follow the notation and terminology of [Utah].

The results of the present paper have been announced in [Résumé].

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§ 2. Known results concerning base point freeness

Concerning base point freeness of linear systems on higher dimensional algebraic varieties, the following results are known.

THEOREM 1 (“eventual freedom”) (Kawamata-Shokurov [4, Theorem 3-1-1]). *If L is a nef divisor on X and $aL - (K_X + \mathcal{A})$ is ample for some $a \geq 0$, then $|lL|$*

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is base point free for every $l \gg 0$.

THEOREM 2 (“effective freeness”) (Kollár [5]). *Notation as in Theorem 1. There exists a natural number l_0 depending only on n and a such that $|l_0 L|$ is base point free.*

Proof. For $0 < d \ll 1$, $aL - (K_X + (1-d)\Delta)$ is ample and $(X, (1-d)\Delta)$ is Kawamata-log-terminal. Thus the assertion immediately follows from Kollár [5, 1.1].
Q. E. D.

THEOREM 3 (“log eventual freedom”) (Reid [8, 10.4]). *If L is a nef divisor on X and $aL - (K_X + \Delta)$ is nef and log big on (X, Δ) for some $a \geq 0$, then $|lL|$ is base point free for every $l \gg 0$.*

Remark. This theorem is stated in Reid [8] with the idea for its proof. We give the theorem a proof based on this idea in the next section.

This theorem is not valid under the condition that $aL - (K_X + \Delta)$ is not “nef and log big” on (X, Δ) but “nef and big” (Zariski [4, Remark 3-1-2], Kawamata [3, Introduction]).

The Main Theorem of this paper is the generalization of Theorem 2 to “log effective freeness”.

§ 3. Norimatsu Type Vanishing and proof of Theorem 3

THEOREM 4 (cf. Ein-Lazarsfeld [1, 2.4] and Norimatsu [7]) (“Norimatsu Type Vanishing”). *Let R be a nef and log big divisor on (X, Δ) . Then*

$$H^i(X, \mathcal{O}_X(K_X + \Delta + R)) = 0 \quad \text{for } i > 0.$$

Proof (similar to the proof of [1, 2.4]). The assertion follows from the following exact sequence by induction on (n, s) , by virtue of Kawamata-Viehweg vanishing theorem:

$$\begin{aligned} 0 &\longrightarrow \mathcal{O}_X(K_X + \sum_{j < s} \Delta_j + R) \longrightarrow \mathcal{O}_X(K_X + \Delta + R) \\ &\longrightarrow \mathcal{O}_{\Delta_s}(K_{\Delta_s} + \sum_{j < s} \Delta_j|_{\Delta_s} + R|_{\Delta_s}) \longrightarrow 0, \end{aligned}$$

where $\Delta = \sum_{j=1}^s \Delta_j$.

Q. E. D.

Now we give Theorem 3 a proof relying on “Norimatsu Type Vanishing” and the method of Kawamata [3].

Proof of Theorem 3 (based on Reid’s idea [8, 10.4]). We shall prove the theorem by induction on $n = \dim X$. In the case where $\Delta = 0$, this is a Kawamata-Shokurov result [4, Remark 3-1-2]. So we may assume that $\Delta \neq 0$.

For any $i \in \{1, 2, \dots, s\}$, we consider the following exact sequence:

$$0 \longrightarrow \mathcal{O}_X(lL - \Delta_i) \longrightarrow \mathcal{O}_X(lL) \longrightarrow \mathcal{O}_{\Delta_i}(lL) \longrightarrow 0.$$

We assume that $l \geq a$. Thus $lL - \Delta_i - (K_X + \sum_{j \neq i} \Delta_j) = lL - (K_X + \Delta)$ is nef and log big on $(X, \sum_{j \neq i} \Delta_j)$.

Hence $H^1(X, \mathcal{O}_X(lL - \Delta_i)) = 0$ from Theorem 4.

And $aL|_{\Delta_i} - (K_{\Delta_i} + \sum_{j \neq i} \Delta_j|_{\Delta_i}) = (aL - (K_X + \Delta))|_{\Delta_i}$ is nef and log big on $(\Delta_i, \sum_{j \neq i} \Delta_j|_{\Delta_i})$. Thus by induction hypothesis, $\text{Bs}|lL|_{\Delta_i} = \emptyset$ for every $l \gg 0$.

Therefore $\text{Bs}|lL| \cap \text{Supp } \Delta = \emptyset$ for every $l \gg 0$.

From now, we use the technique of Kawamata [3, Proof of Lemma 2].

Now fix a prime number p . We claim that $\text{Bs}|p^m L| = \emptyset$ for $m \gg 0$.

Take a sufficient large natural number m_0 such that $\text{Bs}|p^{m_0} L| \cap \text{Supp } \Delta = \emptyset$.

We assume that $\text{Bs}|p^{m_0} L| \neq \emptyset$.

Take a log resolution $\mu: Y \rightarrow X$ (all relevant divisors F_j are smooth and cross normally) such that

- (a) $K_Y = \mu^*(K_X + \Delta) + \sum_j a_j F_j$,
- (b) $|\mu^*(p^{m_0} L)| = |M| + \sum_j r_j F_j$ ($|M|$ is the movable part and is free), and
- (c) $\mu^*(aL - (K_X + \Delta)) - \sum_j \delta_j F_j$ is ample (where $\delta_j \in \mathbf{Q}$ and $0 \leq \delta_j \ll 1$).

Let $c := \min_{r_j \neq 0} ((a_j + 1 - \delta_j) / r_j)$.

Here $c > 0$, because if $a_j = -1$ then $\mu(F_j) \subset \text{Supp } \Delta$ and therefore $r_j = 0$.

We may assume that the minimum is attained at exactly one value $j = j_0$. Note that $F_{j_0} \cap \mu^{-1}(\Delta) = \emptyset$.

Put $A := \sum_j (-cr_j + a_j - \delta_j) F_j$ and $A' := \lceil (A + F_{j_0} + \mu^{-1}(\Delta)) \rceil = \lceil A \rceil + F_{j_0} + \mu^{-1}(\Delta)$ (where $\mu^{-1}(\Delta)$ denotes the set theoretical inverse image with a reduced structure).

Then $A' \geq 0$ and $\mu_* A' = 0$.

We consider a \mathbf{Q} -divisor $N := \mu^*(p^m L) + A - K_Y \equiv cM + \mu^*((p^m - c p^{m_0})L - (K_X + \Delta)) - \sum_j \delta_j F_j$.

If $p^m - c p^{m_0} \geq a$, then N is ample and hence $H^1(Y, \mathcal{O}_Y(\mu^*(p^m L) + \lceil A \rceil)) = 0$.

We consider the following exact sequence:

$$\begin{aligned} 0 &\longrightarrow \mathcal{O}_Y(\mu^*(p^m L) + \lceil A \rceil) \longrightarrow \mathcal{O}_Y(\mu^*(p^m L) + A') \\ &\longrightarrow \mathcal{O}_{F_{j_0}}(\mu^*(p^m L) + A') \oplus \mathcal{O}_{\mu^{-1}(\Delta)}(\mu^*(p^m L) + A') \longrightarrow 0. \end{aligned}$$

By the Nonvanishing theorem of Shokurov,

$$H^0(F_{j_0}, \mathcal{O}_{F_{j_0}}(\mu^*(p^m L) + A')) \neq 0 \quad \text{for } m \gg 0,$$

because $p^m \mu^* L|_{F_{j_0}} + (A + F_{j_0} + \mu^{-1}(\Delta))|_{F_{j_0}} - K_{F_{j_0}} = (\mu^*(p^m L) + A - K_Y)|_{F_{j_0}} = N|_{F_{j_0}}$ is ample.

Thus, for $m \gg 0$, $\text{Bs}|p^m L|$ does not include $\mu(F_{j_0})$ but $\text{Bs}|p^{m_0} L|$ includes $\mu(F_{j_0})$. Hence the method of KMM [4, Theorem 3-1-1] implies the theorem.

Q. E. D.

§4. Main theorem (log effective freeness)

MAIN THEOREM (“log effective freeness”). Assume that L is a nef divisor on X and that $aL - (K_X + \Delta)$ is nef and log big on (X, Δ) for some $a \geq 0$. Then there exists a natural number $f(n, a)$ (which is $\geq a$), depending only on n and a , such that the complete linear system $|f(n, a)L|$ is base point free.

LEMMA 1. Let L be a nef divisor on X such that $aL - (K_X + \Delta)$ is nef and log big on (X, Δ) for an integer $a \geq 0$. Then $\text{Bs}|f(n-1, a)L| \cap \Delta = \emptyset$.

Proof. Put $m_1 := f(n-1, a)$. We consider the exact sequence

$$0 \longrightarrow \mathcal{O}_X(m_1L - \Delta_i) \longrightarrow \mathcal{O}_X(m_1L) \longrightarrow \mathcal{O}_{\Delta_i}(m_1L) \longrightarrow 0$$

for all $1 \leq i \leq s$. By Theorem 4, $H^1(X, \mathcal{O}_X(m_1L - \Delta_i)) = 0$ because $m_1L - \Delta_i - (K_X + \sum_{j \neq i} \Delta_j)$ is nef and log big on $(X, \sum_{j \neq i} \Delta_j)$. Here $\text{Bs}|m_1L|_{\Delta_i} = \emptyset$, because $aL|_{\Delta_i} - (K_{\Delta_i} + \sum_{j \neq i} \Delta_j|_{\Delta_i})$ is nef and log big on $(\Delta_i, \sum_{j \neq i} \Delta_j|_{\Delta_i})$. Thus $\text{Bs}|m_1L| \cap \Delta = \emptyset$. Q. E. D.

Let $g: X \rightarrow S$ be the morphism defined by the linear system $|lL|$ for $l \gg 0$ (Theorem 3). Here there exists a Cartier divisor L_S on S such that L is linearly equivalent to g^*L_S , because $\Phi_{|lL|} = \Phi_{|(l+1)L|} = g$ for $l \gg 0$. We may assume that $L = g^*L_S$.

For any effective divisor H on X which is linearly equivalent to mL for $m \in \mathbb{N}$, there exists a divisor H_S on S such that $H = g^*H_S$, because $\Phi_{|lH|} = \Phi_{|(l+1)H|} = g$ for $l \gg 0$.

Because $g_*\mathcal{O}_X = \mathcal{O}_S$, for any Cartier divisor D on S such that g^*D is linearly equivalent to 0, $\mathcal{O}_S \cong g_*g^*\mathcal{O}_S(D) = g_*\mathcal{O}_X \otimes_{\mathcal{O}_S} \mathcal{O}_S(D) = \mathcal{O}_S(D)$.

Thus $|mL| = g^*|mL_S|$ for any $m \in \mathbb{N}$.

LEMMA 2. Let m be a natural number such that $|mL| \neq \emptyset$ and $\text{Bs}|mL| \cap \Delta = \emptyset$.

Let Z_S be an irreducible component of $\text{Bs}|mL_S|$ and set $k = \text{codim}(Z_S, S)$. Then $\text{Bs}|(km + j + a + 1)L_S|$ does not include Z_S for $j \geq 0$ except at most $\dim Z_S$ different values of j .

Proof (using Kawamata-Shokurov-Kollár’s method [5]). Taking general elements $B_i \in |mL|$, put $B = (1/2m)B_0 + B_1 + \dots + B_k$. Then $(X, \Delta + B)$ is log canonical outside $\text{Bs}|mL|$ and $(X, \Delta + B)$ is not log canonical at the points belonging to the inverse image of the generic point of Z_S by g (by the argument of Kollár [5, 2.2.1]).

Let $M_0 := aL - (K_X + \Delta) + (1/2)L$.

Take a log resolution $f: Y \rightarrow X$ (i.e. Y is smooth and all relevant divisors are smooth and cross normally). Let

$$K_Y = f^*(K_X + \Delta) + \sum_i e_i E_i \quad (e_i \geq -1);$$

$$f^*B = \sum_i b_i E_i;$$

$$f^*M_0 = A + \sum_i p_i E_i \quad (A \text{ is an ample } \mathbf{Q}\text{-divisor and } 0 \leq p_i < 1).$$

Put $c := \min \{(e_i + 1 - p_i)/b_i \mid Z_S \subset gf(E_i); b_i > 0\}$. By changing the p_i slightly, we may assume that the minimum is achieved for exactly one index. Let us denote the corresponding divisor by E_0 .

Put $W := \bigcup_{e_i - b_i < -1} gf(E_i)$.

CLAIM 1. $0 < c < 1$.

Proof of Claim 1. We prove $c > 0$. If $Z_S \subset gf(E_i)$, $b_i > 0$ and $e_i = -1$, then $f(E_i) \subset \Delta$. But this can not occur, because $g^{-1}(Z_S) \cap \Delta = \emptyset$ from $\text{Bs}|mL| \cap \Delta = \emptyset$.

Next we prove $c < 1$. Let z be a scheme-theoretic point on X such that $g(z)$ is the generic point of Z_S . Because $(X, \Delta + B)$ is not log canonical at z , $z \in f(E_j)$ and $e_j - b_j < -1$ for some j . Q. E. D.

CLAIM 2. Put $c' := \max \{(e_i + 1)/b_i \mid e_i + 1 < b_i\}$. Then $c \leq c' < 1$ and c' is not affected by p_i 's.

Proof of Claim 2. From the proof of Claim 1, $(e_j + 1)/b_j < 1$ and $gf(E_j) \supset Z_S$ for some j . Q. E. D.

CLAIM 3. If W does not include $gf(E_i)$, then $cb_i - e_i + p_i < 1$ or $f(E_i) \subset \Delta$.

Proof of Claim 3. Here $e_i - b_i \geq -1$. If $b_i \neq 0$, then $e_i - cb_i \geq e_i - c'b_i > -1$ by Claim 2. If $b_i = 0$ and $e_i > -1$, then $e_i - cb_i = e_i > -1$. If $b_i = 0$ and $e_i = -1$, then $f(E_i) \subset \Delta$. Q. E. D.

CLAIM 4. $gf(E_0) = Z_S$. If $cb_i - e_i + p_i \geq 1$ and $i \neq 0$, then $gf(E_i)$ does not include Z_S .

Proof of Claim 4. If $cb_i - e_i + p_i \geq 1$, then $gf(E_i) \subset W$ or $f(E_i) \subset \Delta$, by Claim 3. Because $cb_0 - e_0 + p_0 = 1$, $Z_S \subset gf(E_0)$ and $g^{-1}(Z_S) \cap \Delta = \emptyset$, we get $gf(E_0) \subset W$. Here Z_S is an irreducible component of W . Thus $gf(E_0) = Z_S$. If $cb_i - e_i + p_i \geq 1$ and $Z_S \subset gf(E_i)$, then $p_i < 1 + e_i$, because Δ does not include $f(E_i)$. So $b_i > 0$. Thus $c = (e_i + 1 - p_i)/b_i$ by the definition of c . Hence $i = 0$. Q. E. D.

CLAIM 5. If $cb_i - e_i + p_i < 0$, then E_i is f -exceptional.

Proof of Claim 5. Because $c > 0$ (Claim 1), $e_i > 0$. Thus E_i is f -exceptional. Q. E. D.

Proof of Lemma 2 Continued. Let $N_j := (km + j + a + 1)L$ and $N'_j := f^*N_j - \sum_{i \neq 0} \lfloor cb_i - e_i + p_i \rfloor E_i$.

We consider the exact sequence

$$0 \longrightarrow \mathcal{O}_Y(N'_j - E_0) \longrightarrow \mathcal{O}_Y(N'_j) \longrightarrow \mathcal{O}_{E_0}(N'_j) \longrightarrow 0.$$

Note that

$$N'_j - E_0 \equiv K_Y + A + (1 - c)f^*B + jf^*L + \sum_{i \neq 0} \{cb_i - e_i + p_i\} E_i$$

and

$$N'_j|_{E_0} \equiv K_{E_0} + (A + (1 - c)f^*B + jf^*L)|_{E_0} + \sum_{i \neq 0} \{cb_i - e_i + p_i\} E_i|_{E_0}.$$

By Claim 1, Kawamata-Viehweg vanishing implies $H^1(Y, \mathcal{O}_Y(N'_j - E_0)) = 0$ and $h^0(E_0, \mathcal{O}_{E_0}(N'_j)) = \chi(\mathcal{O}_{E_0}(N'_j))$.

$h^0(E_0, \mathcal{O}_{E_0}(N'_j))$ is a nonzero polynomial of degree $\dim Z_S$ in j for $j \geq 0$ (by the argument of Kollár [5, p. 600]).

By Claim 4 and Claim 5, $\text{Bs}|N_j|$ does not include $f(E_0)$ for all $j \geq 0$ except at most $\dim Z_S$ different values of j .

Noting that $\text{Bs}|N_j| = g^{-1}\text{Bs}|(km + j + a + 1)L_S|$, we end the proof of this lemma. Q. E. D.

Proof of Main Theorem. By using Lemma 1 and Lemma 2, the same argument as in Kollár [5, 2.3] implies the theorem. Q. E. D.

Remark. The function $f(n, a)$ is given as follows.

$$\text{When } n = 1, \text{ we put } f(1, a) := 2^a(1+1)!(a+1).$$

$$\text{When } n \geq 2, \text{ we put } f(n, a) := 2^{n+1}(n+1)!(a+n)f(n-1, a).$$

$$\text{Thus } f(n, a) = \prod_{i=1}^n (2^{i+1}(i+1)!(a+i)).$$

Proof of the Remark Above. When $n = 1$, if $\Delta \neq 0$, then $\text{Bs}|2(a+1)L| \cap \Delta = \emptyset$ from the proof of Lemma 1, because $2(a+1) \geq a$. If $\Delta = 0$, then $|2(a+1)L| \neq \emptyset$ from Kollár [5, 2.4]. Therefore $\text{Bs}|2^1(1+1)!2(a+1)L| = \emptyset$ from the argument of Kollár [5, 2.3], by using Lemma 2.

When $n \geq 2$, if $\Delta \neq 0$, then $\text{Bs}|2(a+n)f(n-1, a)L| \cap \Delta = \emptyset$ from Lemma 1. If $\Delta = 0$, then $|2(a+n)L| \neq \emptyset$ from Kollár [5, 2.4], thus $|2(a+n)f(n-1, a)L| \neq \emptyset$. Therefore $\text{Bs}|2^n(n+1)!2(a+n)f(n-1, a)L| = \emptyset$ from the argument of Kollár [5, 2.3], by using Lemma 2. Q. E. D.

§ 5. Appendix

In this section we show some results concerning the log canonical divisor $K_X + \Delta$.

THEOREM 5. Assume that L is a nef divisor on X and that $aL - (K_X + \Delta)$ is nef and log big on (X, Δ) for some $a \geq 0$. Let Γ be a member of **Strata**(Δ) and $d := \dim \Gamma$. Then $\text{Bs}|mL|$ does not include Γ for every $m \geq 2(d+a)$.

Remark. For the proof, we use the following two propositions.

PROPOSITION 1. Let R be a nef and log big divisor on (X, Δ) and Γ a member of **Strata**(Δ). Let $\pi: \tilde{X} \rightarrow X$ be the blow up with center Γ and E the exceptional divisor. Then

$$\begin{aligned} 0 &\longrightarrow H^0(\tilde{X}, \mathcal{O}_{\tilde{X}}(\pi^*(K_X + \Delta + R) - E)) \longrightarrow H^0(\tilde{X}, \mathcal{O}_{\tilde{X}}(\pi^*(K_X + \Delta + R))) \\ &\longrightarrow H^0(E, \mathcal{O}_E(\pi^*(K_X + \Delta + R))) \longrightarrow 0 \end{aligned}$$

is exact and $\mathcal{O}_E(\pi^*(K_X + \Delta + R)) \cong \pi|_E^* \mathcal{O}_\Gamma(K_X + \Delta + R)$.

Proof. Let $e := \text{codim}(\Gamma, X)$. We may assume that $\Gamma \subset \Delta_1 \cap \Delta_2 \cap \dots \cap \Delta_e$. Let Δ'_i be the strict transform of Δ_i . Thus $\pi^*(K_X + \Delta + R) - E = K_{\tilde{X}} - (e-1)E + \pi^*(\Delta_1 + \Delta_2 + \dots + \Delta_e) + \sum_{i>e} \Delta'_i + \pi^*R - E = K_{\tilde{X}} - (e-1)E + \sum_i \Delta'_i + eE + \pi^*R - E = K_{\tilde{X}} + \sum_i \Delta'_i + \pi^*R$ and π^*R is nef and log big on $(\tilde{X}, \sum_i \Delta'_i)$ (where $\sum_i \Delta'_i$ is with only simple normal crossings). Therefore, by Theorem 4, $H^1(\tilde{X}, \mathcal{O}_{\tilde{X}}(\pi^*(K_X + \Delta + R) - E)) = 0$. Q. E. D.

Remark. Let Γ be a member of **Strata**(Δ). We may assume that

$$(*) \quad \Gamma \subset \Delta_1 \cap \Delta_2 \cap \dots \cap \Delta_e \text{ (where } e = \text{codim}(\Gamma, X)\text{)}.$$

Then $(K_X + \Delta)|_\Gamma = (K_X + \Delta_1 + \Delta_2 + \dots + \Delta_e + \sum_{i>e} \Delta_i)|_\Gamma = K_\Gamma + \sum_{i>e} \Delta_i|_\Gamma$.

PROPOSITION 2. Let L be a nef divisor on X such that $aL - (K_X + \Delta)$ is nef and log big on (X, Δ) for some $a \geq 0$. Let Γ be a member of **Strata**(Δ) and $d := \dim \Gamma$. Then $H^0(\Gamma, \mathcal{O}_\Gamma(mL)) \neq 0$ for every $m \geq 2(d+a)$.

Proof. We may assume (*) in the remark above. If $m \geq a$, then

$$(mL)|_\Gamma = K_\Gamma + \sum_{i>e} \Delta_i|_\Gamma + (mL - (K_X + \Delta))|_\Gamma,$$

where $(mL - (K_X + \Delta))|_\Gamma$ is nef and log big on $(\Gamma, \sum_{i>e} \Delta_i|_\Gamma)$. Thus $h^0(\Gamma, \mathcal{O}_\Gamma(mL))$ is a polynomial in m for $m \geq a$, of degree at most d , by Theorem 4. Note that $h^0(\Gamma, \mathcal{O}_\Gamma(mL))$ is a nonzero polynomial by Theorem 3. Thus the same argument as in Kollár [5, 2.4] implies the assertion. Q. E. D.

Proof of Theorem 5. Let $R := mL - (K_X + \Delta)$. Then the assertion follows from Proposition 1 and Proposition 2. Q. E. D.

THEOREM 6 (Kollár-Matsuki [6, 4.12.1.2], cf. Iitaka [2, Example 11.6]). Let $f: Y \rightarrow X$ be a birational morphism between non-singular projective varieties.

Suppose that $K_Y = f^*(K_X + \mathcal{A}) + \sum_{i=1}^t e_i E_i$ and that the union of $\text{Supp } \sum_{i=1}^t E_i$ and $\text{Exc}(f)$ is a divisor with only simple normal crossings. Then $f(E_i) \in \mathbf{Strata}(\mathcal{A})$ for all i such that $e_i = -1$.

Remark. We give Theorem 6 an alternative proof, using Iitaka's Logarithmic Ramification formula [2].

Proof. We assume that $e_j = -1$ and that $f(E_j)$ is not a member of $\mathbf{Strata}(\mathcal{A})$, and we shall derive a contradiction.

Note that $f(E_j) \subset \text{Supp}(\mathcal{A})$. Let Γ be the minimal member of $\mathbf{Strata}(\mathcal{A})$ which includes $f(E_j)$. We may assume that Γ is an irreducible component of $\mathcal{A}_1 \cap \mathcal{A}_2 \cap \cdots \cap \mathcal{A}_e$ (where $e = \text{codim}(\Gamma, X)$). Note that $f(E_j) \not\subseteq \Gamma$.

Take a general smooth point p of $f(E_j)$. From now in the proof of this theorem, we consider the problem analytically. There exists a neighborhood U of p in X and a smooth prime divisor \mathcal{A}_0 on U , such that $\mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_e$ are crossing normally and that $f(E_j) \cap U \subset \mathcal{A}_0$. We may assume that $\mathcal{A}_l \cap U = \emptyset$ for any $l > e$.

If we write

$$K_{f^{-1}(U)} = (f|_{f^{-1}(U)})^*(K_U + \mathcal{A}|_U + \mathcal{A}_0) + \left(\sum_{i=1}^t d_i E_i|_{f^{-1}(U)} - E_0 \right)$$

(where E_0 is the strict transform of \mathcal{A}_0 by f), then $d_j \geq -1$ from Iitaka's Logarithmic Ramification Formula ([2, Theorem 11.5]). Therefore $e_j > -1$ because $(f|_{f^{-1}(U)})^* \mathcal{A}_0 \geq E_j|_{f^{-1}(U)}$. This is a contradiction. Q. E. D.

Remark. At first the author thought that these two theorems in this section are useful to get an estimate concerning Main Theorem.

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