

# **Sustainable Constant Consumption in a Semi-open Economy with Exhaustible Resources<sup>†</sup>**

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**Abstract.** To sustain constant consumption, Hartwick's rule prescribes reinvesting all resource rents in reproducible capital. However, Hartwick's rule is not necessarily the result of optimization. In this paper, we address this insufficiency by deriving a constant consumption path endogenously in a semi-open economy with an exhaustible resource, which has full access to world goods and capital markets, while the resource flows are not internationally tradable. Our findings show that, due to the essentiality of both capital and resource to the production process, the economy transforms its domestic assets into foreign ones, ending up consuming a constant interest flow from the latter.

**Key words:** exhaustible resources; Hartwick's rule; international finance; steady state; sustainable constant consumption

**JEL classification:** D90; Q32; F32

## 1. Introduction

The literature concerning intergenerational equity in an economy with exhaustible resources dates from the seminal paper of Solow (1974), and can also be found in Hartwick (1977) and Dixit et al. (1980). Solow (1974) suggests that the Rawlsian maximin principle should be applied to the problem of optimal capital accumulation, and defines intergenerational equity as constant per capita consumption over time. Hartwick (1977, p. 973) shows that such a Rawlsian path can be achieved by “investing all net returns from exhaustible resources in reproducible capital”, implying that investment in reproducible capital equals the rents from exhaustible resources under competitive pricing at each date yields. This investment rule is known as Hartwick’s rule. Dixit et al. (1980) extends Hartwick’s rule by suggesting that keeping the present discounted value of total net investment constant over time is necessary and sufficient for constant utility.

However, it has to be noticed that Hartwick’s rule does not have a sound microeconomic foundation. It is not necessarily the result of optimization and, to a certain extent, it is an ad hoc arrangement. In this paper, we shall explicitly address this insufficiency by considering a model of a small semi-open economy, which has full access to world goods and capital markets, while resource flows are not internationally tradable. We believe that such an arrangement is capable of emphasizing the limitedness of resource endowments within individual nations. Our purpose is to study the implications of an endogenously determined constant consumption path and the related paths of capital, output, foreign assets, and resource. Moreover, we show that based on

the optimum solution of the model, we can derive endogenously the generalized Hartwick's rule for the semi-open economy.

We start out in the following section by setting up the intertemporal optimizing model for the economy. Section 3 examines the relation between consumption and current account. In Section 4, we use a Cobb-Douglas production function to characterize the equilibrium paths and the initial movements of capital, resource and foreign assets. We observe that along the path converging to the steady state, the economy transforms its domestic assets into foreign ones, ending up consuming a constant interest flow from the latter. We provide a detailed analysis for the underlying mechanism. Our concluding remarks are in Section 5.

## 2. Model Specification

We consider a small country that produces one type of homogeneous good that is allocated between consumption and investment. Its production requires capital, labour and an exhaustible resource from a private sector that consists of a representative household-producer. There are perfect international goods and capital markets; however, the exhaustible resource cannot be traded among nations.

### 2.1 Notations

At time  $t$ , let  $Q_t$  = the output,  $K_t$  = the stock of capital,  $C_t$  = the consumption,  $I_t$  = the net investment,  $B_t$  = the stock of foreign assets,  $S_t$  = the stock of the exhaustible resource,  $R_t$  = the amount of exhaustible resource extracted (at zero

extraction cost) as production input,  $\delta =$  the rate of instantaneous time preference (a constant), and finally,  $r =$  the world rate of interest (a constant).

## 2.2 *The Optimization Problem*

The representative household-producer forms an immortal extended family, which chooses the optimal paths of production, consumption and investment needed to maximize the discounted sum of instantaneous utilities over an infinite time horizon. For the sake of simplicity, it is assumed that the labour force is fixed and it taken to be one. The optimization problem of the representative household-producer is:

$$(1) \quad \text{Max} \int_0^{\infty} U(C_t) e^{-\delta t} dt,$$

subject to

$$(2) \quad \dot{B}_t = rB_t + F(K_t, R_t) - C_t - I_t,$$

$$(3) \quad \dot{K}_t = I_t,$$

$$(4) \quad \dot{S}_t = -R_t,$$

$$(5) \quad \int_0^{\infty} (C_t + I_t) e^{-rt} dt \leq \int_0^{\infty} F(K_t, R_t) e^{-rt} dt + B_0,$$

$$(6) \quad \int_0^{\infty} R_t dt = S_0,$$

where  $B_0$ ,  $K_0$  and  $S_0$  are exogenously given. Equation (2) is the instantaneous budget constraint of the representative household-producer, which defines the evolution of the stock of foreign assets. Equation (3) states the relation between capital accumulation and net investment. Equation (4) is the transition equation for the exhaustible resource

stock. Equation (5) is the intertemporal budget constraint that rules out Ponzi games. Equation (6) shows that the economy is endowed with a finite stock of exhaustible resource, and the sum of extraction at each time equals the initial endowment.

The utility function  $U(C_t)$  and the production function  $F(K_t, R_t)$  of the representative household-producer are subject to the following conditions:

$$(7) \quad U' > 0, \quad U'' < 0, \quad \lim_{C \rightarrow 0} U'(C) = \infty, \quad \lim_{C \rightarrow \infty} U'(C) = 0,$$

and,

$$(7') \quad F_K > 0, \quad F_R > 0, \quad F_{KK} < 0, \quad F_{RR} < 0, \quad F_{KR} > 0,$$

$$\lim_{K \rightarrow 0} F_K = \infty, \quad \lim_{R \rightarrow \infty} F_R = \infty, \quad \lim_{K \rightarrow \infty} F_K = 0, \quad \lim_{R \rightarrow 0} F_R = 0,$$

where  $F_K$  and  $F_R$  denote the partial derivatives of  $F$  with respect to  $K$  and  $R$ , respectively. Similarly,  $F_{KK}$ ,  $F_{RR}$  and  $F_{KR}$  are the second-order partial derivatives. The positive sign of the cross-partial derivative implies that capital and resource are complements. Finally, we assume that the constant rate of instantaneous time preference equals the world rate of interest:  $\delta = r$ .

### 2.3 *The Optimum Solution*

To solve the optimization problem, we define the current-value Hamiltonian as

$$(8) \quad H = U(C_t) + \lambda_t [F(K_t, R_t) + rB_t - C_t - I_t] + \mu_t I_t - \nu_t R_t,$$

where  $\lambda_t$ ,  $\mu_t$ , and  $\nu_t$  are the shadow prices of saving, net investment, and resource input, respectively.

The first-order conditions for an interior maximum can be expressed as

$$(9) \quad \frac{\partial H}{\partial C_t} = U'(C_t) - \lambda_t = 0,$$

$$(10) \quad \frac{\partial H}{\partial I_t} = -\lambda_t + \mu_t = 0,$$

$$(11) \quad \frac{\partial H}{\partial R_t} = \lambda_t F_R - v_t = 0,$$

$$(12) \quad \dot{\lambda}_t = (\delta - r)\lambda_t,$$

$$(13) \quad \dot{\mu}_t = \delta\mu_t - \lambda_t F_K,$$

$$(14) \quad \dot{v}_t = \delta v_t - \frac{\partial H}{\partial S} = \delta v_t, \quad \text{i.e., } v_t = v_0 e^{\delta t}.$$

The optimum paths of consumption  $C_t$ , net investment  $I_t$ , resource input  $R_t$ , capital stock  $K_t$ , foreign assets  $B_t$ , resource stock  $S_t$ , and shadow prices  $\lambda_t$ ,  $\mu_t$ , and  $v_t$  are determined as solutions to the system of differential equations of (2)-(4) and (9)-(14).

From (14), it can be verified that the shadow prices of the resource exhibit the properties as described by Hotelling's rule, i.e., the present value of the shadow prices of the resource has to be the same in each period as long as a positive amount is extracted. Also, owing to the assumption of  $\delta = r$ , the constancy of  $\lambda_t$ ,  $C_t$ , and  $\mu_t$  can be readily derived using (9), (10), and (12). Also, from (13), we have

$$(15) \quad F_K(K_t, R_t) = r.$$

Equations (11) and (14) imply

$$(16) \quad \dot{F}_R / F_R = r,$$

here  $F_R = v_t / \bar{\lambda}$ . Equations (15) and (16) together shows that the resource use is dynamically efficient. They are the core results of this section and will be used repeatedly in the following analysis.

In the following analysis, we show that the net investment  $I_t$  trends negative, despite the fact that its shadow price  $\mu_t$  remains non-negative over time. This is so

since the net investment has both the positive effect of increasing productivity and the negative effect of reducing saving. When the condition stated in equation (10) is satisfied, these two effects reach a state of equilibrium, and, accordingly, the net investment level that leads to equilibrium is determined. Moreover, in the following analysis, it is also shown that the resource input decreases over time. Since capital and resource are complements, in order to keep the marginal productivity of capital constant (equation (15)), it is imperative that the capital should also decrease over time.<sup>1</sup>

Since along the optimum path, consumption is maximized and satisfies the intertemporal budget constraint, and since the level of consumption is constant throughout the planning period, (5) can be rearranged as

$$(17) \quad \bar{C} = r \left\{ \int_0^{\infty} \{F(K_t, R_t) - I_t\} e^{-rt} dt + B_0 \right\}.$$

Thus, the constant optimal consumption can be perceived as the annuity equivalent of the accumulated discounted value of overall production plus the initial stock of foreign assets. Furthermore, by solving (2), we obtain the path of foreign assets:

$$(18) \quad B_t = e^{rt} \left\{ B_0 + \int_0^t [F(K_s, R_s) - \bar{C} - I_s] e^{-rs} ds \right\},$$

where

$$(19) \quad \lim_{t \rightarrow \infty} e^{-rt} B_t = B_0 + \int_0^{\infty} [F(K_t, R_t) - \bar{C} - I_t] e^{-rt} dt$$

is zero by the intertemporal budget constraint (5). This implies that the optimum solution satisfies the transversality condition.

### 3. Constant Consumption, Current Account and the Generalized Hartwick's



## Rule

In this section, we first show how to derive the generalized Hartwick's rule from the optimum solution. Differentiating (2) with respect to time, we have

$$(20) \quad \ddot{B} = r\dot{B} + F_K \dot{K} + F_R \dot{R} - \dot{C} - \dot{I}.$$

Using (15) and (16), and noting that

$$(21) \quad d(F_R R) / dt = \dot{F}_R R + F_R \dot{R},$$

we can restate (20) as

$$(22) \quad \ddot{B}_t + \dot{C}_t + \dot{I}_t - \frac{d}{dt}(F_R R_t) = r(\dot{B}_t + I_t - F_R R_t).$$

Multiplying both sides of (22) by  $e^{-rt}$ , rearranging terms, we obtain

$$(23) \quad \dot{C}_t e^{-rt} + \frac{d}{dt}[(\dot{B}_t + I_t - F_R R_t)e^{-rt}] = 0.$$

From (23), we have  $\dot{C}_t = 0$  when

$$(24) \quad \dot{B}_t + I_t = F_R R_t, \text{ all } t.$$

This is Hartwick's rule in the semi-open economy, which states that the total net investment (the change rate of foreign assets (current account) and the domestic investment) equals the rents from the exhaustible resource at each date.

Moreover,  $\dot{C}_t = 0$  can also be derived when

$$(25) \quad (\dot{B} + I - F_R R)e^{-rt} = \text{constant, all } t.$$

This is the generalized Hartwick's rule in the semi-open economy, which states that the present discounted value of total net investment is constant at each date. However, it has to be noticed that the constant consumption  $\dot{C}_t = 0$  is not obtained endogenously here, but is based on the ad hoc assumption that the economy follows either Hartwick's rule

or the generalized Hartwick's rule. This is in contrast with the semi-open economy model, in which  $\dot{C}_t = 0$  occurs endogenously as a result of optimization, and (23) can be obtained conversely from the constancy of consumption flow.

Next, we study the implications of the constant consumption flow by characterizing the path of current account. According to Bellman's optimality principle, the optimum solution paths are still optimum if they are re-valued at any time  $t > 0$ .<sup>2</sup> Therefore, (17) can be restated as

$$(26) \quad \bar{C} = r \left\{ \int_t^{\infty} [F(K_s, R_s) - I_s] e^{-r(s-t)} ds + B_t \right\} \\ = r \left\{ \int_{t+\Delta t}^{\infty} [F(K_s, R_s) - I_s] e^{-r(s-t-\Delta t)} ds + B_{t+\Delta t} \right\}.$$

Equation (26) implies that the access to international capital market provides an endogenously determined mechanism of "smoothing out" the consumption flows over time by adjusting gap between the output and consumption intertemporally. From (26), we obtain

$$(27) \quad B_{t+\Delta t} - B_t = \int_t^{\infty} [F(K_s, R_s) - F(K_{s+\Delta t}, R_{s+\Delta t}) - I_s + I_{s+\Delta t}] e^{-r(s-t)} ds.$$

Dividing (27) by  $\Delta t$  and taking the limits, we have

$$(28) \quad \dot{B}_t = - \int_t^{\infty} (F_K \dot{K}_t + F_R \dot{R}_t - \dot{I}_t) e^{-r(s-t)} ds.$$

Equation (28) shows how the current account evolves over time. It is clear that the path of current account depends on the relation between the change rate of net investment,  $\dot{I}_t$ , and the change rate of output,  $F_K \dot{K}_t + F_R \dot{R}_t$ . In other words, the increase of the foreign assets in period  $t$  corresponds to the discounted value of future current account

deficit. Hence the constant consumption path is achieved since open economies are incorporated with an endogenous mechanism that automatically offsets the gap between the change rates of output and investment demand at each date through the adjustment of the current account.

#### 4. Capital and Resource Input in a Semi-open Economy: The Case of a Cobb-Douglas Production Function

In this section, we characterize capital and resource input along the optimal path by considering a Cobb-Douglas in which the flow of resource,  $R$ , is combined with a stock of man-made capital,  $K$ , to produce a consumption good,  $Q$ , that can also be used as an investment good. The production function is given as follows:

$$(29) \quad Q = F(K, R) = K^\alpha R^\beta, \text{ with } \alpha + \beta < 1.$$

Note that under the Cobb-Douglas technology, both inputs are essential to production, that is, it is *impossible* to maintain a positive output when either of them is exhausted.

##### 4.1 Equilibrium Paths of Capital and Resource Input

From (15), we have  $F_K = \alpha K^{\alpha-1} R^\beta = r$ . Differentiating with respect to time, we obtain

$$(30) \quad (\alpha - 1) \frac{\dot{K}}{K} + \beta \frac{\dot{R}}{R} = 0.$$

Moreover, since  $F_R = \beta K^\alpha R^{\beta-1}$ , differentiating it with respect to time, together with (16), we have

$$(31) \quad \frac{\dot{F}_R}{F_R} = \alpha \frac{\dot{K}}{K} + (\beta - 1) \frac{\dot{R}}{R} = r.$$

Equations (30) and (31) can be solved as simultaneous equations of  $\dot{K}/K$  and  $\dot{R}/R$ , with the solutions as

$$(32) \quad \frac{\dot{K}}{K} = \frac{-\beta r}{1 - \alpha - \beta} = g_K < 0,$$

$$(33) \quad \frac{\dot{R}}{R} = \frac{-(1 - \alpha)r}{1 - \alpha - \beta} = g_R < 0.$$

Therefore, along the optimal path, both the capital and resource input deplete gradually at a fixed rate along the path converging to the steady state, and become zero in the steady state. Also, since  $(1 - \alpha) > \beta$ , we have  $|g_K| < |g_R|$ , which implies that the resource input decreases at a rate faster than that of the capital, while the difference equals the interest rate:

$$(34) \quad \frac{\dot{K}}{K} - \frac{\dot{R}}{R} = r.$$

Moreover, the movement of the output over time is characterized as

$$(35) \quad \frac{\dot{Q}}{Q} = \alpha \frac{\dot{K}}{K} + \beta \frac{\dot{R}}{R} = \frac{-\beta r}{1 - \alpha - \beta} = g_Q,$$

which implies  $g_Q = g_K$ .

We summarize the movements of the variables along the optimal path as follows

$$(36) \quad \frac{\dot{C}}{C} = 0,$$

$$(37) \quad \frac{\dot{Q}}{Q} = g_Q < 0,$$

$$(38) \quad \frac{\dot{K}}{K} = g_K < 0, \quad g_Q = g_K,$$

$$(39) \quad \frac{\dot{R}}{R} = g_R < 0, \quad |g_R| > |g_Q|.$$

#### 4.2 *The Initial Movements of the Capital and Resource Input*

Next we study the initial movements of the output, resource and capital input after the economy is open to the international goods and capital markets. From the growth rate of the resource input (which is constant from (33)), it is easy to see that the initial resource input  $R_0$  is determined by

$$(40) \quad \int_0^{\infty} R_0 e^{g_R t} dt = S_0.$$

The determination of  $R_0$  and the subsequent movements of  $R_t$  are shown in Figure 1.

[INSERT FIGURE 1]

The optimal initial capital is determined by (15') as  $K_{0+}$ :

$$(15') \quad F_K(K_{0+}, R_0) = r.$$

Therefore, depending on the value of  $K_0$ , the capital input may jump upward or downward at time  $0+$ : capital jumps upward initially if  $K_{0+} > K_0$ , and vice versa. As a result, the initial foreign asset  $B_0$  also jumps instantaneously to  $B_{0+}$ , to a direction opposite to that of the initial capital movement, with  $B_{0+} = B_0 - (K_{0+} - K_0)$ .

Since  $\dot{I}_t = \ddot{K}_t$ , we have  $\dot{I}_t = \ddot{K}_t = \left(\frac{\beta r}{1-\alpha-\beta}\right)^2 K > 0$  from (32). Apply this result to (28), some manipulation leads to  $\dot{B}_t > 0$ , which implies that after the initial jump, the amount of foreign asset increases gradually over time.<sup>3</sup>

Figure 2 and 3 depict the case in which the initial capital stock jumps upward while the foreign assets downward. The movements of the capital and foreign assets after the initial jump are determined by (32) and the facts that  $\ddot{K} > 0$ ,  $\dot{B}_t > 0$  and  $\ddot{B} < 0$ .

[INSERT FIGURE 2 and 3]

Finally, the initial output is determined by

$$(41) \quad Q_{0+} = F(K_{0+}, R_0).$$

The initial output jumps in the same direction as that of the capital stock. Following (35), it then decreases monotonically over time. The movement of the output is shown in Figure 4.

[INSERT FIGURE 4]

Observe that (26) can be restated as

$$(26') \quad \bar{C} - rB_t = r \int_t^{\infty} [F(K_s, R_s) - I_s] e^{-r(s-t)} dt,$$

and the terms in the integrand can be rearranged as  $K^\alpha R^\beta + \frac{\beta r}{1-\alpha-\beta} K$ . Thus, as long as

$K_t$  and  $R_t$  remain positive,  $\bar{C} > rB_t$ . The relation between the constant consumption

and the foreign assets along the path converging to the steady state is shown in Figure 5.

[INSERT FIGURE 5]

#### 4.3 *Steady State*

Since  $Q_t$  and  $I_t$  monotonically converges to zero, and also because of the constancy of  $\bar{C}$ ,  $\dot{B}$  is zero in the steady state, from (2), we have

$$(42) \quad \bar{C} = r\bar{B}.$$

In the steady state, due to the exhaustion of both the capital and the exhaustible resource, the economy ceases production and consumes the constant interest flow from its foreign assets. The exhaustion of the exhaustible resource is within expectation, after all, it is “meant” to be depleted. The mechanism that is responsible for the exhaustion of capital can be explained as follows. Having access to international capital markets provides a means to acquire income in the form of interest flow from foreign assets. Indeed, holding assets domestically (reproducible capitals as well as exhaustible resources) also brings forth similar income flows in the form of either capital gains or resources rents. Nevertheless, in the case that both the resource and the capital are essential to the production process, when either of them is exhausted, production stops and such an income flow ceases. Before the eventual exhaustion, however, the entire capital stock will be transformed into other forms of assets capable of bringing forth an income flow (foreign assets, in this case). This explains why the entire capital stock is depleted in our case. In other words, when holding assets domestically no longer leads to an income flow (since *no* production is undertaken at home), as a direct result of optimization,

domestic assets are transformed into foreign ones, and since foreigners are paying the interests, a constant flow of income can still be guaranteed.

Note that in the classical case examined in Hartwick (1977) and Solow (1986), i.e., a closed economy in which the resource and capital are essential to production, the constant (and positive) consumption level cannot be extended into a steady state, in which stock levels of both the capital and the resource should remain constant (which would mean zero resource input, zero output, and accordingly, zero consumption level).<sup>4</sup> We show that connecting the economy into the world capital market is capable of deriving a constant consumption flow that can be sustained into the steady state. Our analysis predicts that an economy with access to world capital markets will end up holding no domestic capital stock, but a large stock of foreign assets, and consuming a constant flow of consumption that equals the annuity income of its foreign assets over time.

## **5. Concluding Remarks**

This paper shows that in a semi-open economy, when both capital and resource are essential to production, a converting process between the domestic assets and foreign ones occurs as a result of optimization. Note that when the exhaustible resource in consideration is applied to a dwindling work force, or the limitedness of land area for productive purposes, our analysis explains, at least in part, why certain countries (Japan, for example) would experience a continuous surplus in their current accounts and a steady increases of their foreign assets.

The most natural way to extend the current analysis would be to incorporate into



the model the amenity value of the resource, which is an important feature of many non-tradable exhaustible resources. It seems clear, however, that the conclusions of this paper regarding the properties of the steady state will change significantly in such a new model.

## Notes

1. Following Takayama (1993) (pp. 516-517), the argument can be stated more formally as follows. We rewrite equation (8) as

$$H = U(C_t) + \lambda_t [F(K_t, R_t) + rB_t - C_t] - \nu_t R_t + (\mu_t - \lambda_t) I_t.$$

Note that the maximization of  $H$  with respect to  $I_t$  leads to

$$I_t \rightarrow \infty \text{ if } \mu_t - \lambda_t > 0; \quad I_t \rightarrow -\infty \text{ if } \mu_t - \lambda_t < 0.$$

This condition implies that the representative household-producer would adjust to the desired stock of capital instantaneously. When the desired capital stock is attained, we have

$$\frac{\partial H}{\partial I_t} = \mu_t - \lambda_t = 0.$$

Since the resource input decreases over time, in order to maintain the marginal productivity of capital constant, the desired capital stock level must also decrease over time.

2. This approach can be found in Matsuyama (1990) and Okumura (1998).
3. From (32) and (35), the terms in the integrand of (28) can be rearranged as

$$(F_K \dot{K} + F_R \dot{R} - \dot{I}) = \dot{Q} - \dot{K} = - \left[ \left( \frac{\beta r}{1 - \alpha - \beta} \right) Q_t + \left( \frac{\beta r}{1 - \alpha - \beta} \right)^2 K \right],$$

since the terms in the square parentheses are strictly positive as long as the stocks of both the capital and

the resource are not exhausted, we have  $\dot{B}_t > 0$ . It is also easy to verify that  $\ddot{B} < 0$ .

4. Hartwick's rule has been argued by many as not indicating sustainability (Asheim et al. (2003), for example, provides a semantic clarification of the rule). Obviously, the picture that Hartwick and Solow have in mind is one in which the resource input level asymptotically approaches zero, and the capital makes up for the deduction of the resource input, while the production somehow continues. Apparently, a steady state that has a positive consumption level does not exist under such a case.

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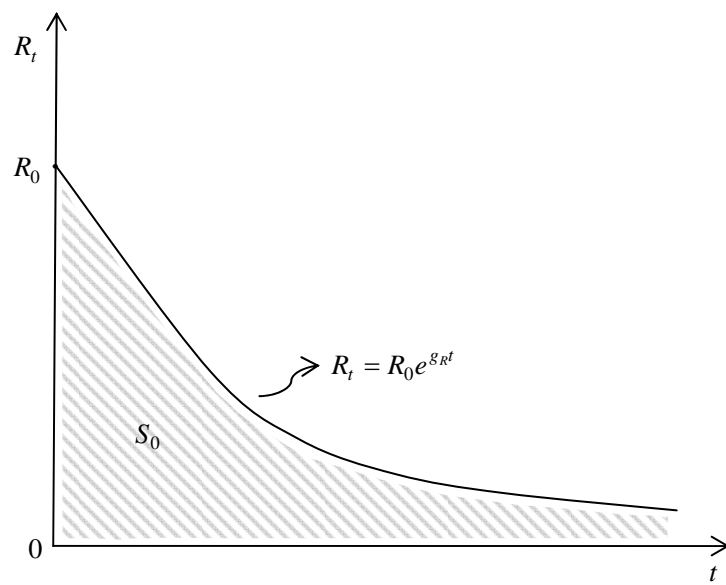


FIGURE 1. Path of the Resource Input

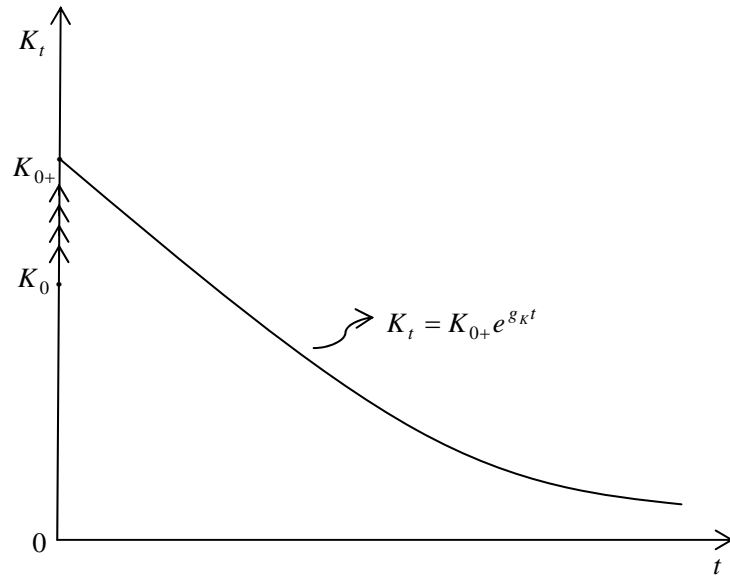


FIGURE 2. Path of the Capital Stock

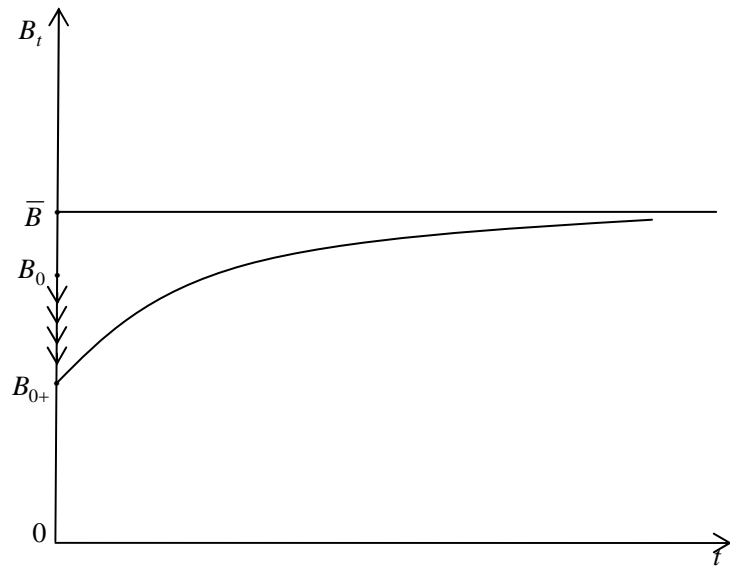


FIGURE 3. Path of the Foreign Assets

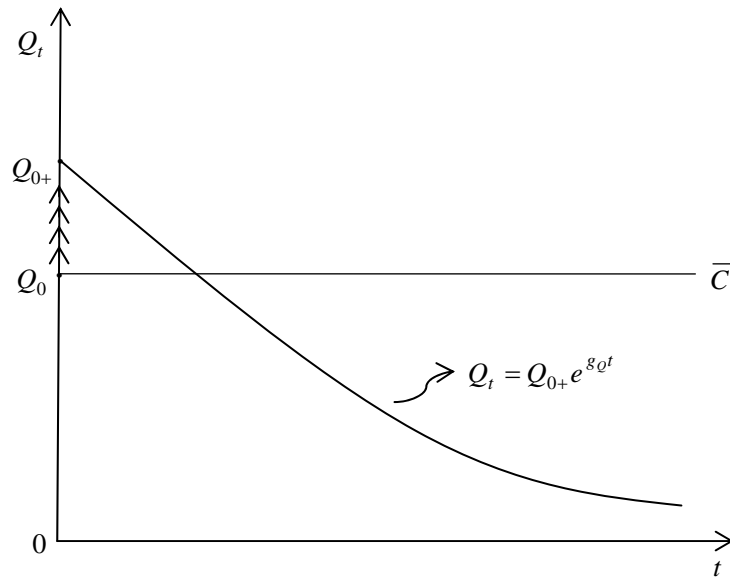


FIGURE 4. Path of Output

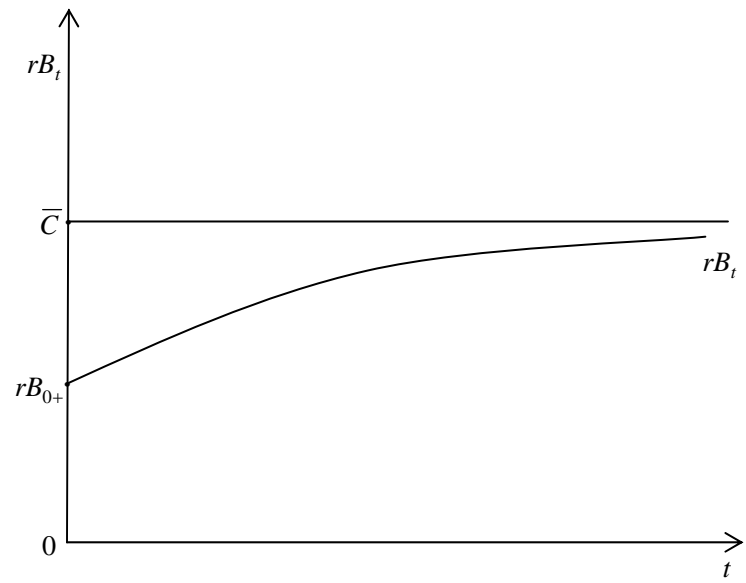


FIGURE 5. The Constant Consumption and the Interest Flow from Foreign Assets